PAST PRESENT AND FUTURE CHALLENGES IN THE DETERMINATION OF THE STRUCTURE OF THE PROTON

LECTURE II

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Grad Days 2019

Exercise I: Z contribution

- Show that, in the Parton model, considering also the contribution of a virtual Z boson and its interference with the photon one obtains:

$$F_2^{\gamma,Z}(x) = x\sum_{i=1}^{n_f} c_i[q_i(x) + \bar{q}_i(x)]$$

$$F_3^{\gamma,Z}(x) = \sum_{i=1}^{n_f} d_i[q_i(x) - \bar{q}_i(x)]$$

Where

$$c_i = e_i^2 - 2e_i V_{eZ} V_{iZ} P_Z + (V_{eZ}^2 + A_{eZ}^2)(V_{iZ}^2 + A_{iZ}^2)P_Z^2$$

$$d_i = -2e_i A_{eZ} A_{iZ} P_Z + 4V_{eZ} A_{eZ} V_{iZ} A_{iZ} P_Z^2$$

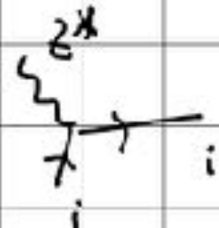
$$P_Z = \frac{Q^2}{(Q^2 + M_Z^2)(4s_w^2 c_w^2)} \longrightarrow \begin{aligned} c_w &= \cos\theta_w \\ s_w &= \sin\theta_w \end{aligned}$$

Solution I

(I) In the lectures we considered

$$\frac{1}{2}\sum_{pol}\Big|$$  $$\Big|^2 = \frac{e^2 e_q^2}{2}\,\mathrm{Tr}\left[\hat{p}\,\gamma^\nu\,\hat{p}'\,\gamma^\mu\right]$$

Add to hadronic tensor

$$\frac{1}{2}\sum_{pol}\Big|$$  $$\Big|^2 = \frac{e^2 k_Z^2}{2}\,\mathrm{Tr}\left[\hat{p}\,\gamma^\nu\,(v_{iZ} + \gamma^5 a_{iZ})\,\hat{p}'\gamma^\mu(v_{iZ} + \gamma_5 a_{iZ})\right]$$

$$= \frac{e^2}{8 s_W^2 c_W^2}\left[(v_{iZ}^2 + a_{iZ}^2)\,\mathrm{Tr}\left[\hat{p}\gamma^\nu\hat{p}'\gamma^\mu\right] + 2 v_{iZ} a_{iZ}\,\mathrm{Tr}\left[\hat{p}\gamma^\nu\hat{p}'\gamma^\mu\gamma^5\right]\right]$$

$$\mathrm{Tr}\left[\hat{p}\gamma^\nu\hat{p}'\gamma^\mu\right] \to 4\left[\hat{p}^\nu\hat{p}'^\mu + \hat{p}^\mu\hat{p}'^\nu - g^{\mu\nu}\,\hat{p}\cdot\hat{p}'\right]$$

$$\mathrm{Tr}\left[\hat{p}\gamma^\nu\hat{p}'\gamma^\mu\gamma^5\right] \to 4 i \epsilon^{\alpha\beta\nu\mu}\,\hat{p}_\alpha \hat{p}'_\beta$$

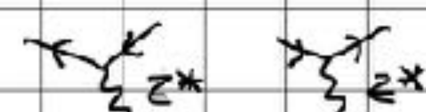
$\neq 0$ when contracted with anti-symm tensor.

Similar contribution in leptonic tensor

$v_{iZ} \to v_{eZ}$

$a_{iZ} \to a_{eZ}$

and propagator $\displaystyle \frac{\overbrace{Q^4}^{P_Z^2}}{(Q^2+M_Z^2)^2} \cdot \frac{1}{Q^4}$



have different sign in front of anti symm. part

Solution I

$\frac{1}{2}\sum_i$

$$= -\frac{1}{2} e^2 k_\gamma k_Z \, \mathrm{Tr}\left[\hat{p}\gamma^\nu (v_{iZ} + a_{iZ}\gamma^5)\hat{p}'\gamma^\mu\right]$$

$$= -\frac{e^2 e_J}{4 s_W c_W}\left[v_{iZ}\, \mathrm{Tr}\left[\hat{p}\gamma^\nu \hat{p}'\gamma^\mu\right] + a_{iZ}\, \mathrm{Tr}\left[\hat{p}\gamma^\nu \hat{p}'\gamma^\mu\gamma^5\right]\right]$$

Similar contribution in leptonic tensor

$v_{iZ} \to v_{eZ}$

$a_{iZ} \to a_{eZ}$

and propagator $\underbrace{\frac{Q^{2}}{(Q^2+M_Z^2)\,Q^2}}_{P_Z}$ $\qquad \frac{1}{Q^4}$

Exercise II: Paschos-Wolfenstein relation

- Show that, in the Parton model, considering a (anti)neutrino-initiated DIS process on a deuteron target – assuming SU(2) isospin symmetry $u_n(x)=d_p(x)$ and $d_n(x) = u_p(x)$ – the ratio R

$$R = \frac{\sigma_{\rm NC}(\nu) - \sigma_{\rm NC}(\bar{\nu})}{\sigma_{\rm CC}(\nu) - \sigma_{\rm CC}(\bar{\nu})}$$

NC (mediated by Z) and CC (mediated by $W^{+/-}$), assuming strange and anti-strange to be equal in the target, is independent of Parton Distribution Functions and can be used to determined the Weinberg angle $\theta_{\rm W}$

$$R = \frac{1}{2}\left(\frac{1}{2} - \sin\theta_w^2\right)$$

You may use (without deriving it) the result (and set c, cbar = 0)

$$\begin{aligned}
F_2^{W^-} &= 2x\left(u + \bar{d} + \bar{s} + c\right), \\
F_3^{W^-} &= 2x\left(u - \bar{d} - \bar{s} + c\right), \\
F_2^{W^+} &= 2x\left(d + \bar{u} + \bar{c} + s\right), \\
F_3^{W^+} &= 2x\left(d - \bar{u} - \bar{c} + s\right),
\end{aligned}$$

Exercise II: Paschos-Wolfenstein relation

July 1972

TESTS FOR NEUTRAL CURRENTS IN NEUTRINO REACTIONS

E. A. PASCHOS
National Accelerator Laboratory
P. O. Box 500, Batavia, Illinois 60510

and

L. WOLFENSTEIN*
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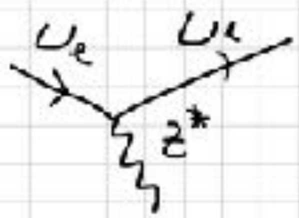
ABSTRACT

Neutral currents predicted by weak interaction models of the type discussed by Weinberg may be detected in neutrino reactions. Limits on the ratio R of $\sigma(\nu + N \rightarrow \nu + x)$ to $\sigma(\nu + N \rightarrow \mu^- + x)$ are obtained independent of any dynamical assumption. For the total cross-section for high energy neutrinos, we find $R \geq 0.18$, provided the Weinberg mixing angle satisfies $\sin^2 \theta_w \leq 0.33$. For the production of a single π^0 we find $R' \geq 0.50$ contrasted with the experimental result $R' \leq 0.14$ using only the assumption of (3, 3) resonance dominance. Applications are also given to anti-neutrino reactions.

Solution II

(II)



$$= \bar{u}'(k) \frac{-ig}{4c_W} \gamma^\mu (1-\gamma_5) u(k)$$

$$L_{\mu\nu}(\nu) = \frac{1}{8c_W^2}\left(Tr[\not k \gamma^\nu \not k' \gamma^\mu] - Tr[\not k \gamma^\nu \not k' \gamma^\mu \gamma^5]\right)$$

$$= \bar{v}(k) \frac{-ig}{4c_W} \gamma^\mu (1-\gamma_5) v(k')$$

$$L_{\mu\nu}(\bar\nu) = \frac{1}{8c_W^2}\left(Tr[\not k' \gamma^\nu \not k \gamma^\mu] - Tr[\not k' \gamma^\nu \not k \gamma^\mu \gamma^5]\right)$$

Because

$$d\sigma = \frac{1}{2s}\frac{g_{\mu\nu}^2 g_{\mu\nu}^2}{(Q^2+M_V^2)^2} L_{\mu\nu} W^{\mu\nu} (4\pi) \frac{d^3k'}{(2\pi)^3 2E'}$$

(only difference in $\nu, \bar\nu$ incoming lepton)

↳ M_V^4 for $Q^2 \ll M_V^2$

$$d\sigma(\nu) - d\sigma(\bar\nu) \propto [L_{\mu\nu}(\nu) - L_{\mu\nu}(\bar\nu)] \frac{g_{\mu\nu}^2 g_{\mu\nu}^2}{M_V^4} W^{\mu\nu} (4\pi) \frac{d^3k'}{(2\pi)^3 2E'}$$

↓ only γ_5 term i.e. $\frac{i}{c_W^2} k_\alpha k'_\beta \varepsilon^{\alpha\beta\mu\nu}$

Only non-zero contribution when contracted with asymm $W_{\mu\nu}$

Solution II

$$[L_{\mu\nu}(\nu) - L_{\mu\nu}(\bar{\nu})] W^{\mu\nu} = -\frac{1}{2(\tilde{p}.q)c_W^2} k_\alpha k'_\beta \, \varepsilon^{\alpha\beta\mu\nu} \varepsilon_{\rho\sigma\mu\nu} \tilde{p}^\rho q^\sigma F_3(x)$$

$$= \frac{ME}{c_W^2} (2-y) \, x F_3^Z(x)$$

$$\Rightarrow \frac{d^2\sigma}{dx\,dy} = \frac{G_F^2 c_W^2 M_n E_\nu}{2\pi^2} \, xy(2-y) \, F_3^{NC}(x)$$

$$\int dy\,dx \ldots = \frac{G_F^2 c_W^2}{3\pi^2} E_\nu M_n \int dx \, x F_3(x)$$

with $F_3(x) \propto \frac{1}{2}\left(\frac{1}{2} - \frac{4}{3}s_W^2\right)(u - \bar{u} + c - \bar{c}) + \frac{1}{2}\left(\frac{2}{3}s_W^2 - \frac{1}{2}\right)(d - \bar{d} + s - \bar{s})$

using $u^P = d^N$, $d^P = u^N$ $\Rightarrow$ $u^P + d^P = u^N + d^N$ $\Rightarrow$ $u = d$, $\bar{u} = \bar{d}$

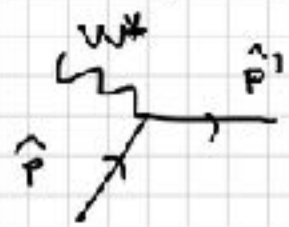
$s = \bar{s}$

$c = \bar{c} \cong 0$

$\rightarrow F_3^{NC}(x) \propto \left(\frac{1}{2} - s_W^2\right)[u(x) - \bar{u}(x)]$

Solution II

Charged current



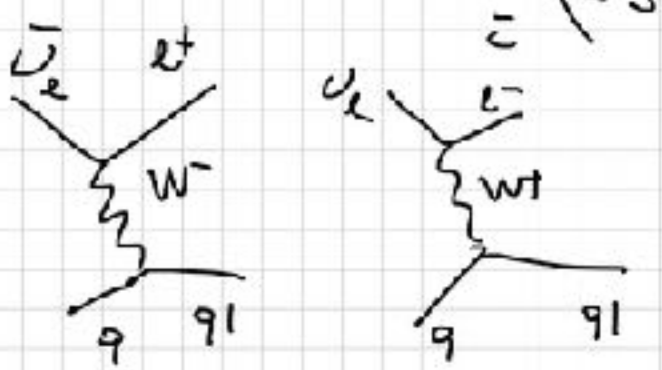
$$\bar{u}_k(p')\left[-\frac{ig}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)\;U_{kj}\right]u_j(p)$$

↓

CKM Matrix (consider only 2 flavor)

$$U_{kj} = \begin{array}{c} \\ u \\ c \end{array}\!\!\begin{array}{c} d \quad\quad\quad s \\ \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}\end{array} = \begin{pmatrix} 1+\theta_c^2 & \theta_c \\ -\theta_c & 1+\theta_c^2 \end{pmatrix}$$

↓ expand in θ_c and keep only $O(1)$



$$\Rightarrow F_3^{W^+} = 2x\,(d-\bar{u} + s-\bar{c})$$

$$F_3^{W^-} = 2x\,(u-\bar{d}-\bar{s}+c)$$

Analogously to NC

$$\sigma_{cc}(\nu) - \sigma_{cc}(\bar{\nu}) = \frac{2G_F^2}{3\pi^2} E_\nu M_N \int_0^1 x\, F_3^{cc}(x)\, dx$$

$$F_3^{cc}(x) = \frac{1}{2}(u+d+s+c-\bar{u}-\bar{d}-\bar{s}-\bar{c}) \longrightarrow (u-\bar{u}) \quad \text{same hypothesis}$$

$$= \frac{2G_F^2}{3\pi^2} E_\nu M_N \int dx\,(u(x)-\bar{u}(x))$$

Solution II

taking ratio

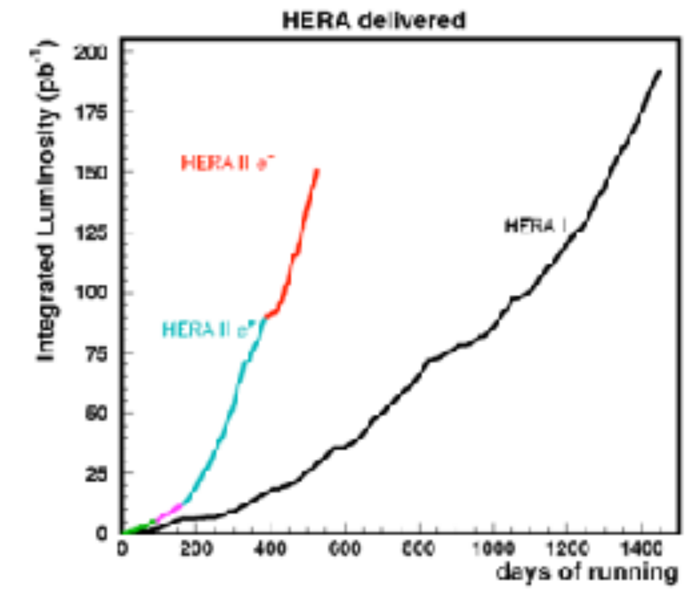
$$R = \frac{\frac{1}{2}(1 - 2 s^2 w) \int_0^1 dx\,[u(x) - \bar{u}(x)]\,dx}{2 \int_0^1 dx\,[u(x) - \bar{u}(x)]}$$

$$= \frac{1}{2}\left(\frac{1}{2} - \sin^2\theta_W\right)$$

Outline

- First lecture (Monday)
    - Motivation: the big picture
    - Parton Model and QCD
    - **Collinear Factorisation**
- **Second lecture (Tuesday)**
    - Experimental Data
    - Disentangling proton's components
    - Heavy quarks and photons
- Third lecture (Wednesday)
    - Fits and methodology
    - The NNPDF approach
- Fourth lecture (Thursday)
    - New frontiers

The HERA collider



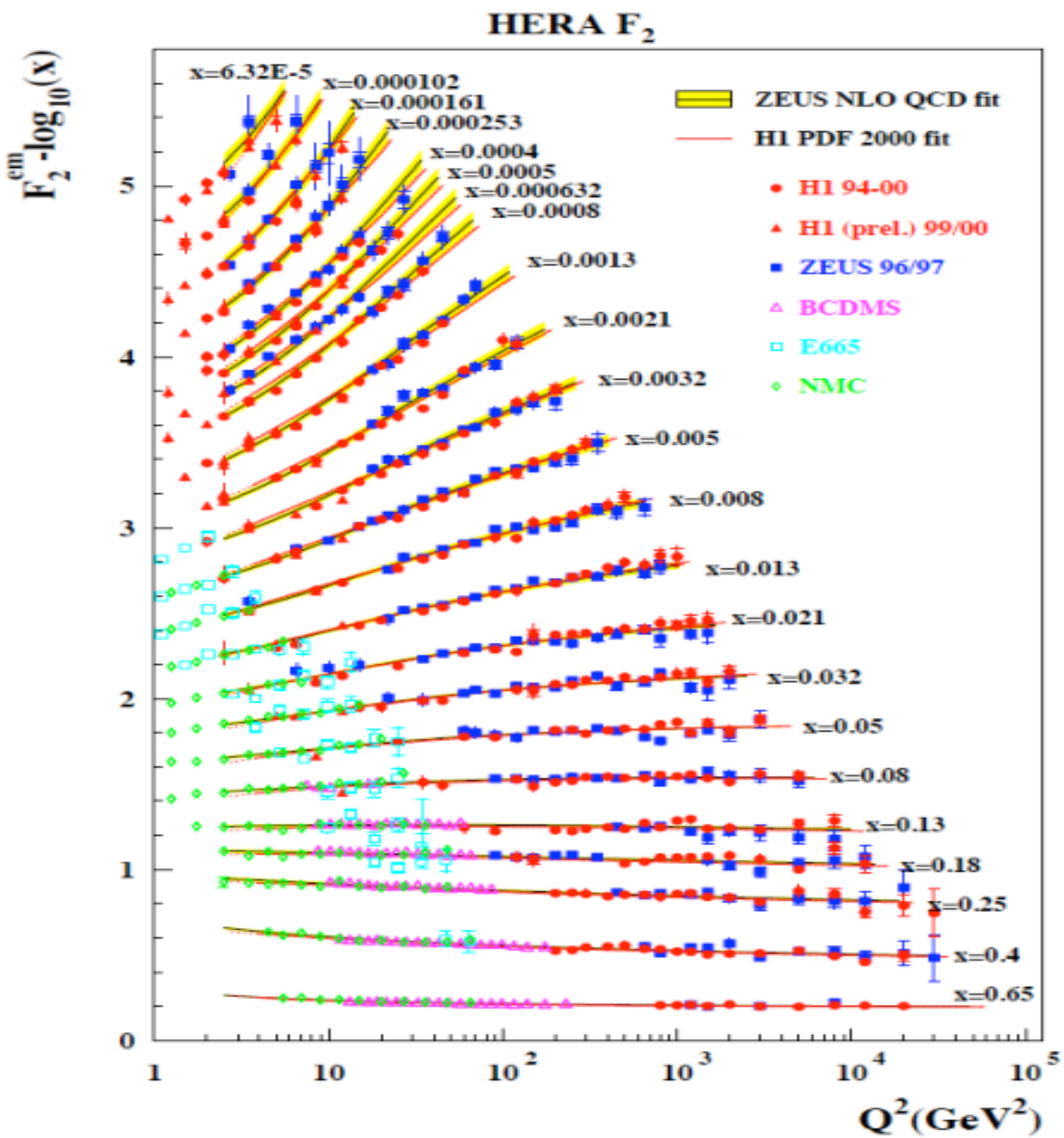
1992-2007

$$\sqrt{S} = 318\,\mathrm{GeV}$$

$$E_e = 27.5\,\mathrm{GeV}$$

$$E_p = 920\,\mathrm{GeV}$$

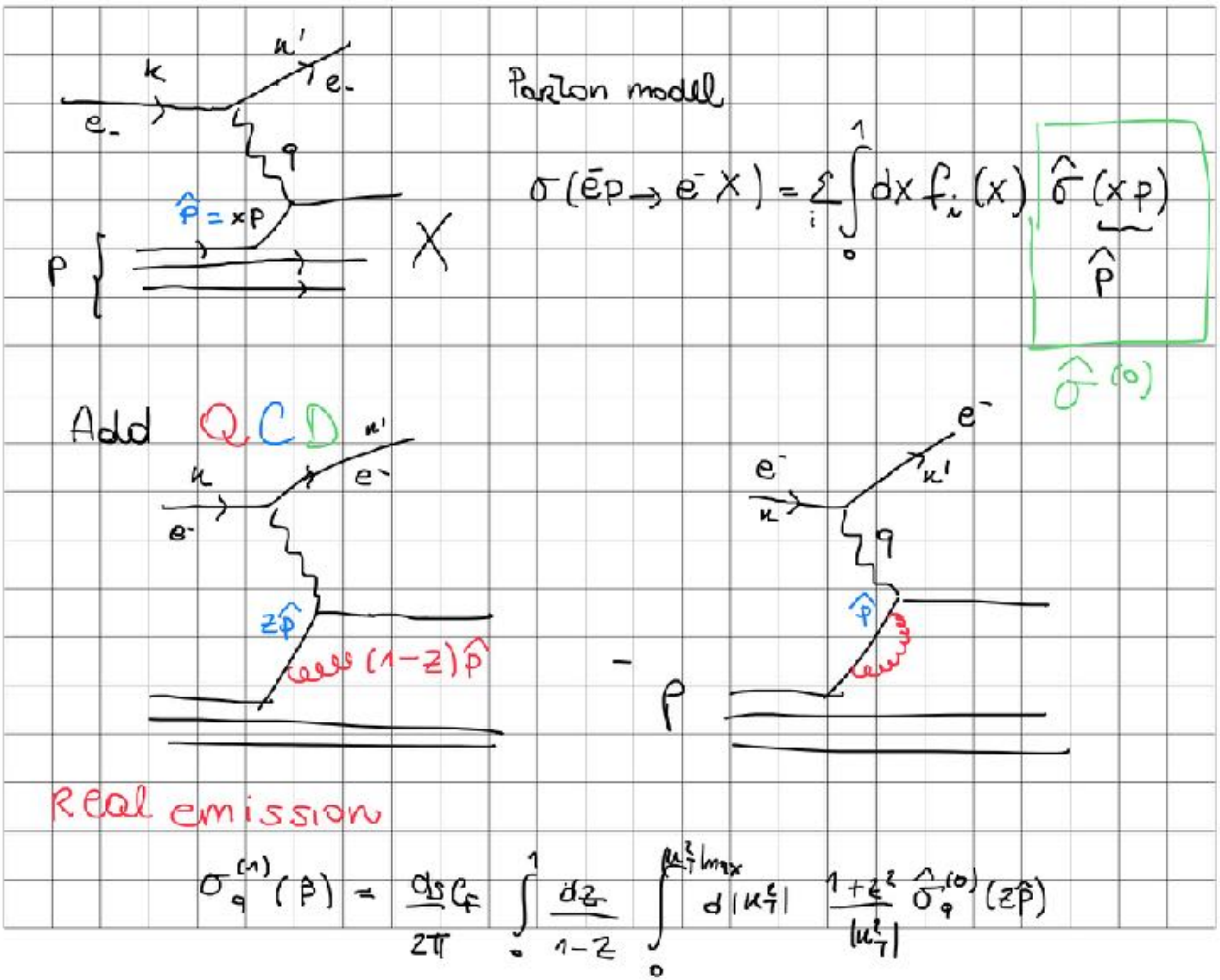
Scaling violation



scaling <u>violation</u>

approx. scaling

QCD and improved parton model



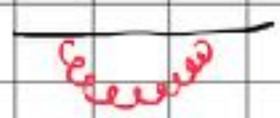
QCD and improved parton model

divergences! SOFT $z \to 1$ Regulator $\varepsilon \to 0$

COLLINEAR $|k_T^2| \to 0$ Regulator $\lambda \to 0$

$$\hat{\sigma}_q^{(1)}(p) = \frac{\alpha_s}{2\pi} C_F \int_0^{1-} \frac{dz}{1-z} \int_{\lambda^2}^{|k_T^2|_{max}} \frac{d|k_T^2|}{|k_T^2|} (1+z^2)\, \hat{\sigma}_q^{(0)}(zp)$$

Adding virtual corrections $\quad - \hat{\sigma}_q^{(0)}(p) \frac{\alpha_s}{2\pi} C_F \int_0^{1-\varepsilon} \frac{dz}{1-z} \int_{\lambda^2}^{|k_T^2|_{max}} \frac{d|k_T^2|}{|k_T^2|} (1+z^2)$



the soft singularity cancels

$\Rightarrow \varepsilon \to 0$

$$\hat{\sigma}_q^{(1)}(p) = \frac{\alpha_s}{2\pi} C_F \int_0^{1} \frac{dz}{1-z} \int_{\lambda^2}^{|k_T^2|_{max}} \frac{d|k_T^2|}{|k_T^2|} (1+z^2) \left[\hat{\sigma}_q^{(0)}(zp) - \hat{\sigma}_q^{(0)}(p) \right]$$

Left with COLLINEAR divergence

QCD and improved parton model

Still left with COLLINEAR divergence!

Introduce μ_F to split integration

$$\int_{\lambda^2}^{|k_T^2|_{max}} \frac{d|k_T^2|}{|k_T^2|} \to \underbrace{\int_{\lambda^2}^{\mu_F^2} \frac{d|k_T^2|}{|k_T^2|}}_{\text{singular}} + \underbrace{\int_{\mu_F^2}^{|k_T^2|_{MAX}} \frac{d|k_T^2|}{|k_T^2|}}_{\text{finite}}$$

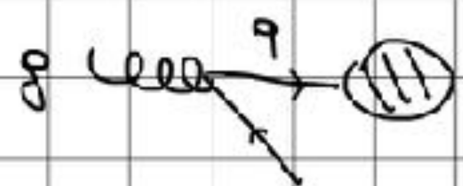
$$\Rightarrow \quad \hat{\sigma}_q(\hat{p}) = \hat{\sigma}_q^{(0)}(\hat{p}) + \hat{\sigma}_q^{(1)}(\hat{p})$$

universal function $P(q \to q)$

$$= \hat{\sigma}_q^{(0)}(\hat{p}) + \frac{\alpha_s}{2\pi} \int_0^1 dz \, P_{qq}(z) \, \hat{\sigma}_q^{(0)}(z\hat{p}) \log \frac{\mu_F^2}{\lambda^2} + \hat{\sigma}_{q,reg}^{(1)}(\hat{p}, \mu_F^2)$$

QCD and improved parton model

Of course quark can come from gluon



Doing the whole calculation, we get

$$\hat{\sigma}_g(p) = \hat{\sigma}_g^{(1)}(p)$$

$$= \frac{\alpha_s}{2\pi} \int_0^1 dz \; P_{qg}(z)\, \hat{\sigma}_q^{(0)}(zp) \log \frac{\mu_F^2}{\lambda^2} \; + \hat{\sigma}^{(1)}_{g\,rep}(p, \mu_F^2)$$

* ↳ in the parton model formula

$$\sigma(p) = \int_0^1 dy \left[f_q(y)\, \hat{\sigma}_q(yp) + f_g(y)\, \hat{\sigma}_g(yp) \right]$$

QCD and improved parton model

$$\sigma(p) = \int_0^1 dy \left[f_q(y)\, \hat{\sigma}_q^{(0)}(yp) \right.$$

$$+ \frac{\alpha_s}{2\pi} \int_0^1 dy\, f_q(y) \int_0^1 dz\, \hat{\sigma}_q^{(0)}(yz\,p)\, P_{qq}(z) \log \frac{\mu_F^2}{\lambda^2}$$

$$+ \frac{\alpha_s}{2\pi} \int_0^1 dy\, f_g(y) \int_0^1 dz\, \hat{\sigma}_g^{(0)}(yzp)\, P_{qg}(z) \log \frac{\mu_F^2}{\lambda^2}$$

$$+ \int_0^1 dy\, f_q(y)\, \hat{\sigma}^{(1)}_{q,reg}(yp, \mu_F^2) + \int_0^1 dy\, f_g(y)\, \hat{\sigma}^{(1)}_{g,reg}(yp, \mu_F^2)$$

terms $\propto \log \frac{\mu_F^2}{\lambda^2}$ can be reabsorbed into redefinition of f_q

$x = yz$

$$f_q(x, \mu_F^2) = \int_x^1 \frac{dy}{y} \left\{ f_q(y) \left[\delta\left(1 - \frac{x}{y}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{y}\right) \log \frac{\mu_F^2}{\lambda^2} \right] + f_g(y) \left[\frac{\alpha_s}{2\pi} P_{qg}\left(\frac{x}{y}\right) \log \frac{\mu_F^2}{\lambda^2} \right] \right\}$$

QCD and improved parton model

So that

$$\sigma(p) = \int_0^1 dx \; f_q(x, \mu_F^2)\, \hat{\sigma}_q(xP, \mu_F^2) + f_g(x)\, \hat{\sigma}_g(xP, \mu_F^2)$$

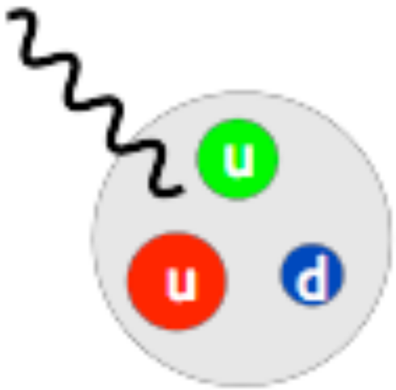
both depend on arbitrary FACTORIZATION scale

Note that however the dependence of $f_{q,g}(x, \mu_F^2)$ is totally fixed by perturbation theory

$$\mu^2 \frac{\partial f_q}{\partial \mu^2}(x, \mu^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[P_{qq}\left(\frac{x}{y}\right) f_q(y, \mu^2) + P_{qg}\left(\frac{x}{y}\right) f_g(y, \mu^2) \right]$$

QCD and improved parton model

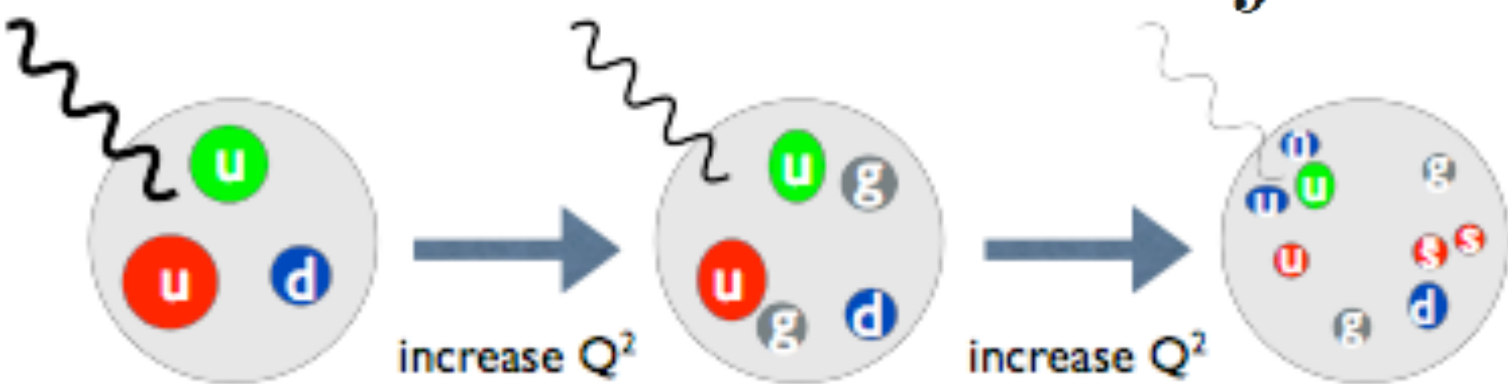
[Parton model picture]



$$\sigma = \int dx \, f_i^{(p)}(x) \, \sigma^{(0)}(xp)$$

[QCD-improved parton model]

$$\sigma = \int dx \, f_i^{(p)}(x, \mu_F^2) \, \sigma(xp, \mu_F^2)$$

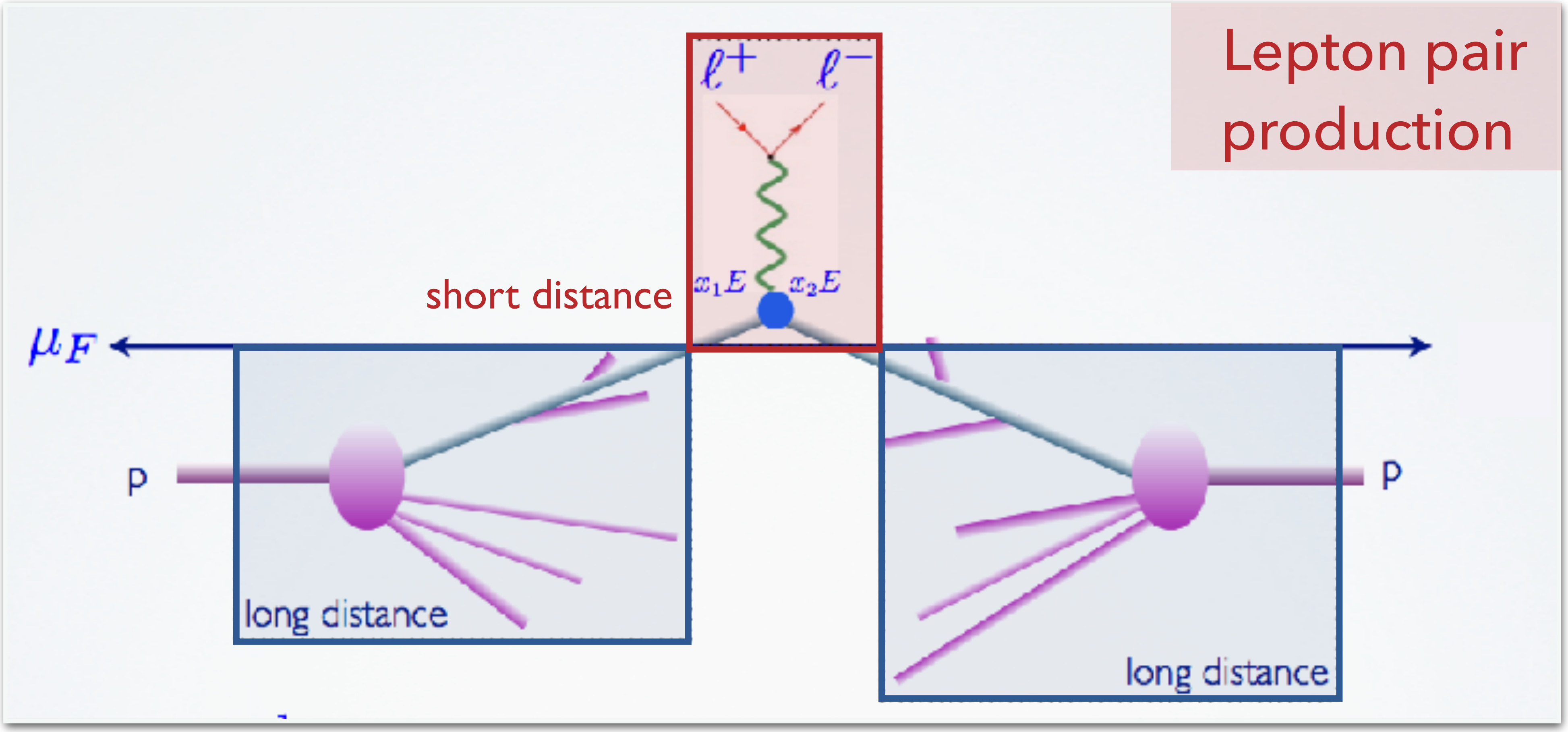


Collinear factorisation theorem

Collinear Factorisation Theorem

$$\frac{d\sigma_H^{ep\to ab}}{dX} = \sum_{i=-n_f}^{+n_f} \int_{x_B}^{1} \frac{dz}{z}\, f_i(z,\mu_F)\, \frac{d\hat{\sigma}_i^{ei}}{dX}(zS, \alpha_s(\mu_R), \mu_F) + \mathcal{O}\left(\frac{\Lambda^n}{S^n}\right)$$

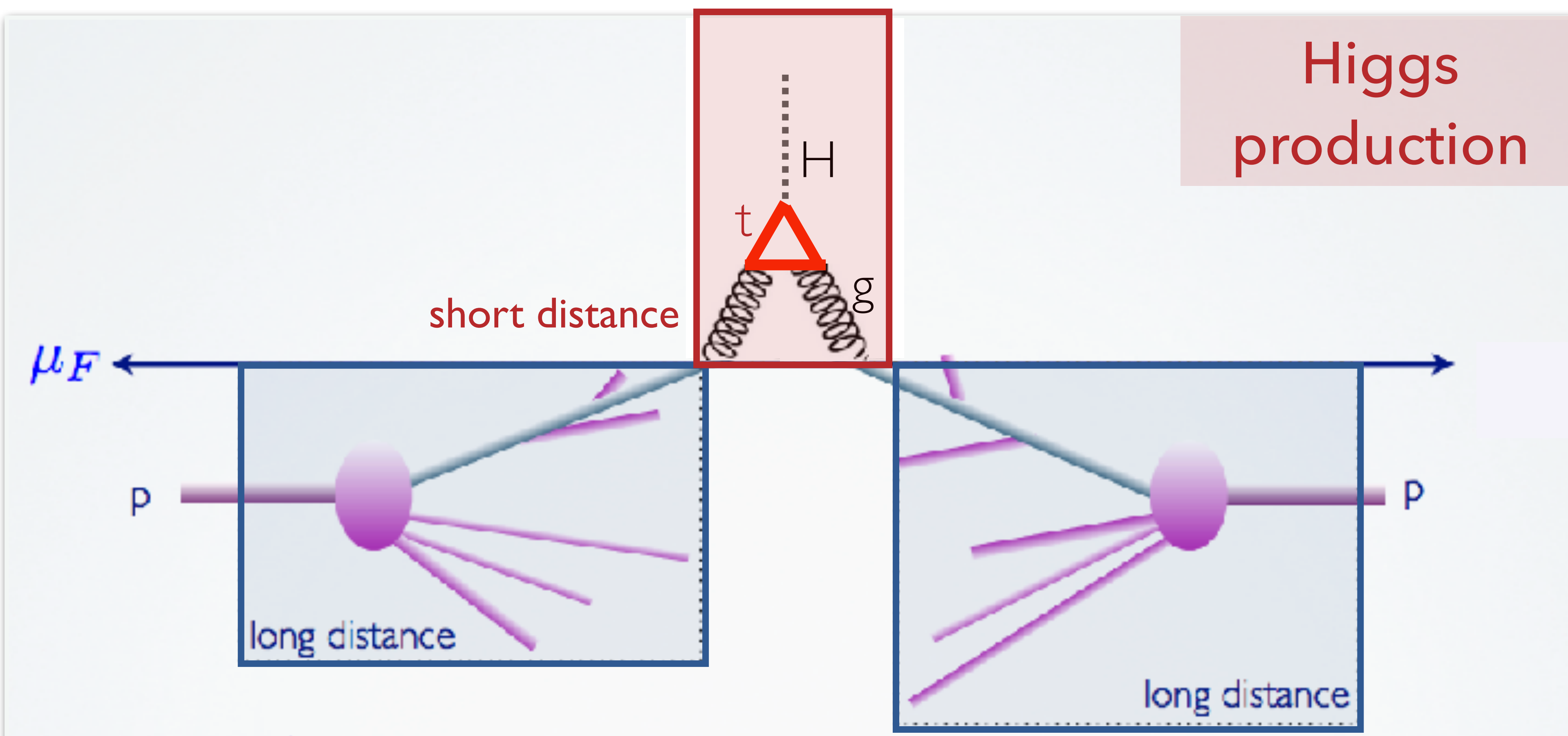
$$\frac{d\sigma_H^{pp\to ab}}{dX} = \sum_{i,j=-n_f}^{+n_f} \int_{\tau_0}^{1} \frac{dz_1}{z_1}\frac{dz_2}{z_2}\, f_i(z_1,\mu_F)\, f_j(z_2,\mu_F)\, \frac{d\hat{\sigma}_i^{ij}}{dX}(zS, \alpha_s(\mu_R), \mu_F) + \mathcal{O}\left(\frac{\Lambda^n}{S^n}\right)$$



Collinear Factorisation Theorem

$$\frac{d\sigma_H^{ep\to ab}}{dX} = \sum_{i=-n_f}^{+n_f} \int_{x_B}^{1} \frac{dz}{z} f_i(z,\mu_F) \frac{d\hat{\sigma}_i^{ei}}{dX}(zS, \alpha_s(\mu_R), \mu_F) + \mathcal{O}\left(\frac{\Lambda^n}{S^n}\right)$$

$$\frac{d\sigma_H^{pp\to ab}}{dX} = \sum_{i,j=-n_f}^{+n_f} \int_{\tau_0}^{1} \frac{dz_1}{z_1} \frac{dz_2}{z_2} f_i(z_1,\mu_F) f_j(z_2,\mu_F) \frac{d\hat{\sigma}_i^{ij}}{dX}(zS, \alpha_s(\mu_R), \mu_F) + \mathcal{O}\left(\frac{\Lambda^n}{S^n}\right)$$

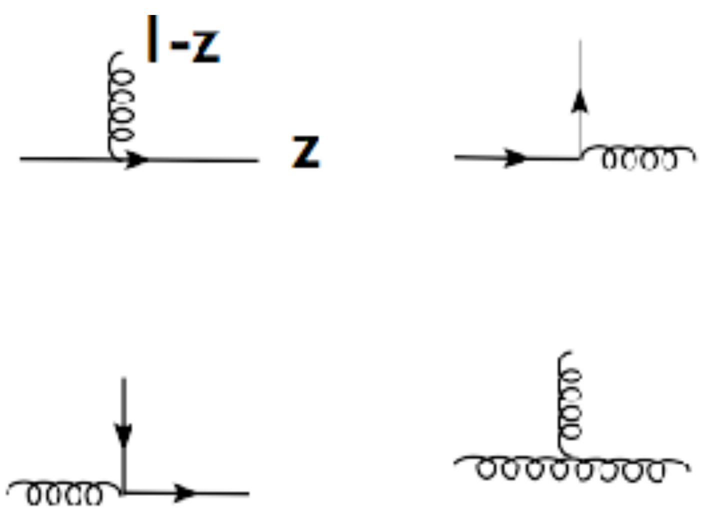


DGLAP evolution equations

In analogy with running coupling, imposing that cross section does not depend on arbitrary scale **μ**, get renormalisation group equations for PDFs

$$t = \log \frac{Q^2}{\mu_F^2} \qquad \frac{d}{dt}\begin{pmatrix} q_i(x,t) \\ g(x,t) \end{pmatrix} = \frac{\alpha_s(t)}{2\pi}\int_x^1 \sum_{j=q,\bar{q}} \frac{d\xi}{\xi} \begin{pmatrix} P_{ij}\left(\frac{x}{\xi},\alpha_s(t)\right) & P_{ig}\left(\frac{x}{\xi},\alpha_s(t)\right) \\ P_{gj}\left(\frac{x}{\xi},\alpha_s(t)\right) & P_{gg}\left(\frac{x}{\xi},\alpha_s(t)\right) \end{pmatrix} \otimes \begin{pmatrix} q_j(\xi,t) \\ g(\xi,t) \end{pmatrix}$$

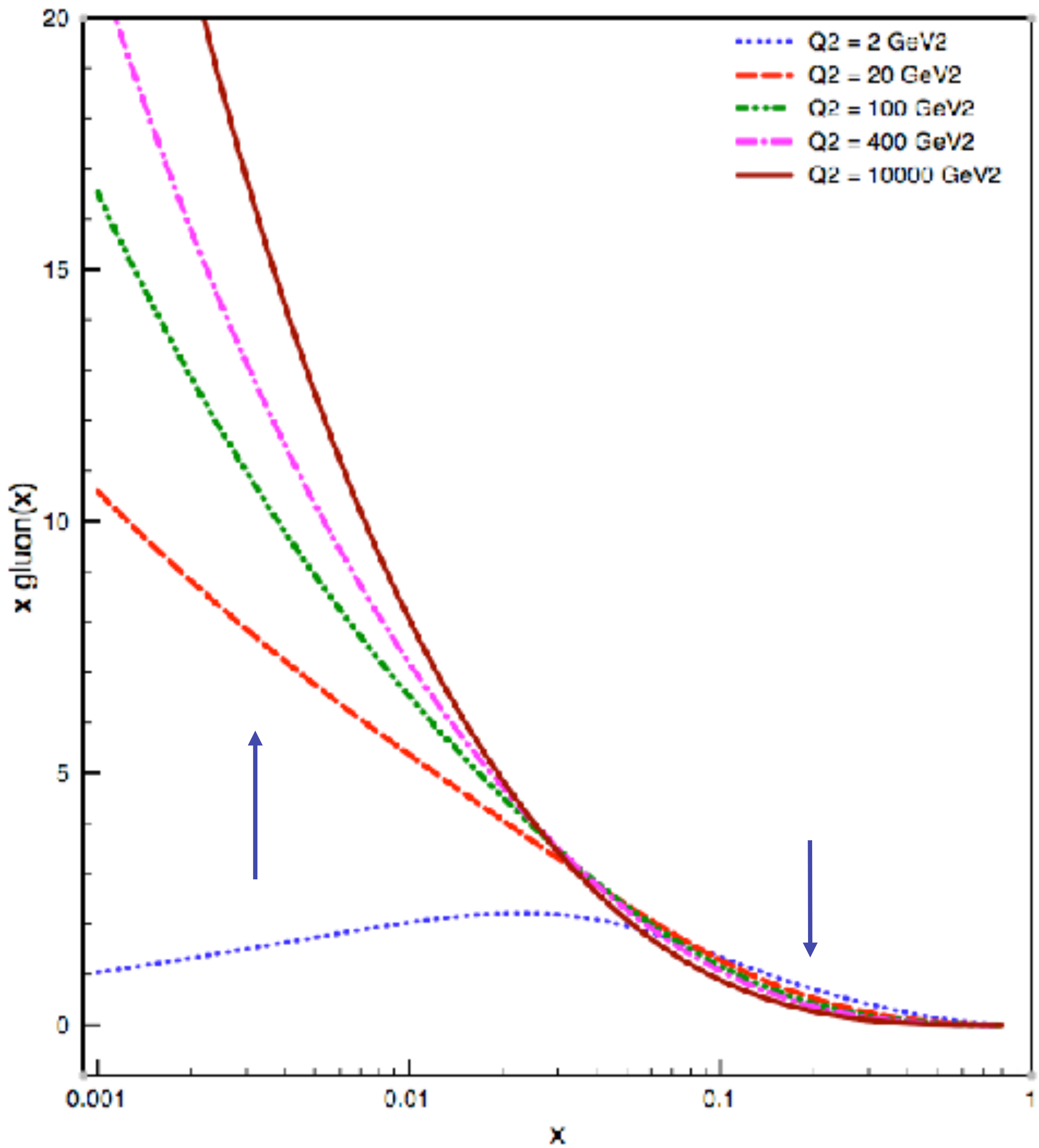
Dokshitzer, Gribov, Lipatov, Altarelli, Parisi equations



- Functional dependence on μ^2 is totally predicted by solving DGLAP evolution eqns

- Splitting functions known up to NNLO:
  LO Dokshitzer; Gribov, Lipatov; Altarelli, Parisi (1977)
  NLO Floratos,Ross,Sachrajda; Floratos, Lacaze, Kounnas, Gonzalez-Arroyo,Lopez,Yndurain; Curci,Furmanski Petronzio, (1981)
  NNLO - Moch, Vermaseren, Vogt, 2004

DGLAP evolution equations



Gluon evolution

$$g(x,\mu^2) = \Gamma_{gq} \otimes \Sigma(x,\mu_0^2) + \Gamma_{gg} \otimes g(x,\mu_0^2)$$

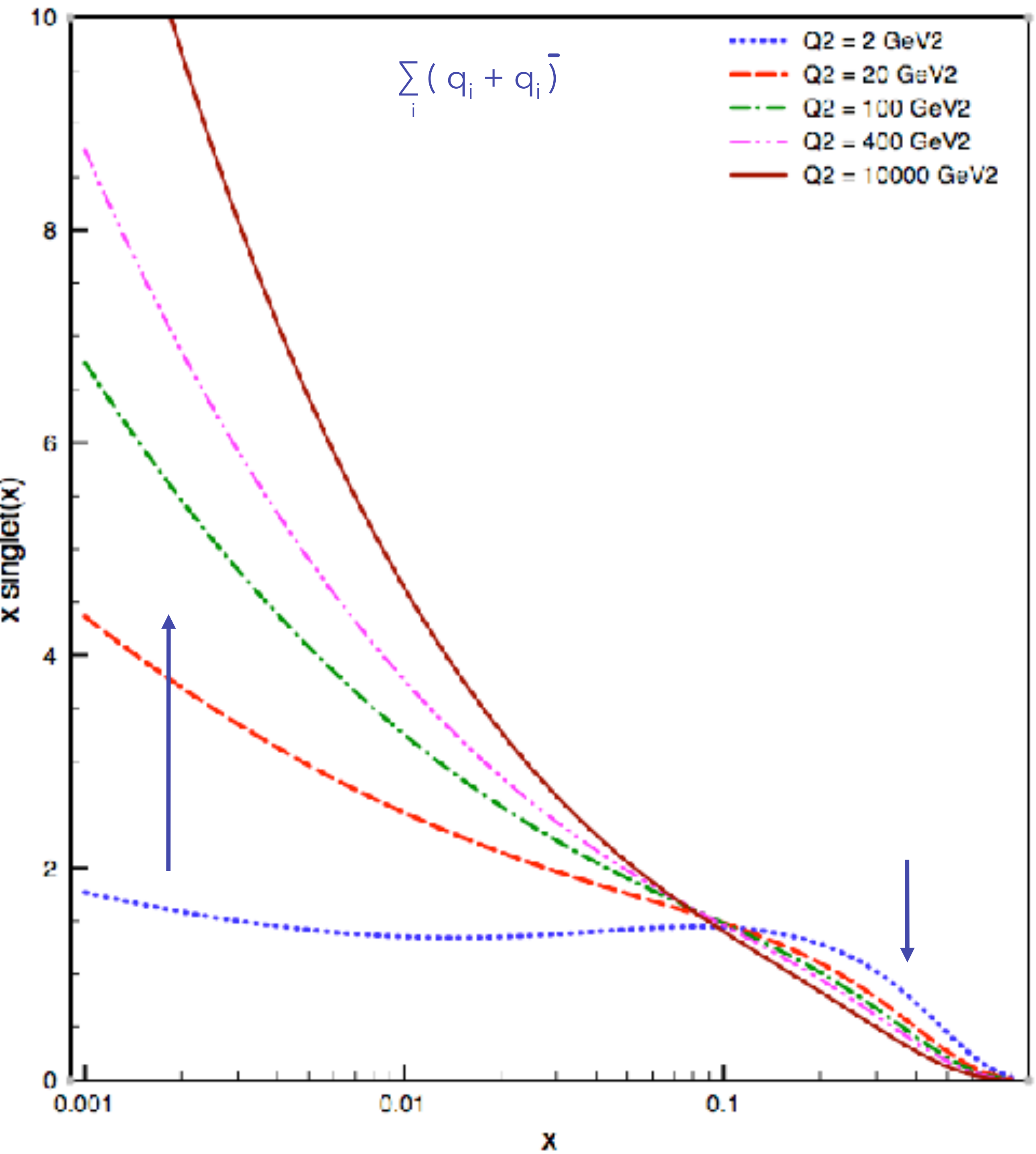
$$P_{gq}^{(0)}(x) = C_F \left[\frac{1+(1-x)^2}{x}\right]$$

$$P_{gg}^{(0)}(x) = 2N\left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right] + \delta(1-x)\frac{(11N - 4n_f T_R)}{6}$$

- Both P_{gg} and P_{gq} diverge for $x \to 0$

- Gluon is depleted at large x

DGLAP evolution equations



Singlet evolution

$$\Sigma(x,\mu^2) = \Gamma_{qq} \otimes \Sigma(x,\mu_0^2) + \Gamma_{qg} \otimes g(x,\mu_0^2)$$

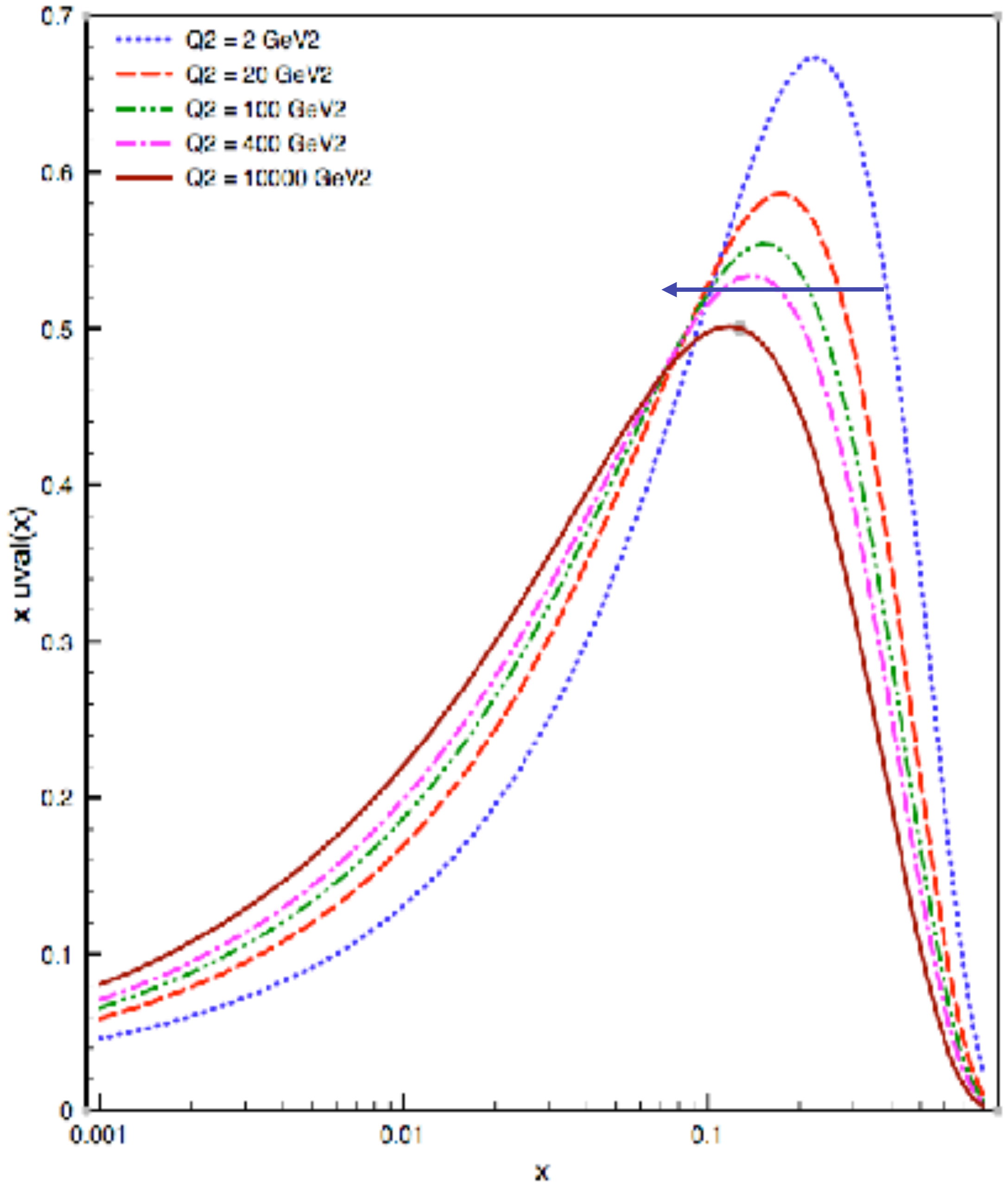
$$P_{qq}^{(0)}(x) = C_F \left[\frac{(1+x^2)}{(1-x)_+} + \frac{3}{2}\delta(1-x) \right]$$

$$P_{qg}^{(0)}(x) = T_R \left[x^2 + (1-x)^2 \right]$$

- High-x gluon feeds growth of small-x gluon and quark

- Gluons can be seen because they help drive the quark evolution

DGLAP evolution equations



Non-singlet valence evolution

$$u_v(x, \mu^2) = \Gamma^v_{NS} \otimes u_v(x, \mu_0^2)$$

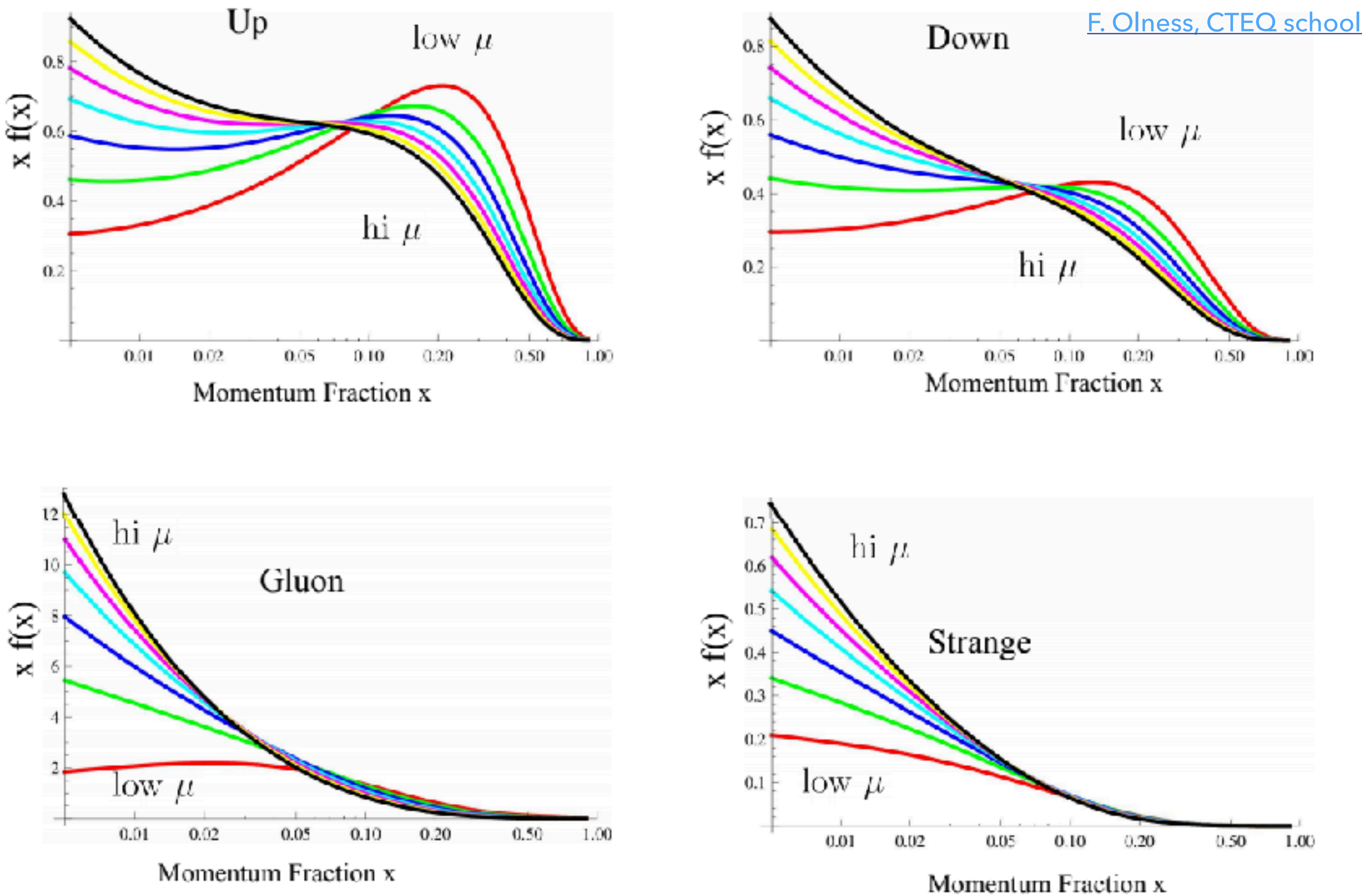
$$P^{(0),v}_{NS} = P^{(0)}_{qq}(x) = C_F \left[\frac{(1+x^2)}{(1-x)_+} + \frac{3}{2}\delta(1-x) \right]$$

- As Q^2 increases partons lose longitudinal momentum; distributions all shift to lower x

- Gluons can be seen because they help drive the quark evolution

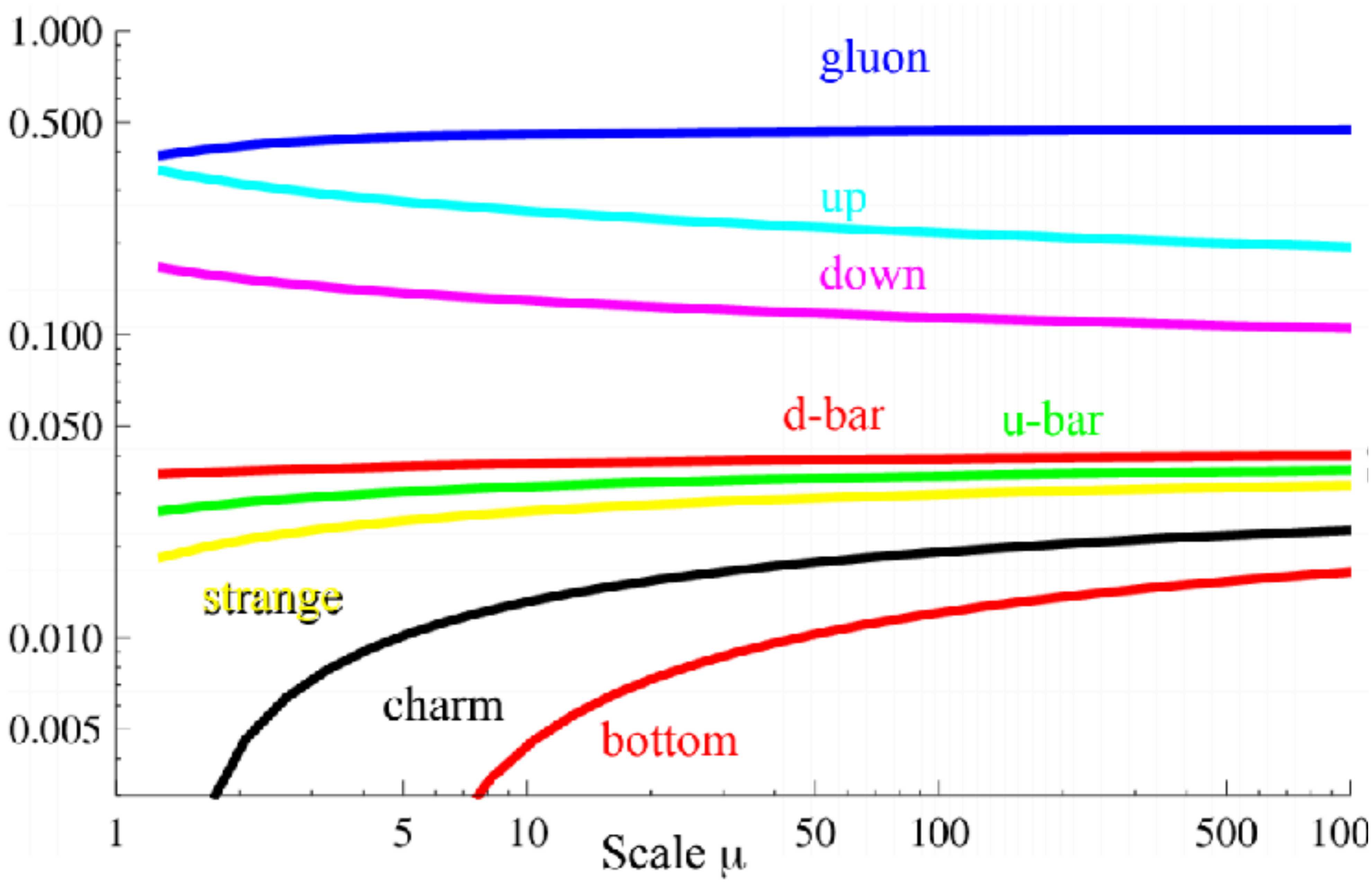
DGLAP evolution equations

F. Olness, CTEQ school 2017



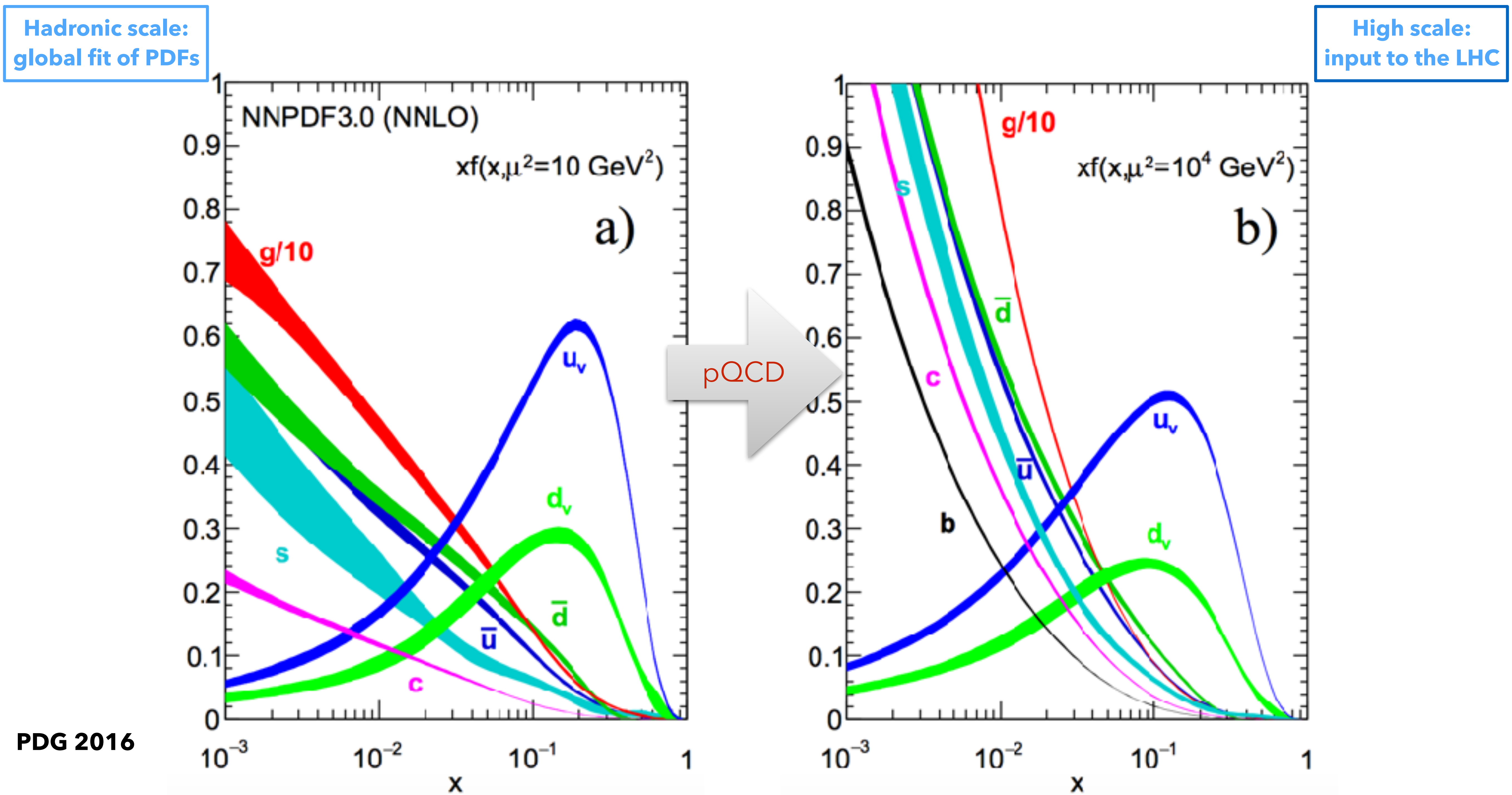
DGLAP evolution equations

F. Olness, CTEQ school 2017



DGLAP evolution equations

Functional dependence of PDFs on the scale is totally predicted up to NNLO accuracy by solving DGLAP evolution equations



PDG 2016

Wrap-up

- ➡ The structure of the proton has been a crucial ingredient to test and verify perturbative QCD and it is now key to the precision challenge that we are facing at the LHC

- ➡ Today's lecture

  - ✓ Parametrisation of the proton in terms of structure functions
  - ✓ Parton model picture
  - ✓ QCD - Improved parton model
  - ✓ DGLAP evolution equations
  - ✓ Collinear Factorisation Theorem

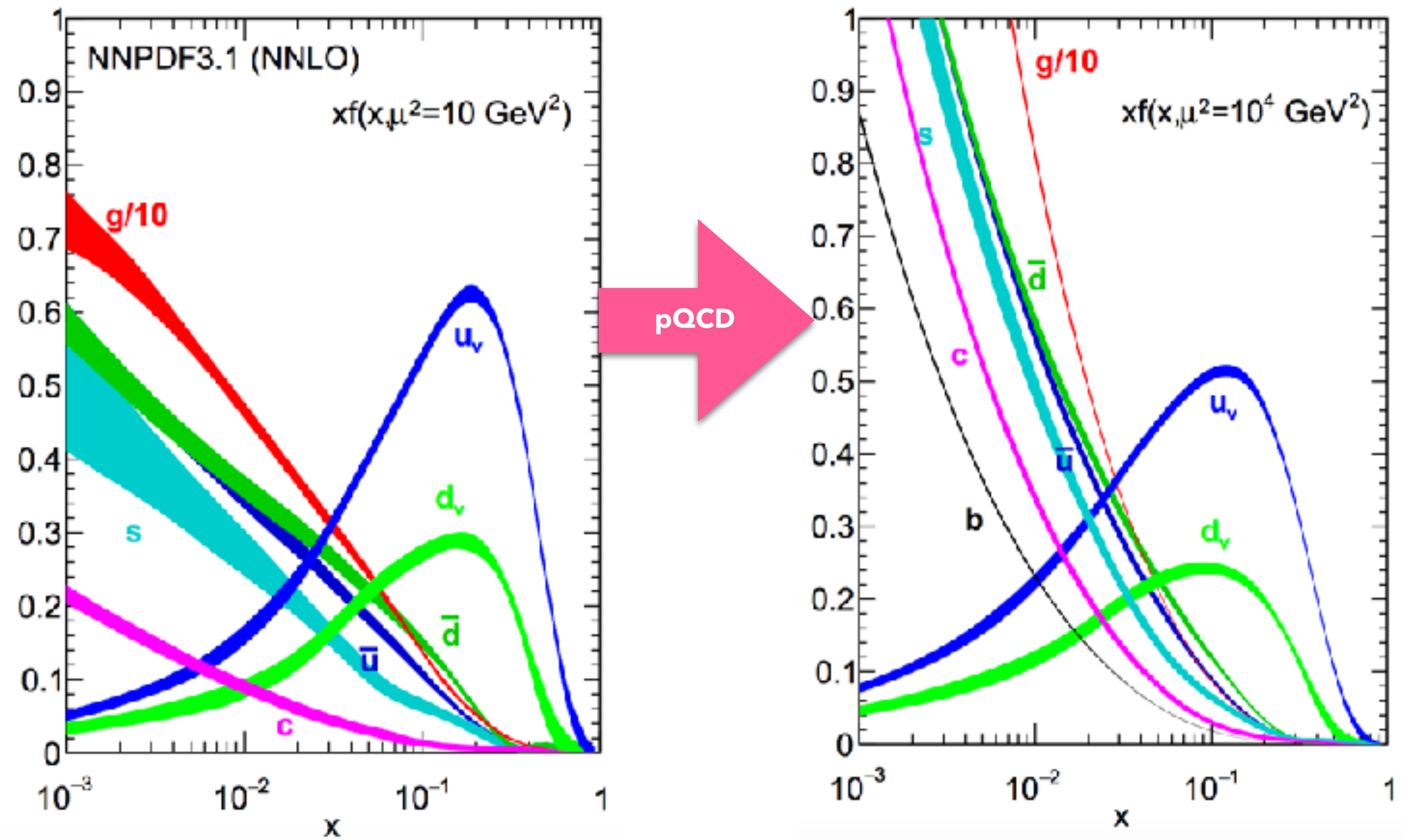
PDF determination

$$f_i(x, \mu)$$

Data

Perturbative QCD

Hadronic scale: global fit of PDFs



High scale: input to the LHC

PDF determination

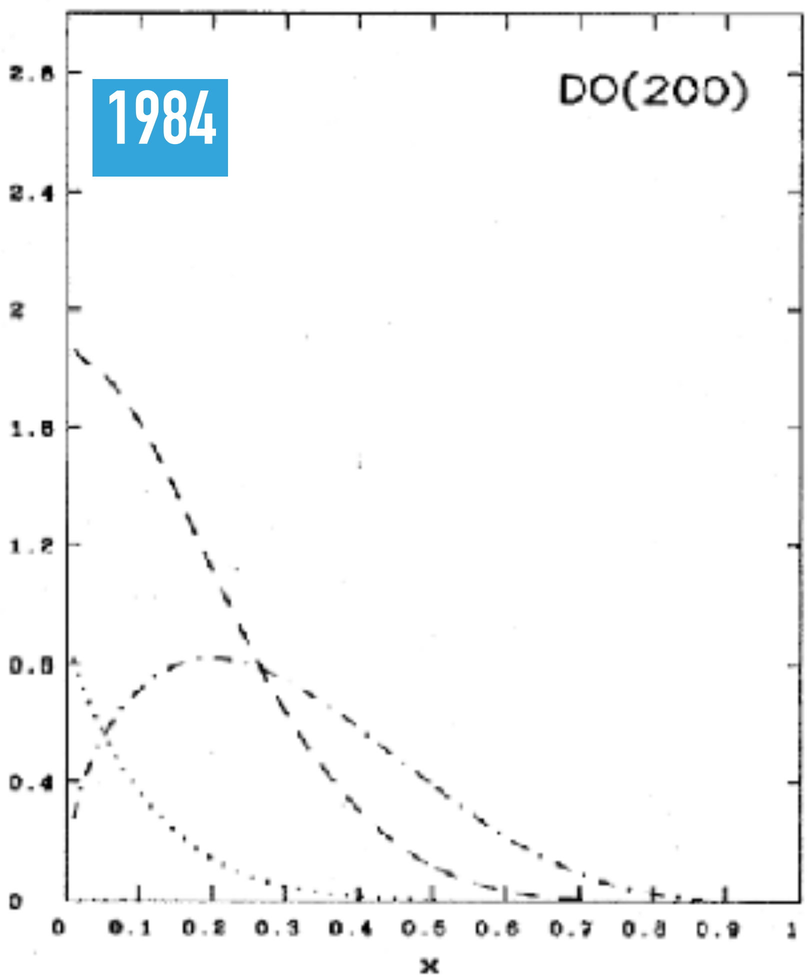
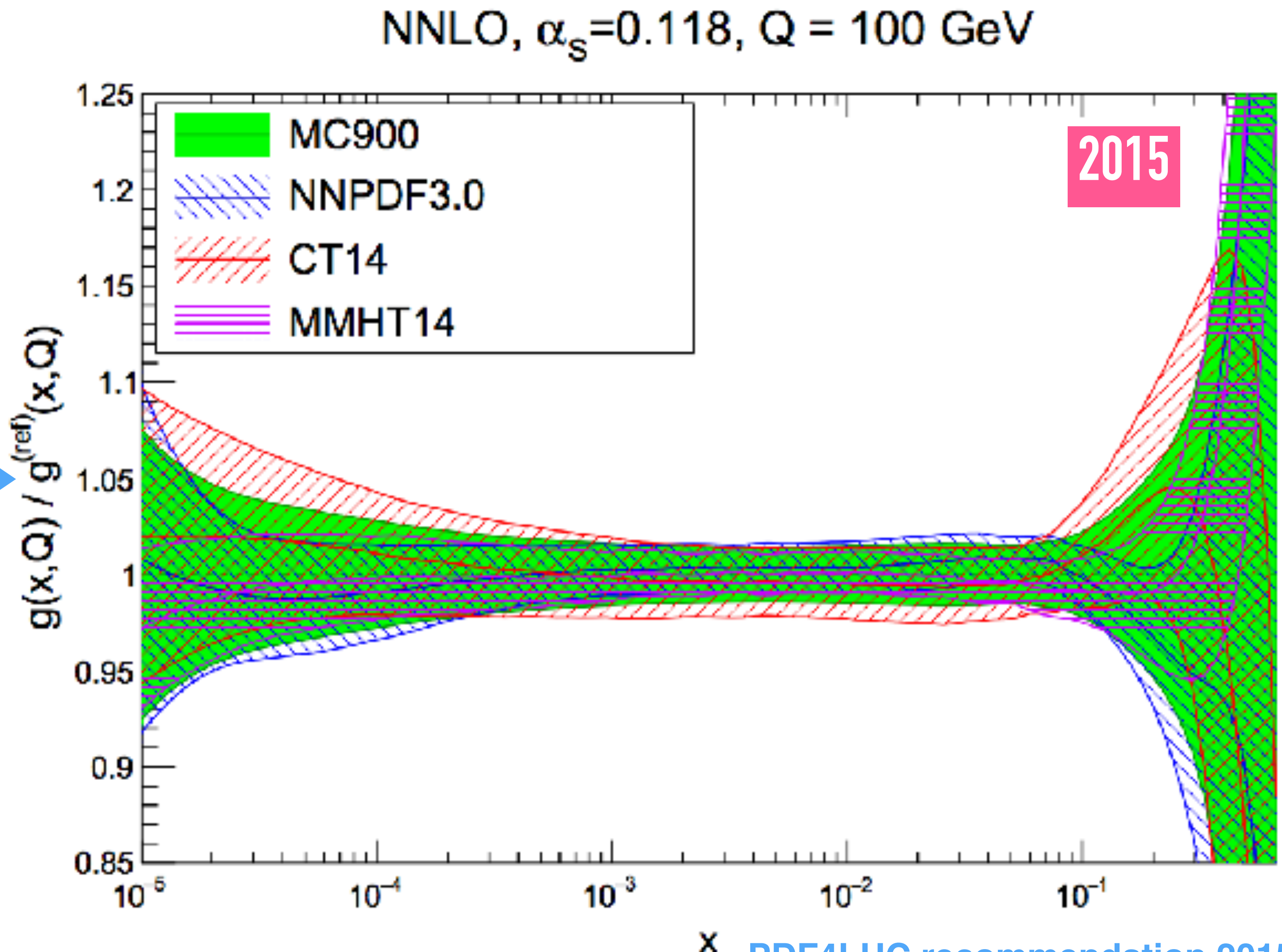


FIG. 27. "Soft-gluon" ($\Lambda = 200$ MeV) parton distributions of Duke and Owens (1984) at $Q^2 = 5$ GeV^2: valence quark distribution $x[u_v(x) + d_v(x)]$ (dotted-dashed line), $xG(x)$ (dashed line), and $q_v(x)$ (dotted line).

Rev.. Mod. Phys. 1984



PDF4LHC recommendation 2015
J.Phys. G43 (2016) 023001

★ 30 years of steady progress in PDF community have produced a huge impact on understanding of proton structure and precision physics

The ingredients

The ingredients

- Choose **experimental data** to fit and include all info on correlations
- **Theory settings**: perturbative order, heavy quark mass scheme, EW corrections, intrinsic heavy quarks, α_s, quark masses value and scheme
- Choose a starting scale Q_0 where pQCD applies
- **Parametrise** independent quarks and gluon distributions at the starting scale
- Solve **DGLAP equations** from initial scale to scales of experimental data and build up observables
- **Fit** PDFs to data
- Provide **error sets** to compute PDF uncertainties

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- Provide PDF **error sets** to compute PDF uncertainties

The ingredients

$$\sigma_{\mathcal{F}} = \left(\sum_{k=1}^{N_{\rm set}} \left(\mathcal{F}[\{f^{(k)}\}] - \mathcal{F}[\{f^{(0)}\}] \right)^2 \right)^{1/2}$$

error sets
mem > 1

central set
mem = 0

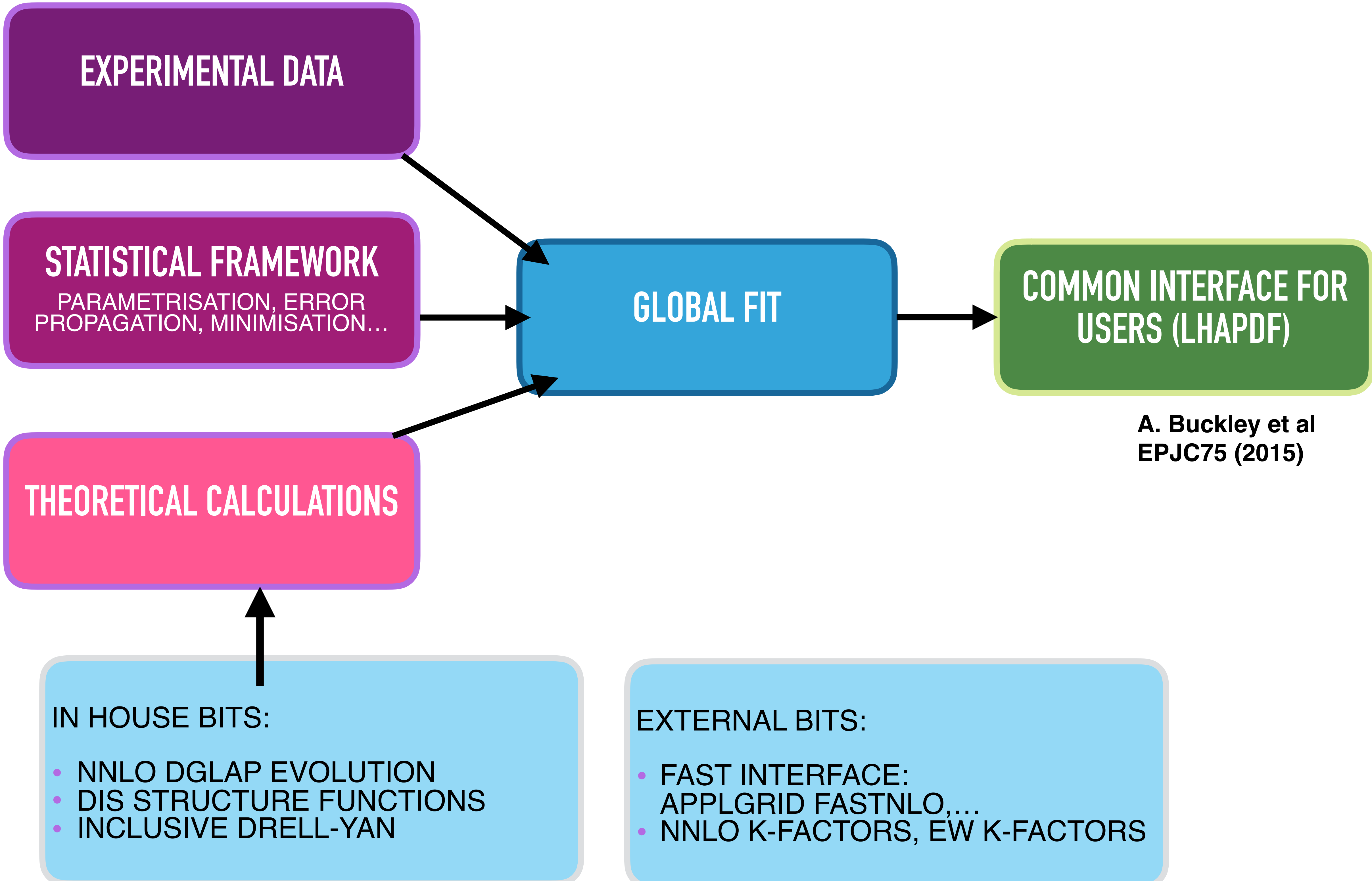
```
call InitPDF(mem)

call evolvePDF(x,Q,f)
```

LHAPDF interface
http://lhapdf.hepforge.org

- Provide PDF **error sets** to compute PDF uncertainties

	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
Parton	tbar	bbar	cbar	sbar	ubar	dbar	g	d	u	s	c	b	t

A complex machinery



Experimental Data

Experimental data

- PDFs are not measurable, we measure observables that convolute PDFs with partonic cross sections

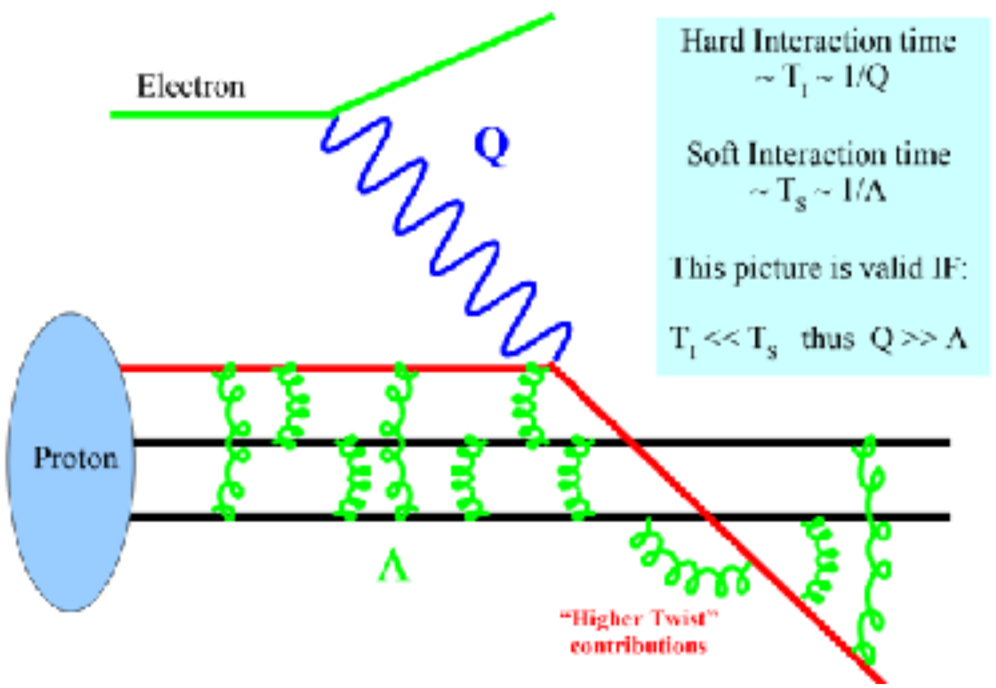
$$\frac{d\sigma_H^{ep\to ab}}{dX} = \sum_{i=-n_f}^{+n_f} \int_{x_B}^{1} \frac{dz}{z}\, f_i(z,\mu_F)\, \frac{d\hat{\sigma}_i^{ei}}{dX}(zS, \alpha_s(\mu_R), \mu_F) + \mathcal{O}\left(\frac{\Lambda^n}{S^n}\right)$$

$$\frac{d\sigma_H^{pp\to ab}}{dX} = \sum_{i,j=-n_f}^{+n_f} \int_{\tau_0}^{1} \frac{dz_1}{z_1}\frac{dz_2}{z_2}\, f_i(z_1,\mu_F)\, f_j(z_2,\mu_F)\, \frac{d\hat{\sigma}_i^{ij}}{dX}(zS, \alpha_s(\mu_R), \mu_F) + \mathcal{O}\left(\frac{\Lambda^n}{S^n}\right)$$

- Must exclude regions where factorisation fails to apply (low Q2 and large x). Typically

$$Q^2_{\rm min} = 2\,{\rm GeV}^2$$

$$W^2_{\rm min} = \left(Q^2\frac{1-x}{x}\right)_{\rm min} = 12.5\,{\rm GeV}^2$$



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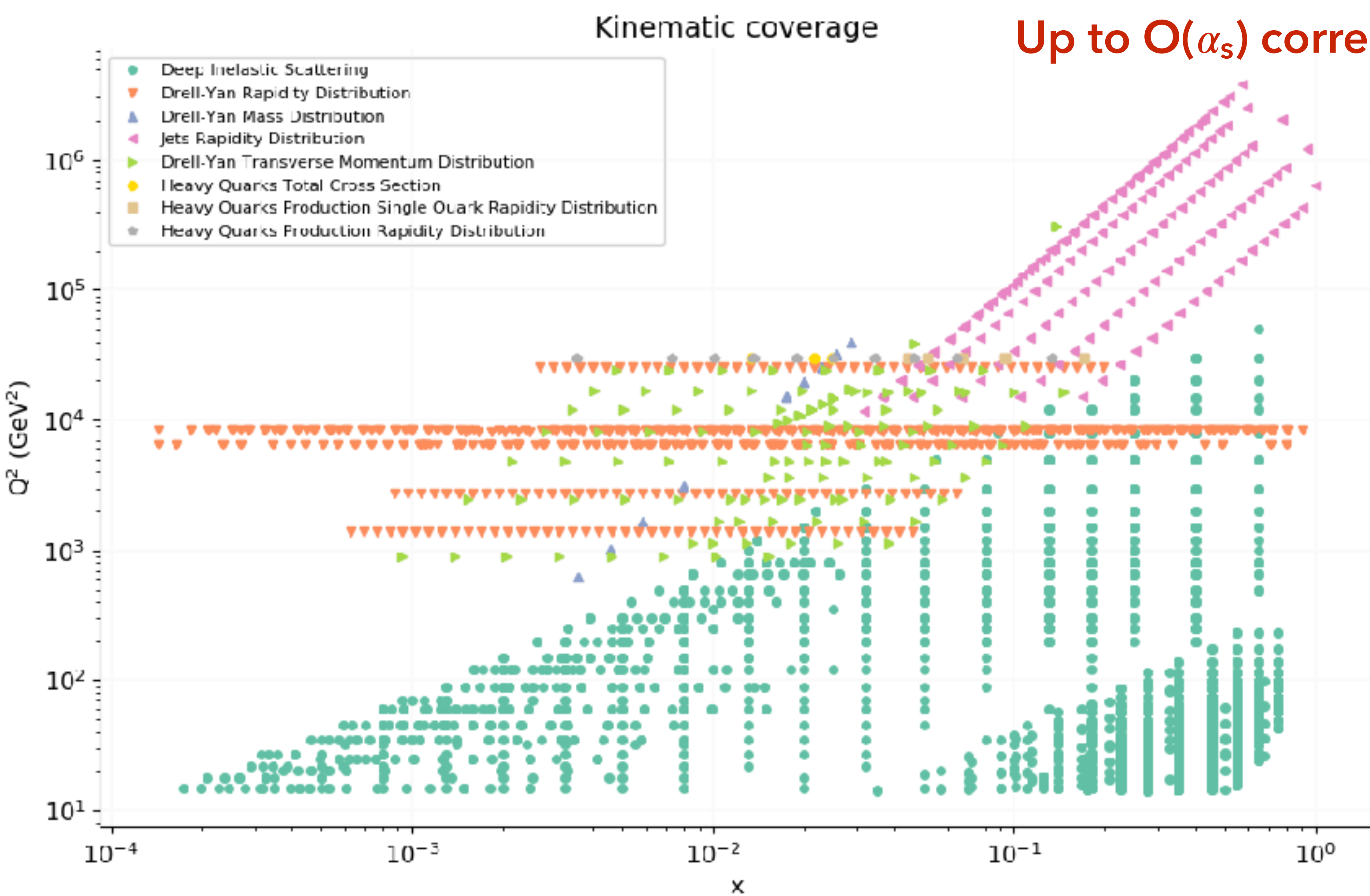
- Different data constrain different PDF combinations in different regions

  ➡ DIS data on proton abundant and precise (HERA)
  ➡ In principle F_2, F_3 CC provide 4 light quark combinations
  F_2, F_3 NC provide 2 extra light quark combinations
  ➡ HERA data only determine four combinations of PDFs
  ➡ Old DIS and Drell-Yan data still used because of isospin symmetry
  ➡ W,Z boson final state provide lot of information, gluon from scale dependence
  ➡ Processes with jets and/or heavy quark in final states direct handle on the gluon

Disentangling PDFs

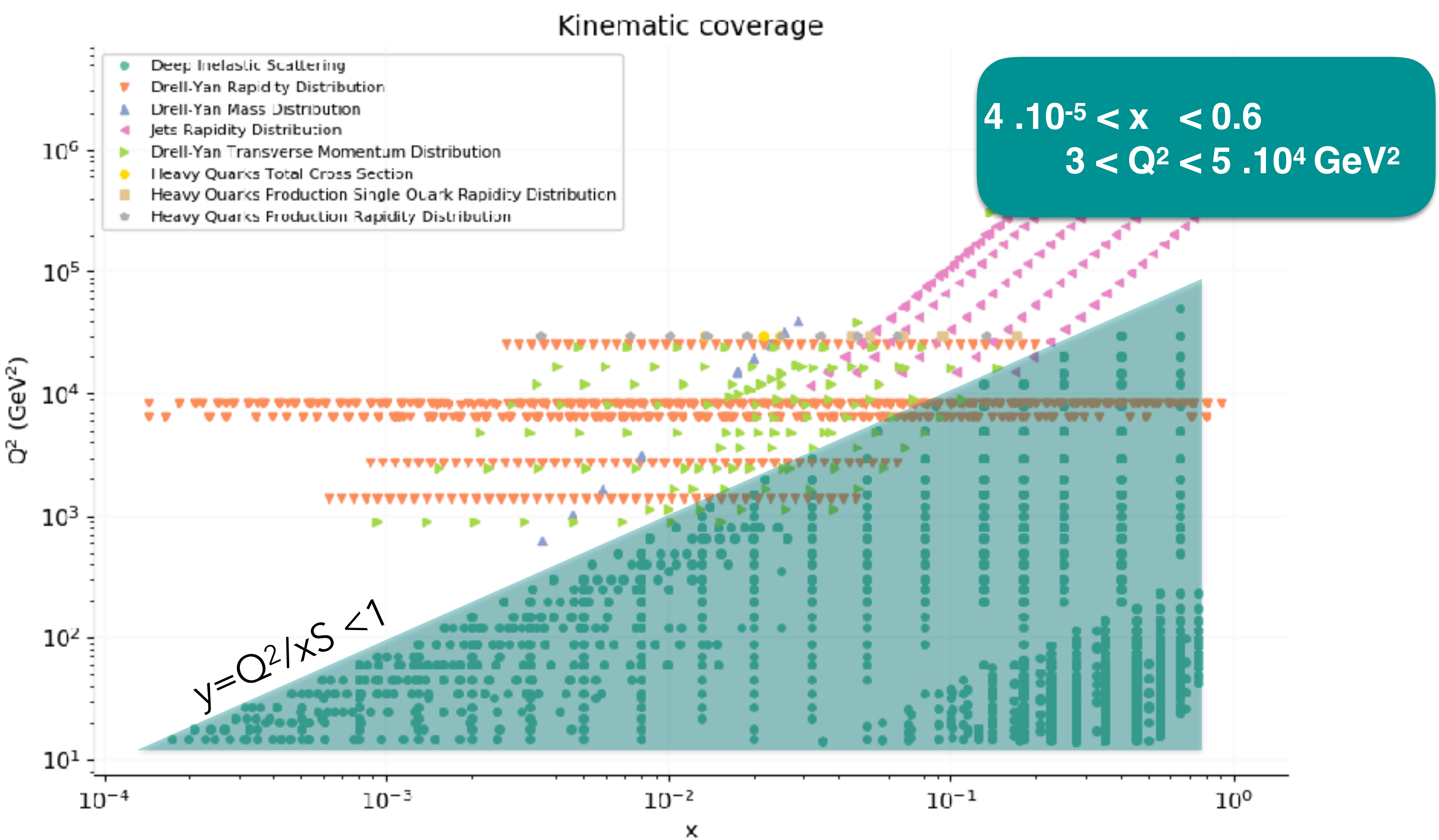


Up to O(α_s) corrections

Q-dependence: pert. theory

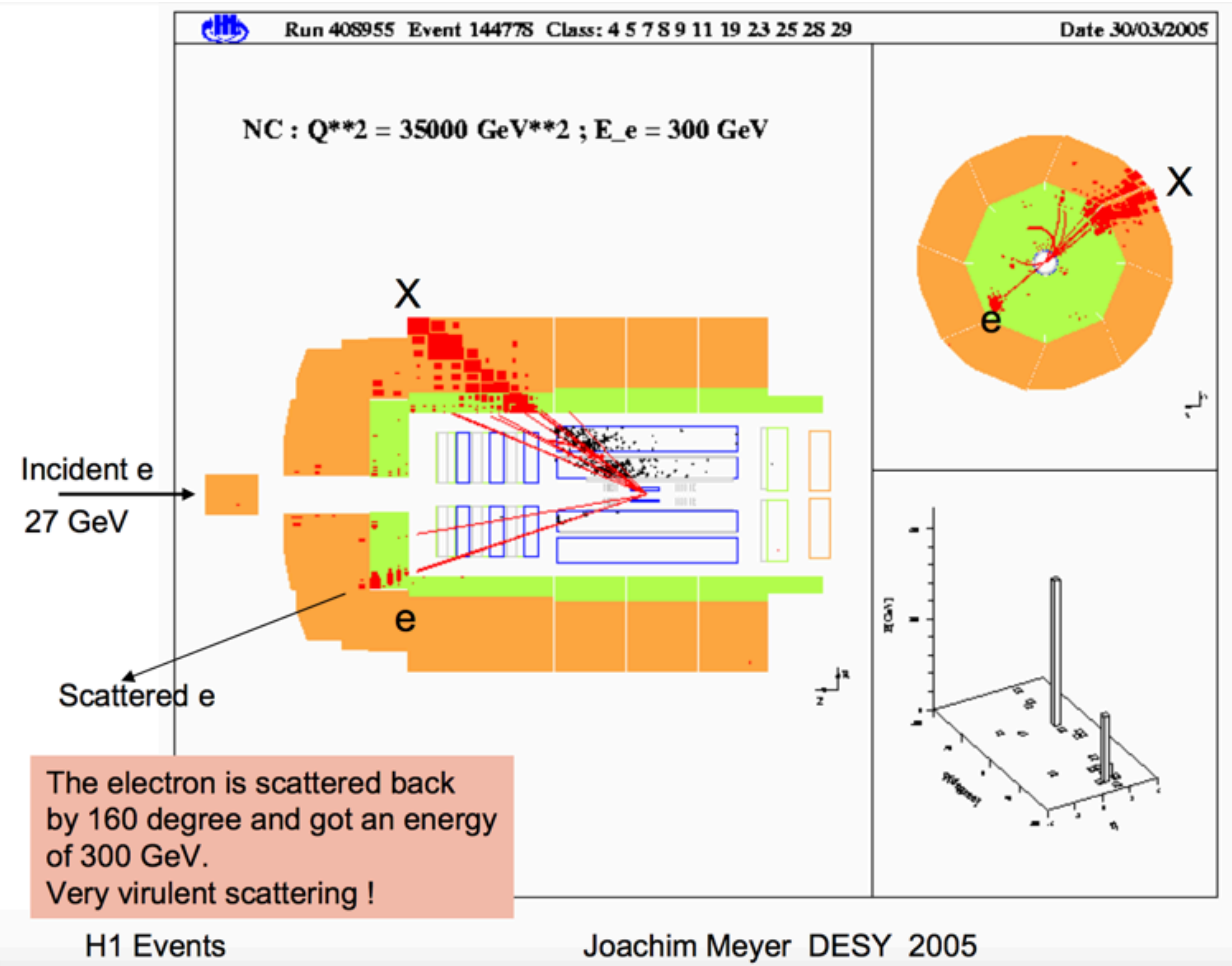
x-dependence: from data

HERA data



Kinematic coverage
Deep Inelastic Scattering
Drell-Yan Rapidity Distribution
Drell-Yan Mass Distribution
Jets Rapidity Distribution
Drell-Yan Transverse Momentum Distribution
Heavy Quarks Total Cross Section
Heavy Quarks Production Single Quark Rapidity Distribution
Heavy Quarks Production Rapidity Distribution
$4 \cdot 10^{-5} < x < 0.6$
$3 < Q^2 < 5 \cdot 10^4$ GeV^2
$y = Q^2/xS < 1$
Q^2 (GeV^2)
x

HERA data

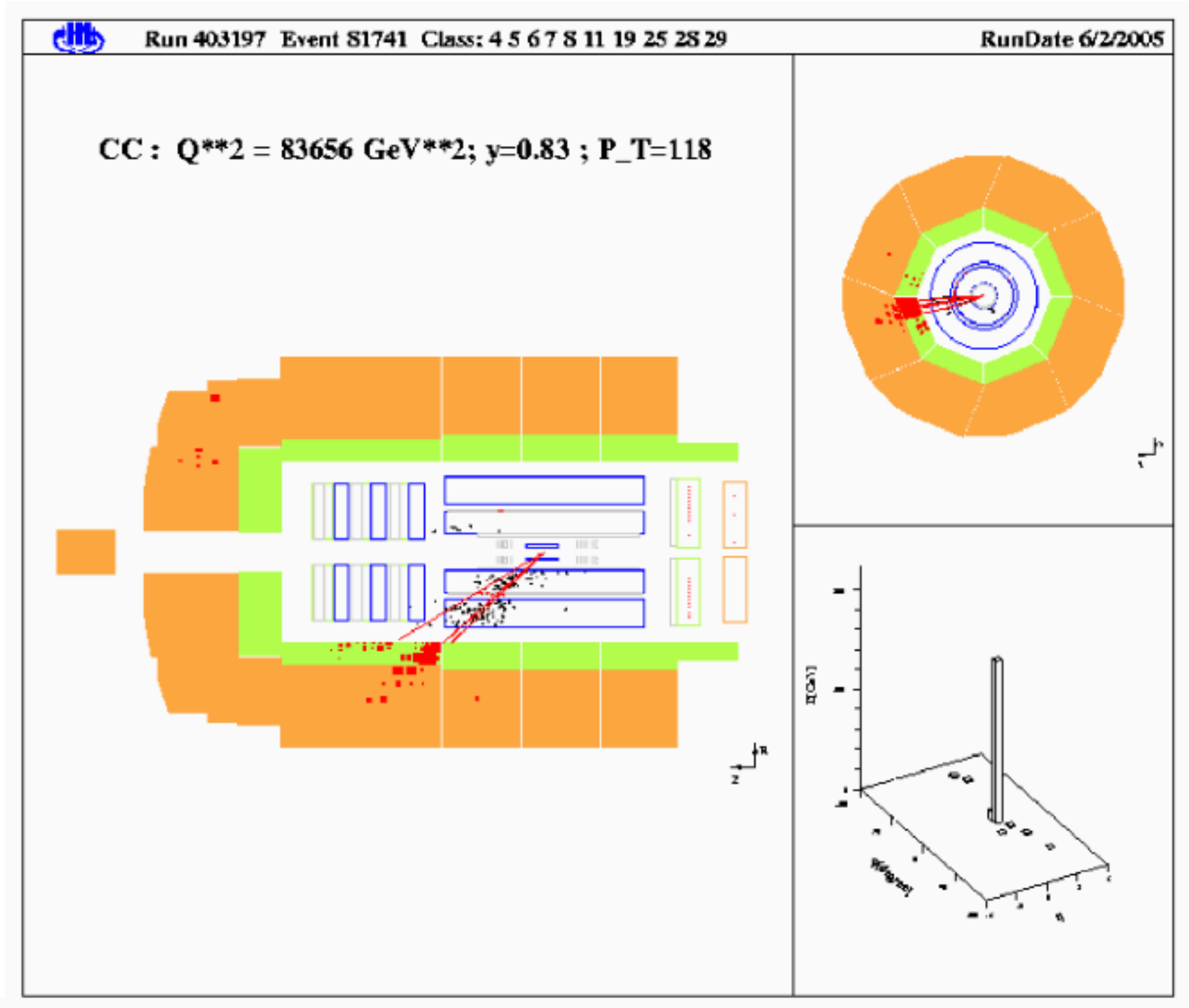


Neutral Current event

$$ep \to e\,X$$

$$Q = 180\,\mathrm{GeV}$$
$$y = 0.66$$
$$x_B = 0.47$$

HERA data



Charged Current event

$$ep \to \nu X$$

$$Q = 289\,\mathrm{GeV}$$

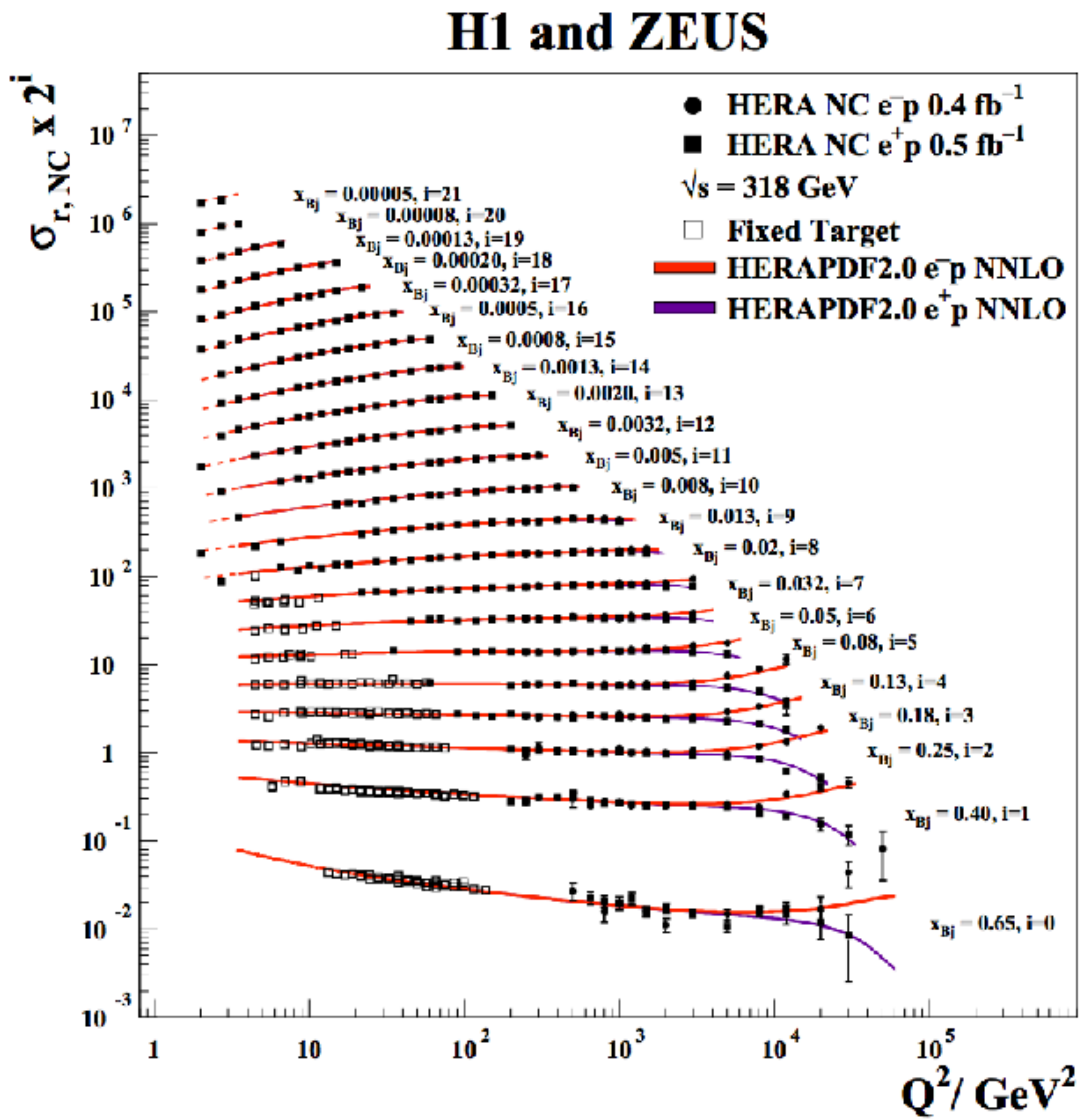
$$y = 0.83$$

$$x_B = 0.91$$

Joachim Meyer DESY 2005

HERA data

- Combination of Run I + Run II data led to very precise measurements of reduced xsec
- F3 contribution visible at larger x and Q~Mz



arXiv: 1506.06042

HERA data

Neutral Current

$$\left[F_2^\gamma, F_2^{\gamma Z}, F_2^Z\right] = x \sum_{i=1}^{n_f} \left[e_i^2, 2e_i g_V^i, (g_V^i)^2 + (g_A^i)^2\right](q_i + \bar{q}_i)$$

③ ④

$$\left[F_3^\gamma, F_3^{\gamma Z}, F_3^Z\right] = x \sum_{i=1}^{n_f} \left[0, 2e_i g_A^i, 2g_V^i g_A^i\right](q_i - \bar{q}_i)$$

Charged Current

①

$$F_2^{W^-} = 2x\left(u + \bar{d} + \bar{s} + c\right),$$
$$F_3^{W^-} = 2x\left(u - \bar{d} - \bar{s} + c\right),$$

②

$$F_2^{W^+} = 2x\left(d + \bar{u} + \bar{c} + s\right),$$
$$F_3^{W^+} = 2x\left(d - \bar{u} - \bar{c} + s\right),$$

$$\frac{d^2\sigma}{dx\,dQ^2} \propto Y_+ F_2(x, Q^2) \mp Y_- x F_3(x, Q^2) - y^2 F_L(x, Q^2)$$

Longitudinal Structure function

$$F_L(x, Q^2) = \frac{\alpha_s(Q^2)}{\pi}\left[\frac{4}{3}\int_x^1 \frac{dy}{y}\left(\frac{x}{y}\right)^2 F_2(y, Q^2) + 2\sum_i e_i^2 \int_x^1 \frac{dy}{y}\left(\frac{x}{y}\right)^2\left(1 - \frac{x}{y}\right) g(y, Q^2)\right]$$

Altarelli, Martiinelli Phys. Lett. 76B (1978)

HERA data

Neutral Current

$$\left[F_2^{\gamma}, F_2^{\gamma Z}, F_2^{Z}\right] = x \sum_{i=1}^{n_f} \left[e_i^2, 2e_i g_V^i, (g_V^i)^2 + (g_A^i)^2\right] (q_i + \bar{q}_i)$$

③ ④

$$\left[F_3^{\gamma}, F_3^{\gamma Z}, F_3^{Z}\right] = x \sum_{i=1}^{n_f} \left[0, 2e_i g_A^i, 2g_V^i g_A^i\right] (q_i - \bar{q}_i)$$

Charged Current

①

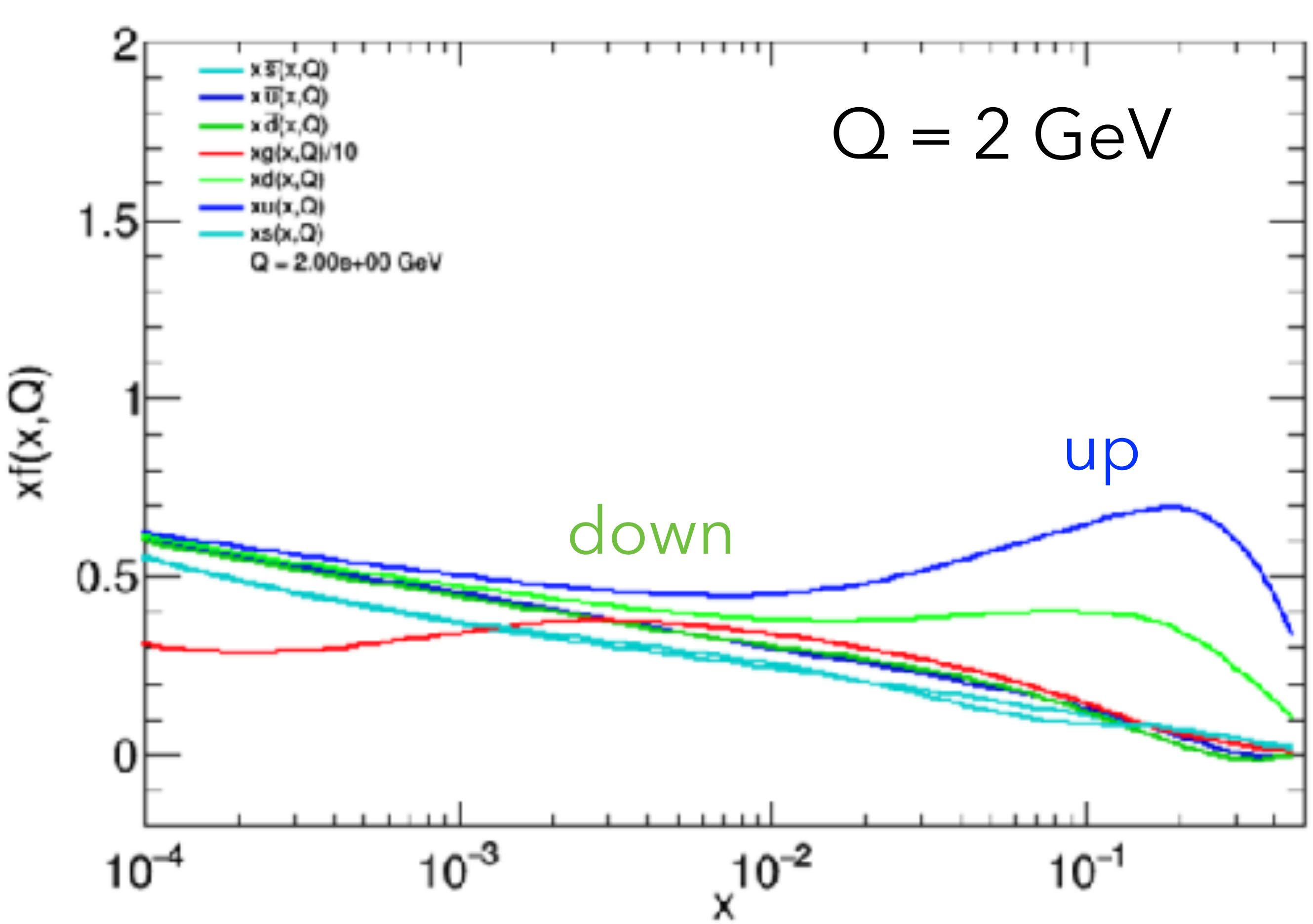
$$F_2^{W^-} = 2x\left(u + \bar{d} + \bar{s} + c\right),$$

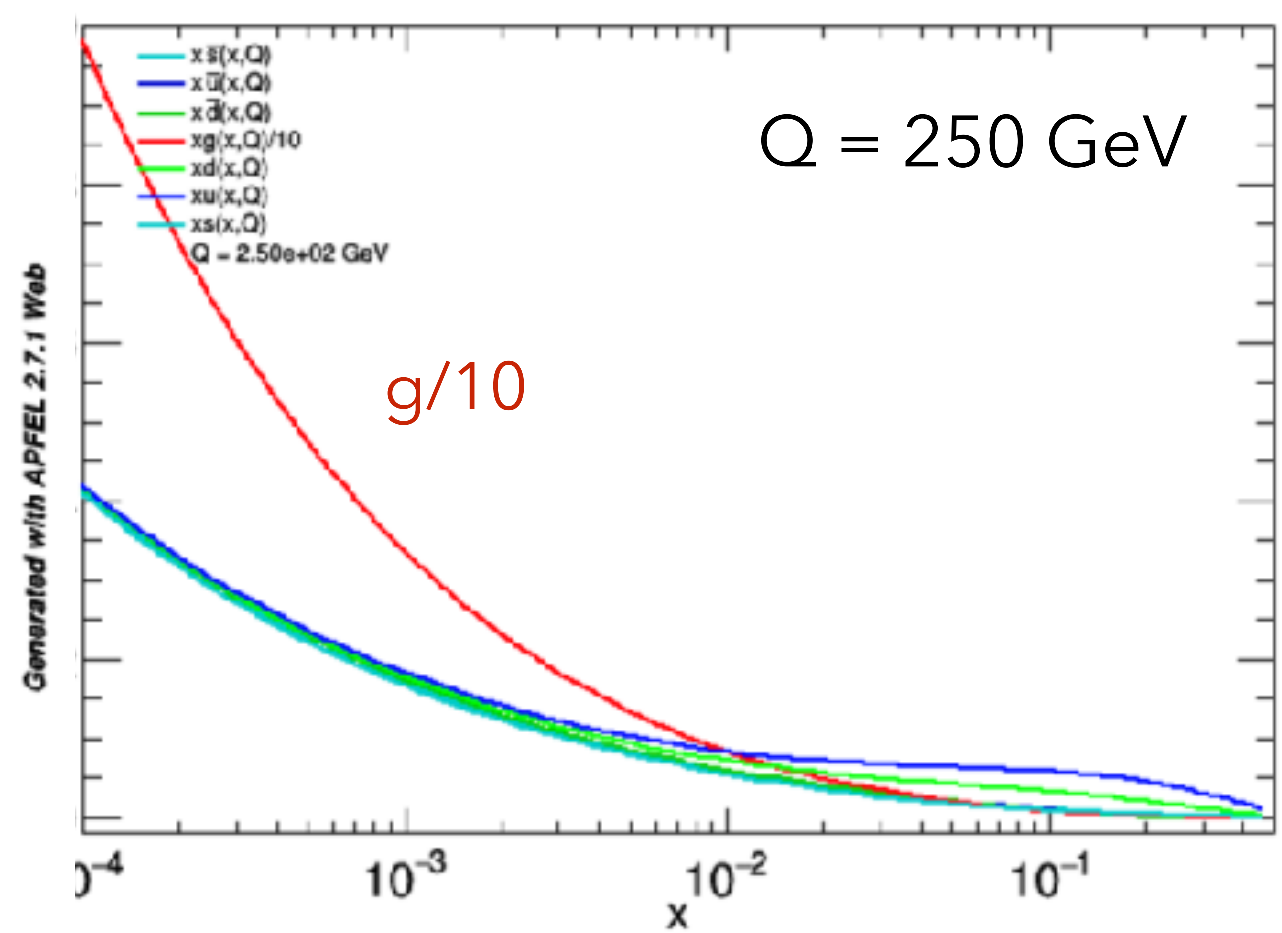
$$F_3^{W^-} = 2x\left(u - \bar{d} - \bar{s} + c\right),$$

②

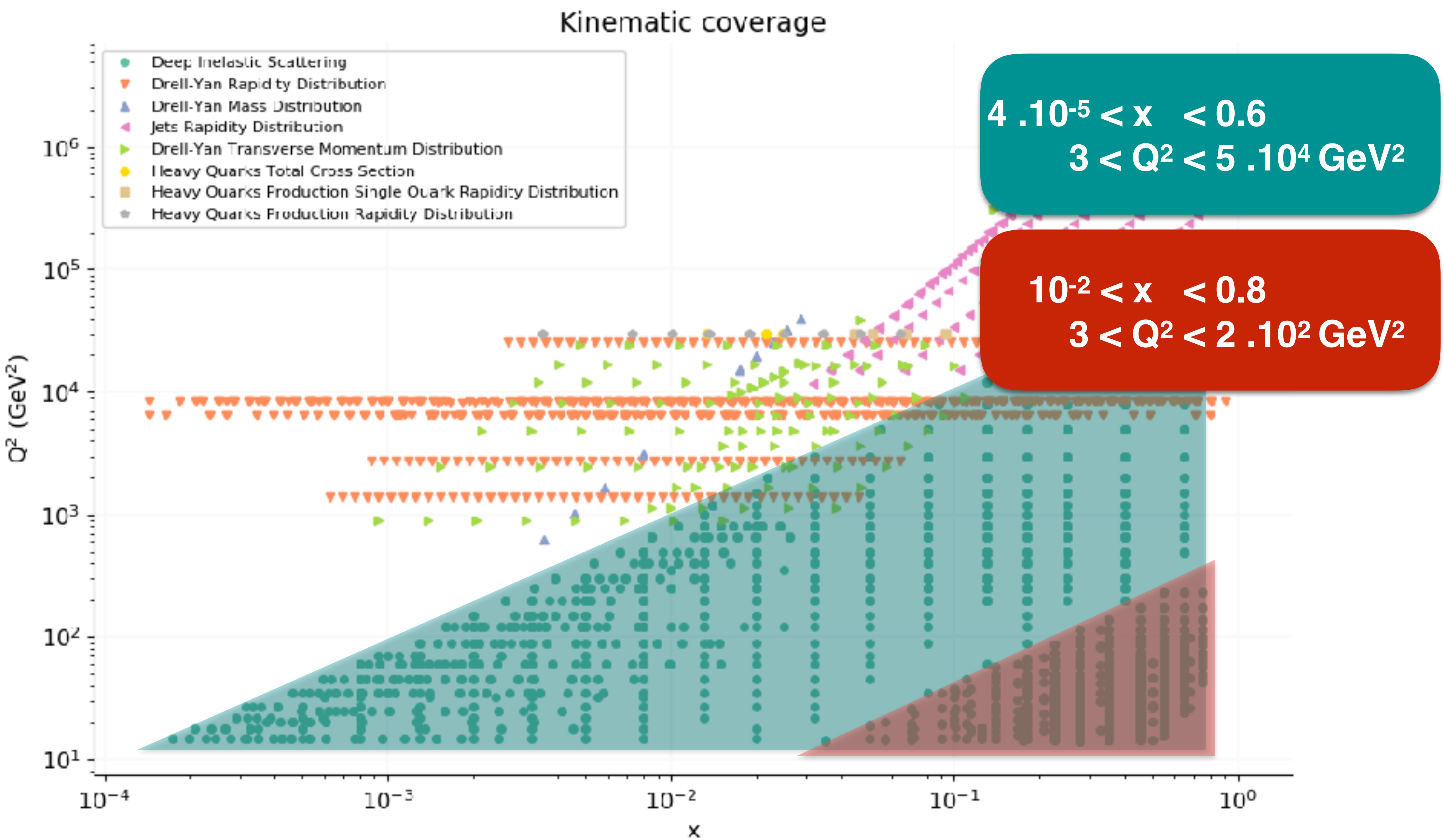
$$F_2^{W^+} = 2x\left(d + \bar{u} + \bar{c} + s\right),$$

$$F_3^{W^+} = 2x\left(d - \bar{u} - \bar{c} + s\right),$$





Fixed target DIS data



Fixed target DIS data

- Assumption (SU(2) isospin): neutron is just like proton with u ⇔ d

  proton = uud
  neutron = ddu

  ⇒ **$u_n(x)=d_p(x)$** and **$d_n(x) = u_p(x)$**

- Linear combinations of F_2^p and F_2^n give separately $u_p(x) \equiv u(x)$ and $d_p(x) \equiv d(x)$,
- Experimentally measured is deuteron structure function

$$F_2^d = (F_2^p + F_2^n)/2$$

$$F_2^p(x,Q^2) - F_2^d(x,Q^2) = \frac{1}{3}(u + \bar{u} - d - \bar{d})$$

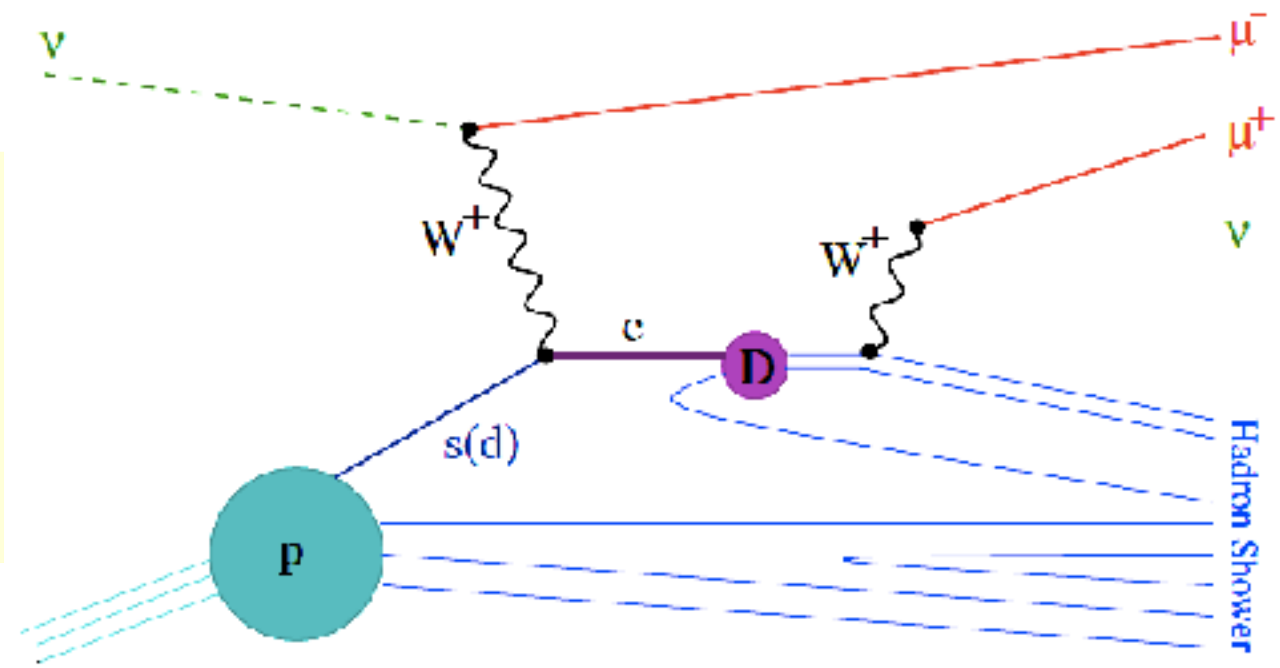
$$\frac{F_2^d(x)}{F_2^p(x)} \sim \frac{u}{d}$$

Fixed target DIS neutrino

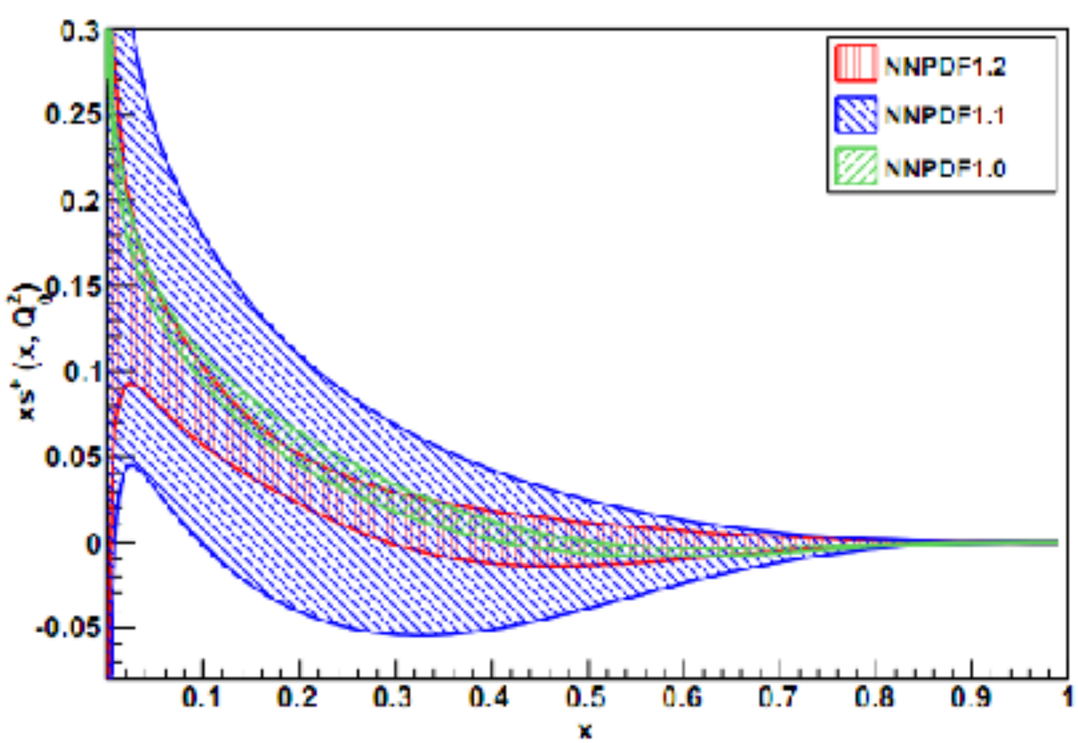
$$\tilde{\sigma}^{\nu(\bar{\nu}),c} \propto \left(F_2^{\nu(\bar{\nu}),c}, F_3^{\nu(\bar{\nu}),c}, F_L^{\nu(\bar{\nu}),c}\right)$$

$$F_2^{\nu,c} = x\left[C_{2,q} \otimes 2|V_{cs}|^2 s + \frac{1}{n_f} C_{2,g} \otimes g\right]$$

$$F_2^{\bar{\nu},c} = x\left[C_{2,q} \otimes 2|V_{cs}|^2 \bar{s} + \frac{1}{n_f} C_{2,g} \otimes g\right]$$



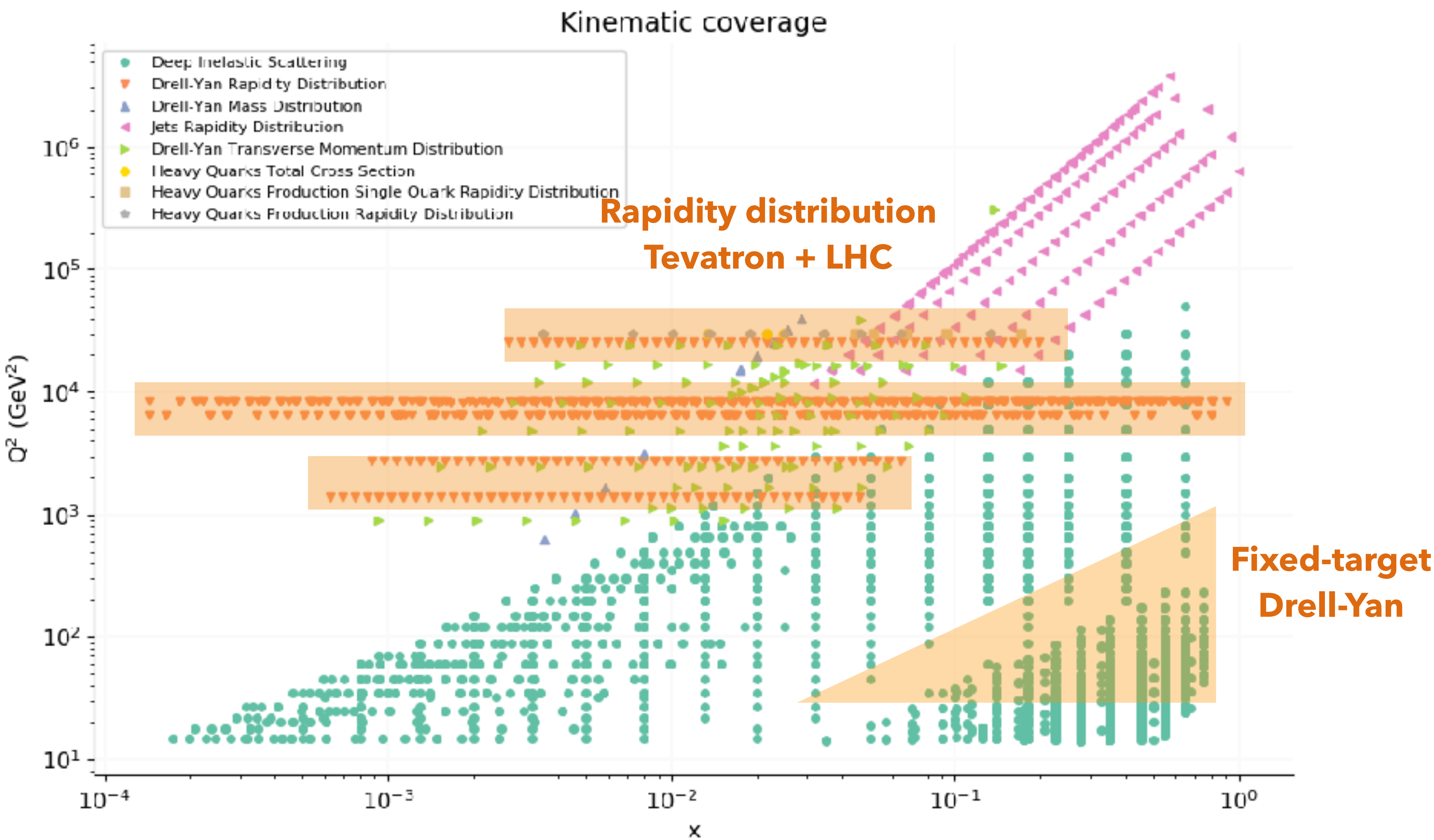
V_{cs} enhancement



$$x = \frac{Q^2}{2 M_n E_\nu y}$$

- Old NuTeV data still provide main constraints on strangeness inside the proton
- Some (mild) tension between fixed target data and W+c data at the LHC

Drell-Yan/V production data



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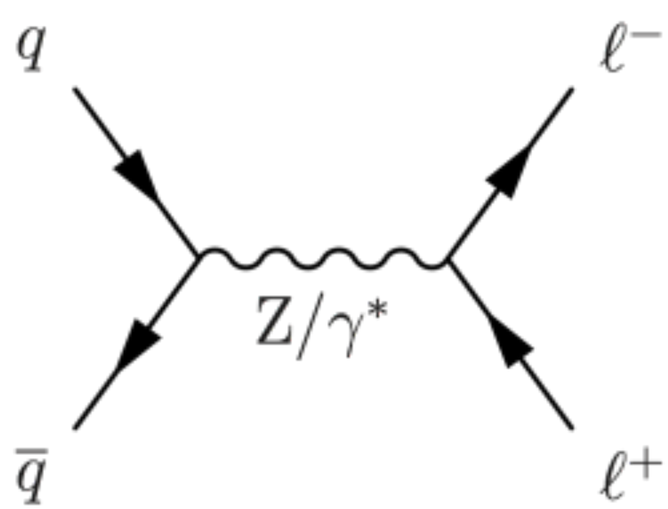
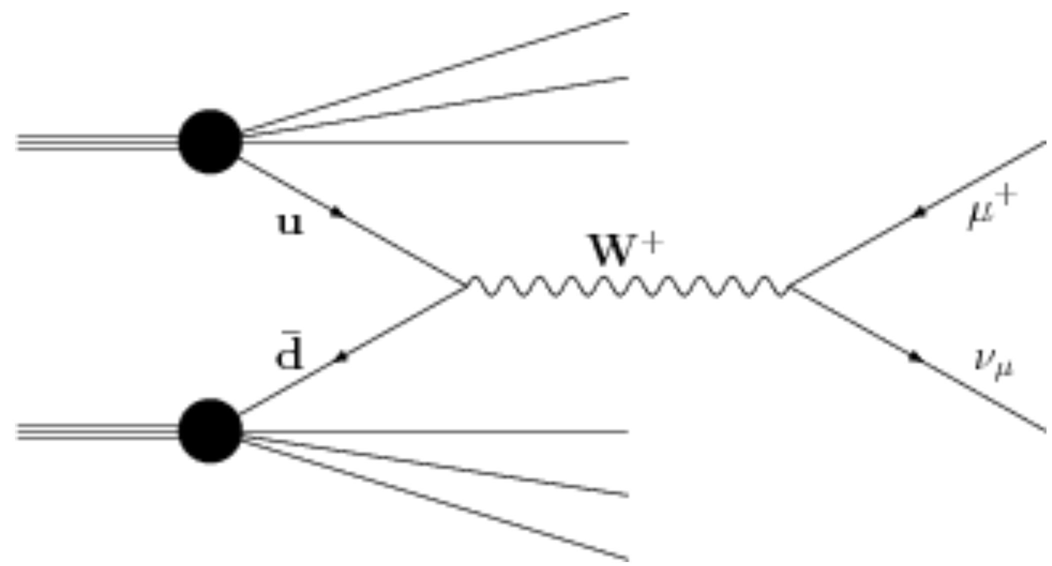




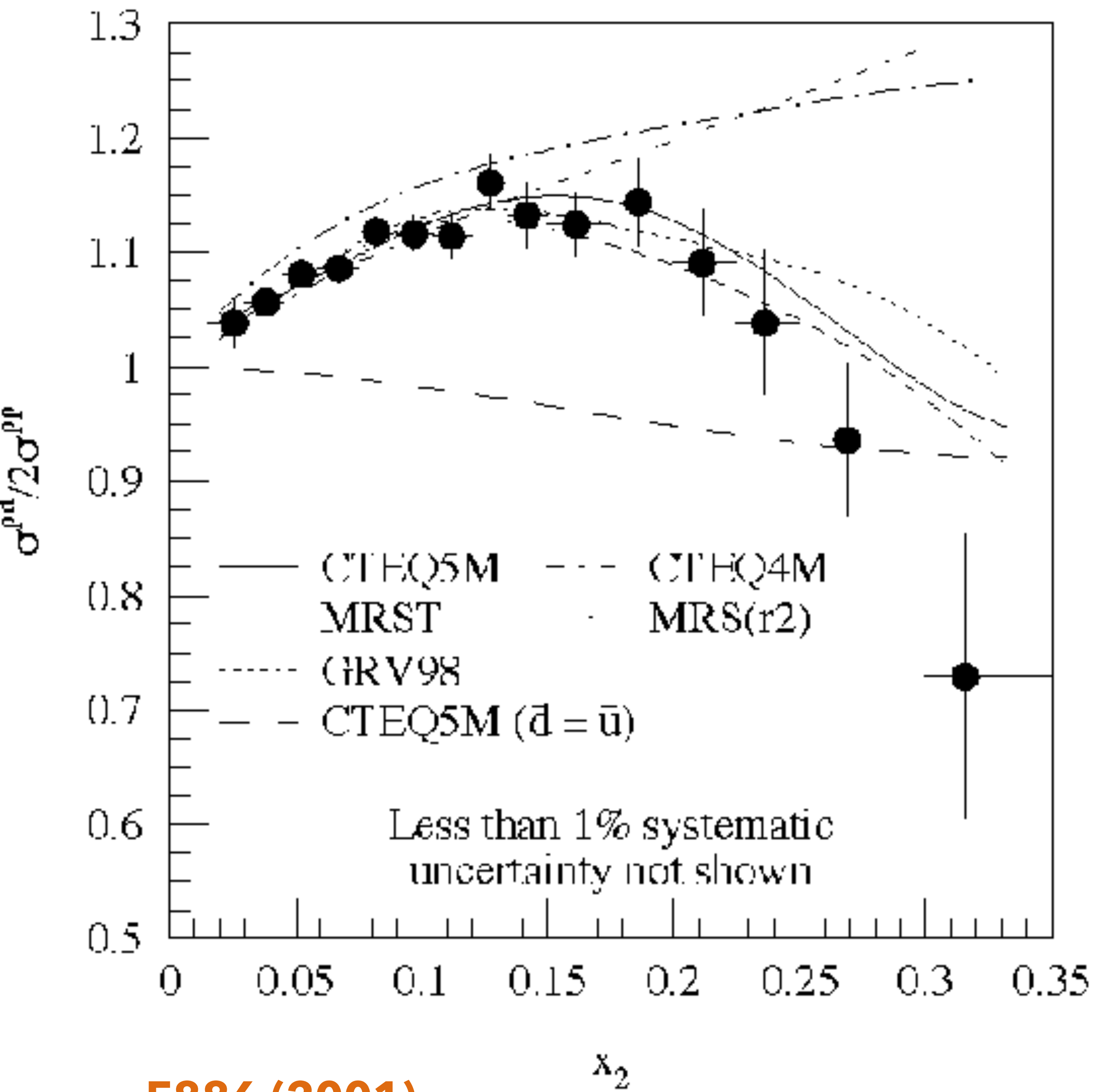
$$L_{ij}(x_1, x_2) = q_i(x_1)\bar{q}_j(x_2)$$

$$\gamma^* : \quad \frac{d\sigma}{dydM^2} = \frac{4\pi\alpha^2}{9M^2 S} \sum_i e_i^2 L_{ij}(x_1, x_2)$$

$$Z : \quad \frac{d\sigma}{dy} = \frac{\pi G_F M_V^2 \sqrt{2}}{3S} \sum_i (v_{iZ}^2 + a_{iZ}^2) L_{ij}(x_1, x_2)$$

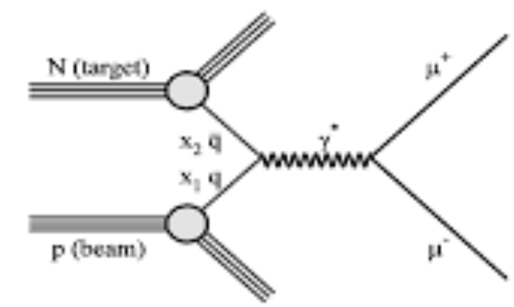
$$W : \quad \frac{d\sigma}{dydM^2} = \frac{\pi G_F M_V^2 \sqrt{2}}{3S} \sum_{ij} |V_{ij}^{\mathrm{CKM}}|^2 L_{ij}(x_1, x_2)$$

Drell-Yan data



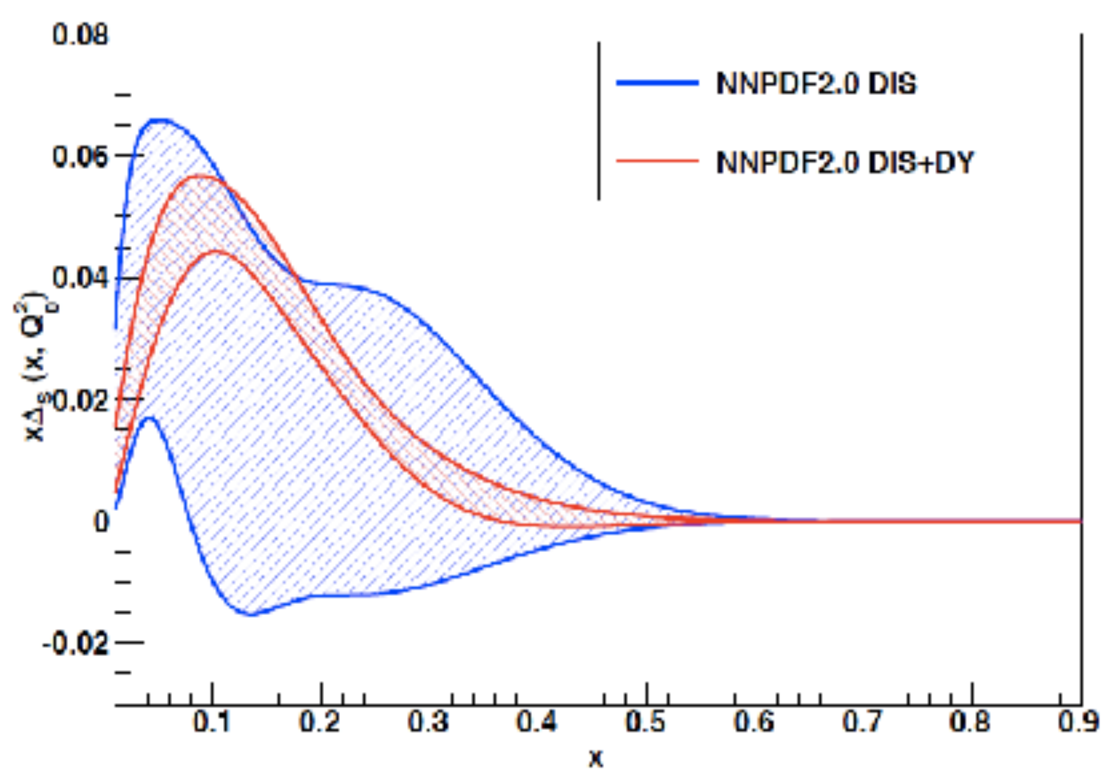
E886 (2001)

The Drell-Yan Process: $pN \to \mu^+\mu^- X$

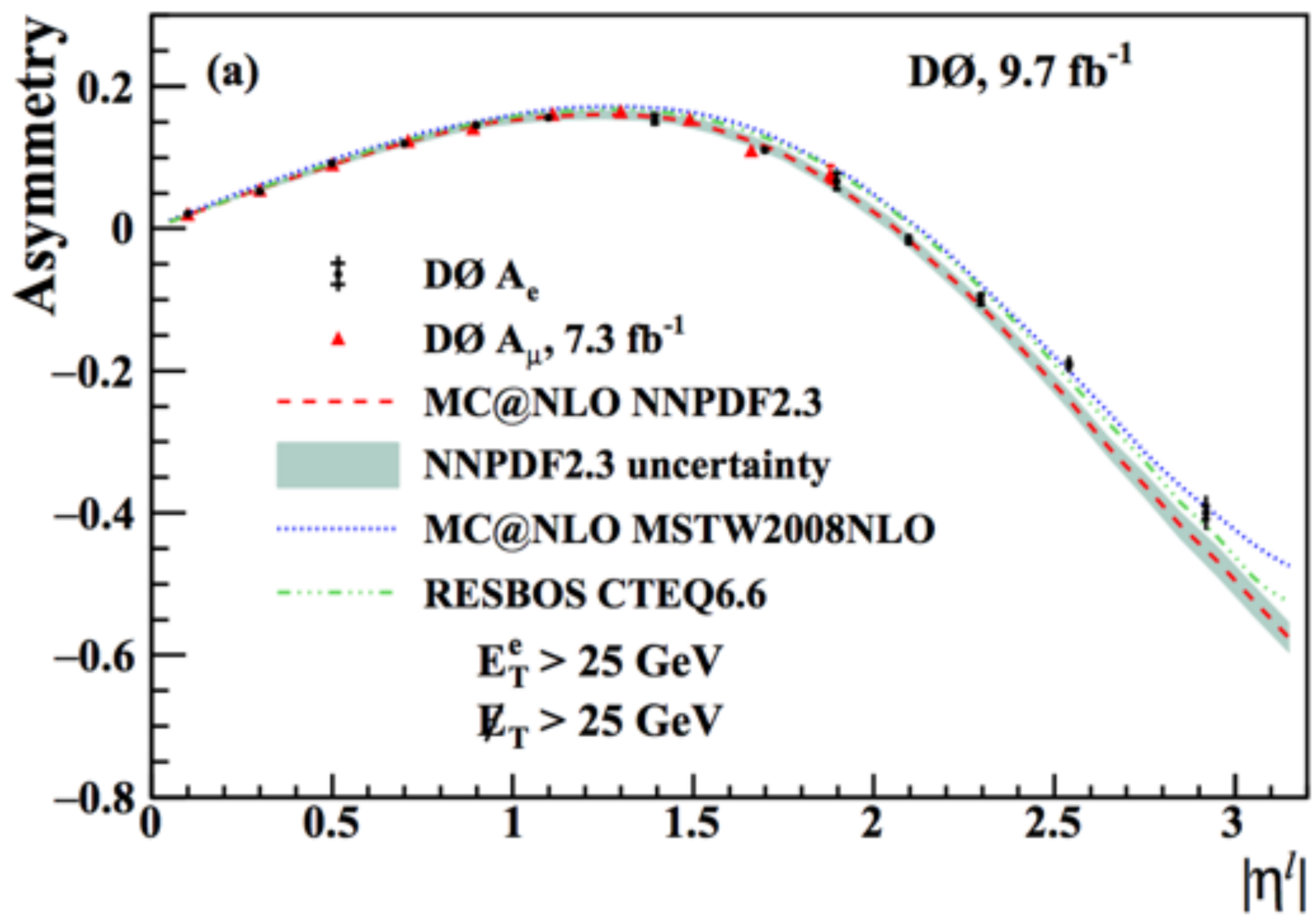


Fixed-target Drell-Yan

$$\frac{\sigma(pd \to \mu^+\mu^-)}{\sigma(pp \to \mu^+\mu^-)} = \frac{\frac{4}{9}u\bar{d} + \frac{1}{9}d\bar{u}}{\frac{4}{9}u\bar{u} + \frac{1}{9}d\bar{d}} \sim \frac{\bar{d}}{\bar{u}}$$



Z/W production data



W asymmetry at Tevatron

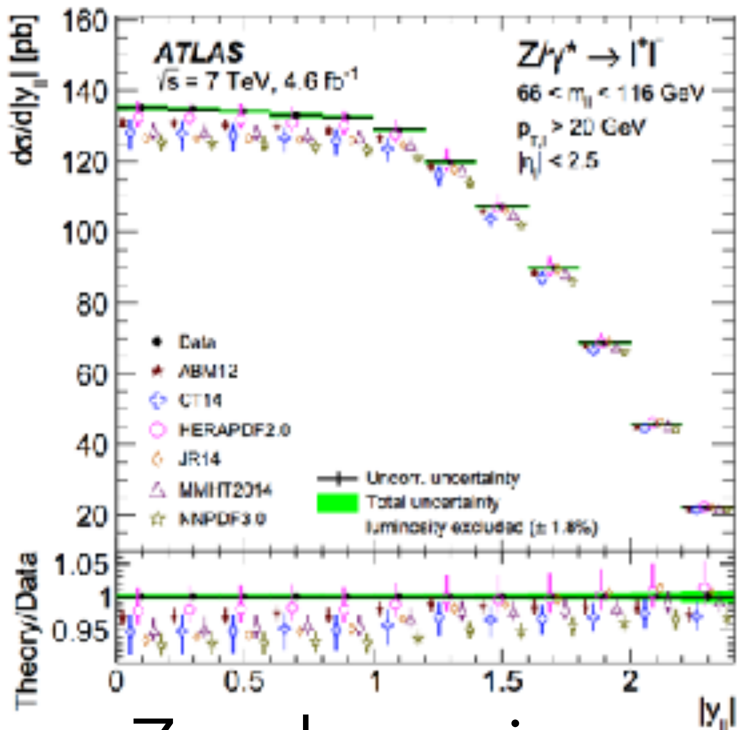
$$u^{\bar{p}} = \bar{u}^{p}$$
$$d^{\bar{p}} = \bar{d}^{p}$$

Charge conjugation

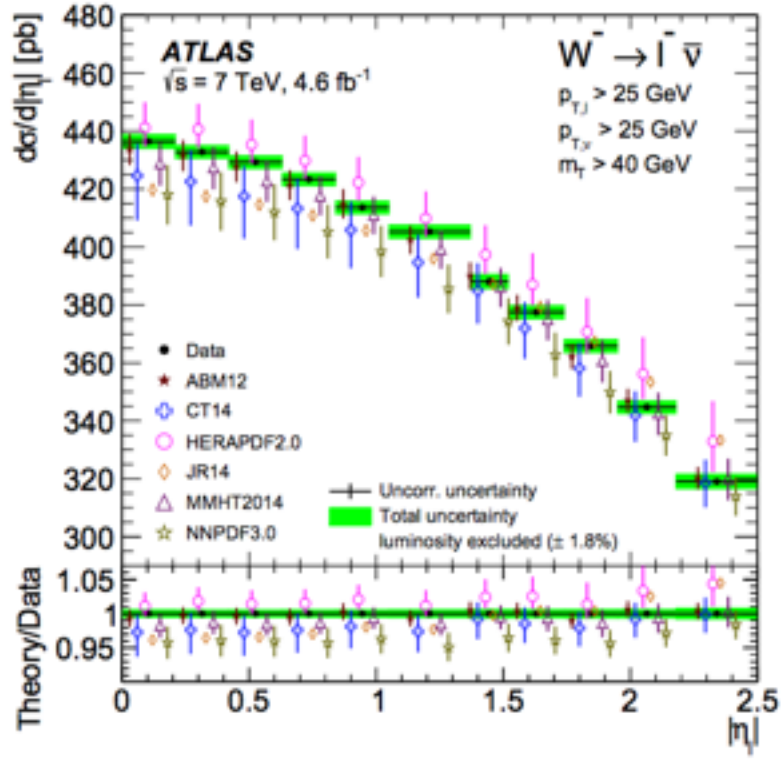
$$\frac{\sigma(p\bar{p} \to W^+)}{\sigma(p\bar{p} \to W^-)} = \frac{u(x_1)d(x_2) + \bar{u}(x_1)\bar{d}(x_2)}{d(x_1)u(x_2) + \bar{d}(x_1)\bar{u}(x_2)} \sim \frac{u}{d}(x_1)\frac{u}{d}(x_2)$$

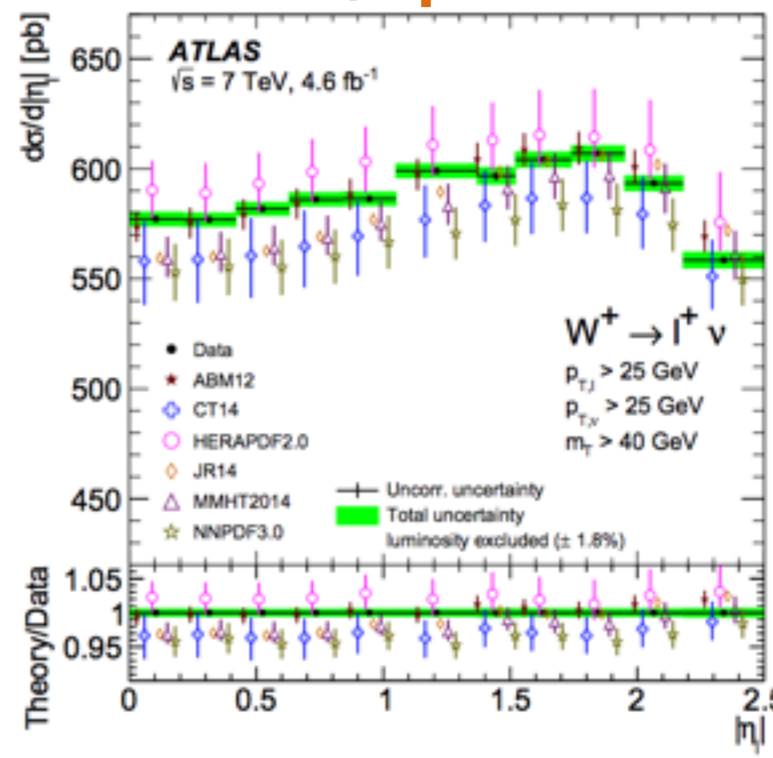
Z/W production data

ATLAS W,Z production 7 TeV



Z pole region





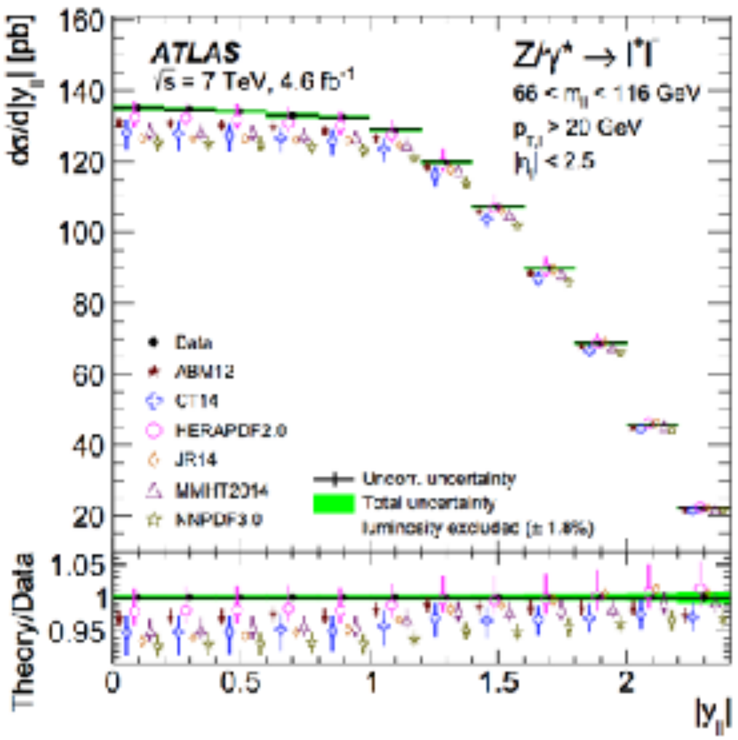
$$\sigma(pp \to Z) = u\bar{u} + d\bar{d} + s\bar{s}$$

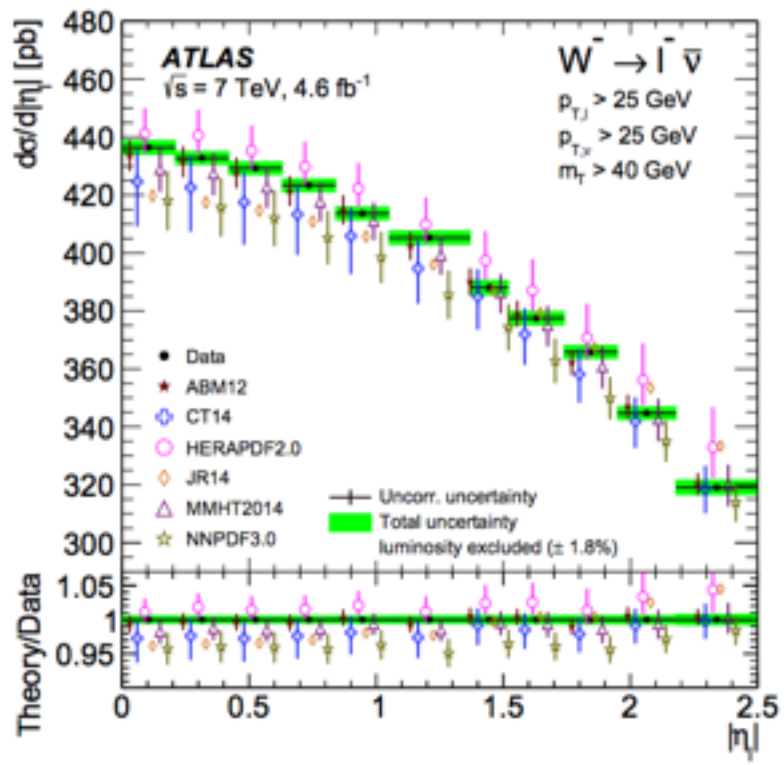
$$\sigma(pp \to W^+) = u\bar{d} + c\bar{s}$$

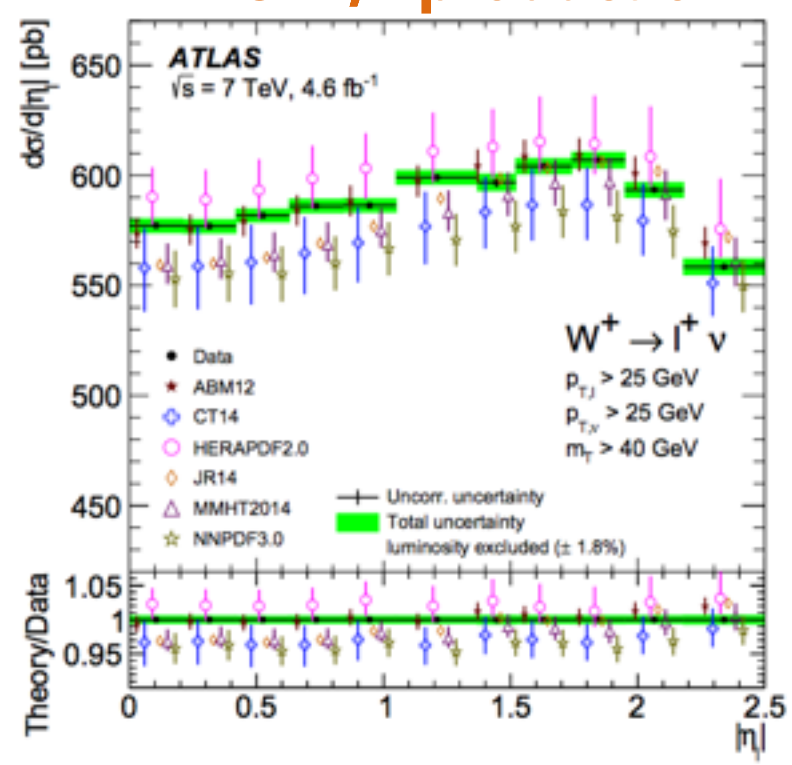
$$\sigma(pp \to W^-) = d\bar{u} + s\bar{c}$$

Z/W production data

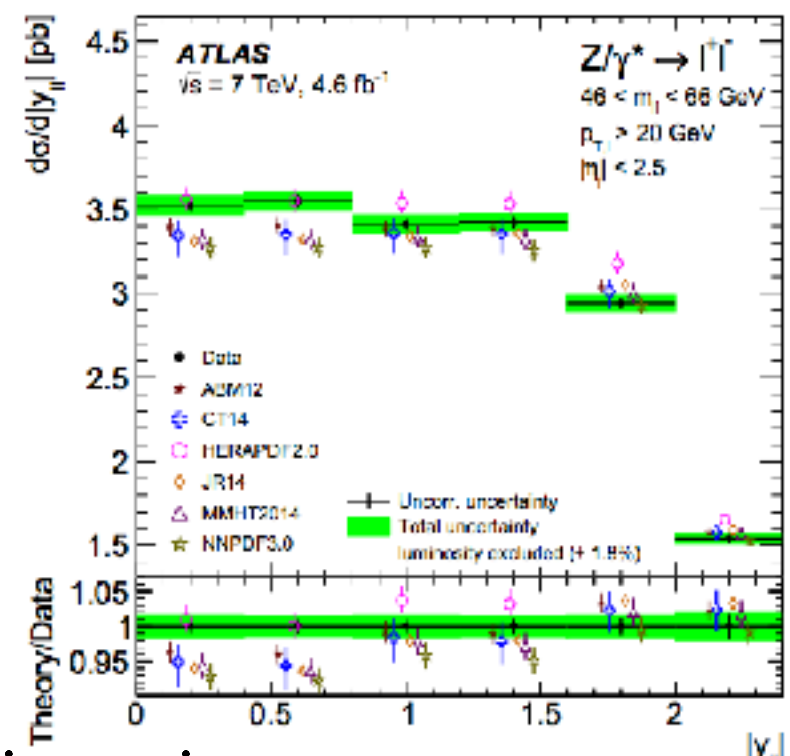
ATLAS W,Z production 7 TeV

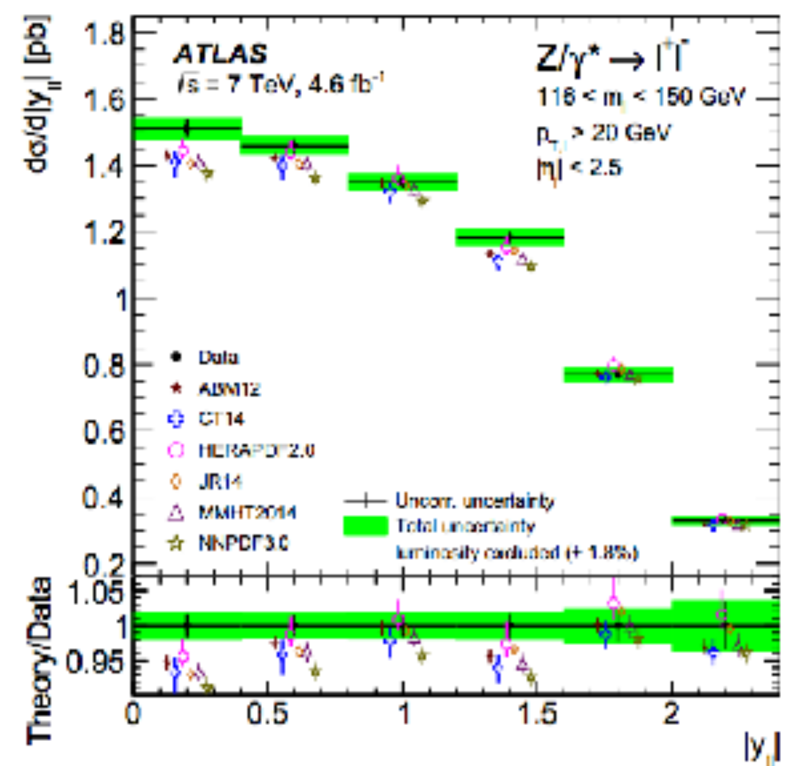






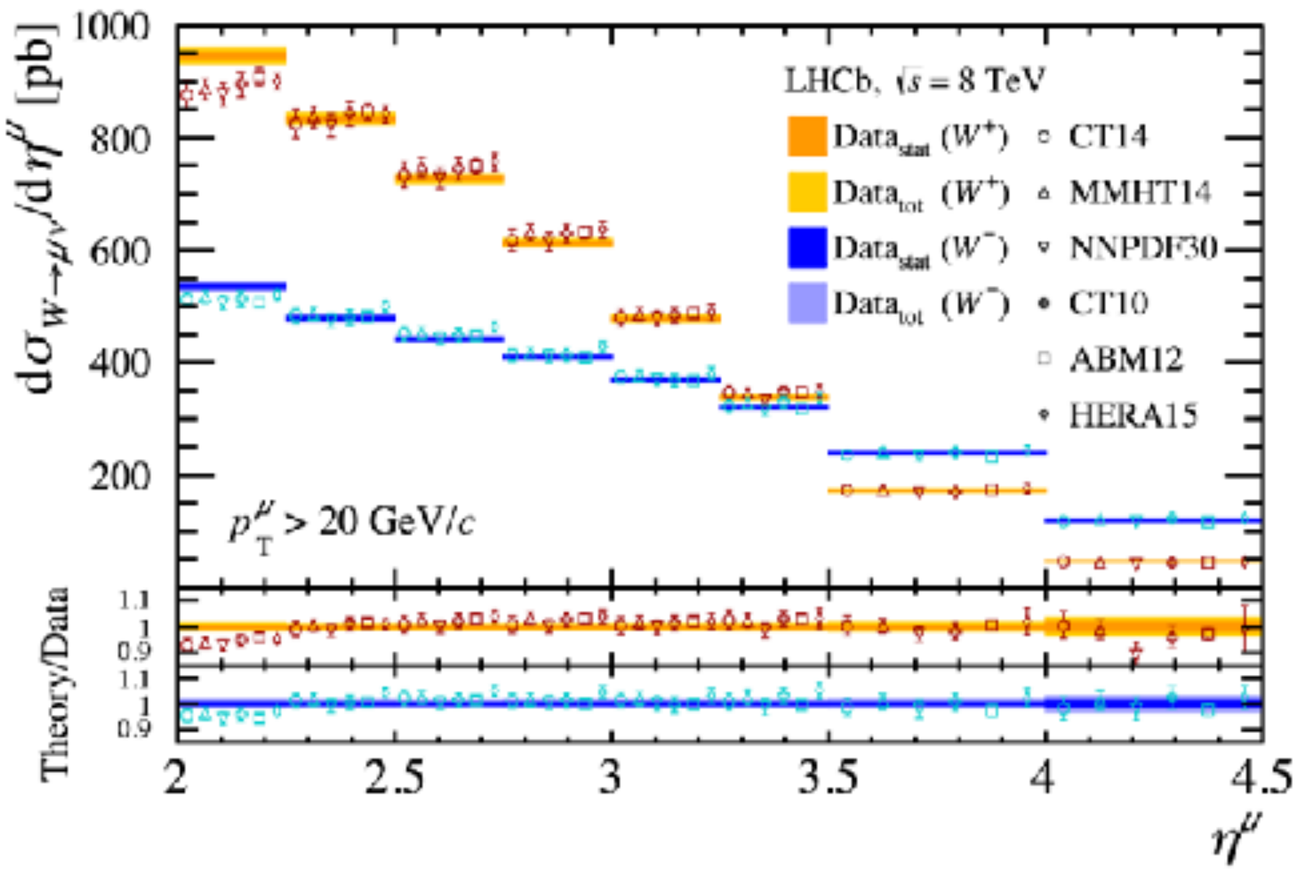
$$x_{1,2} = \frac{M^2}{s} e^{\pm y}$$

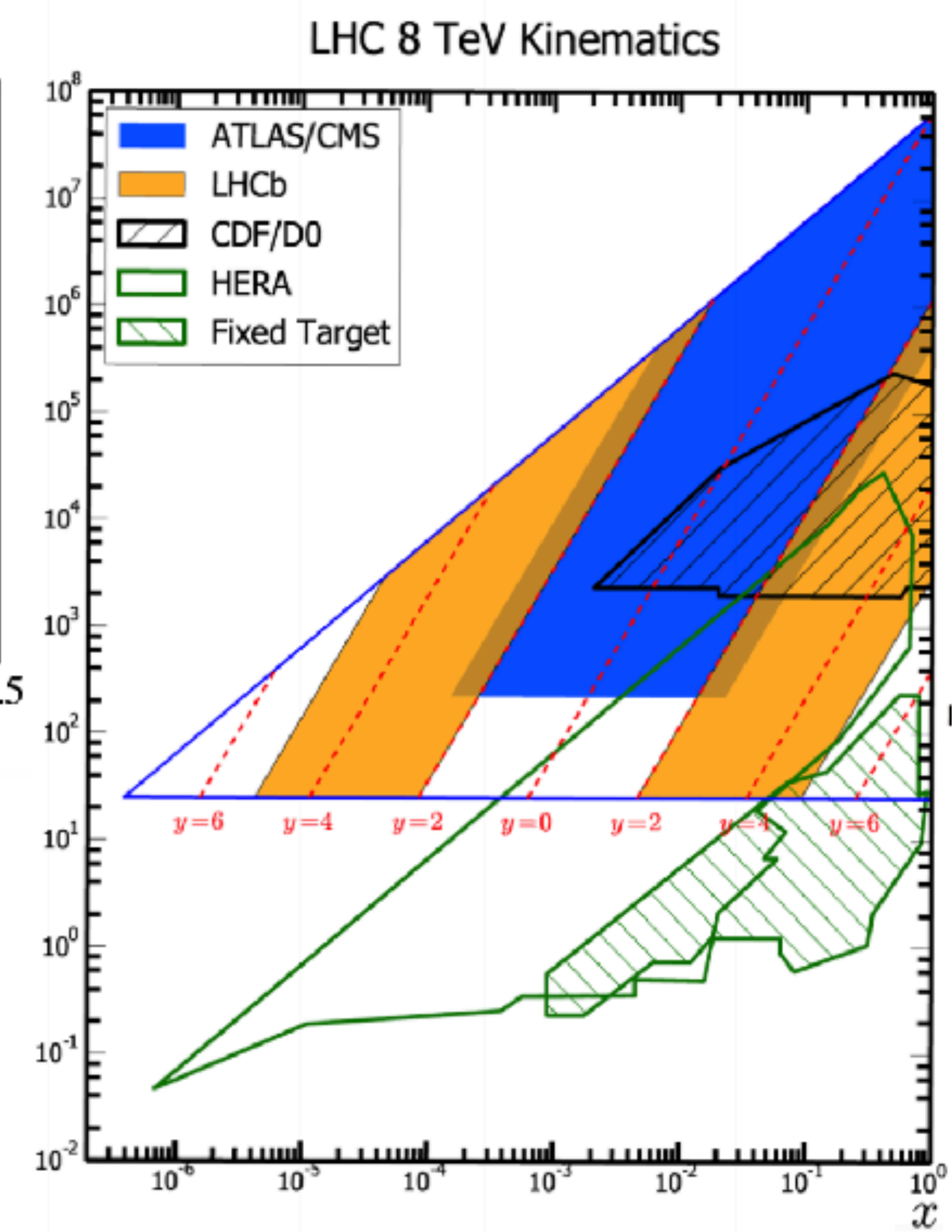




high/low invariant mass

Z/W production data

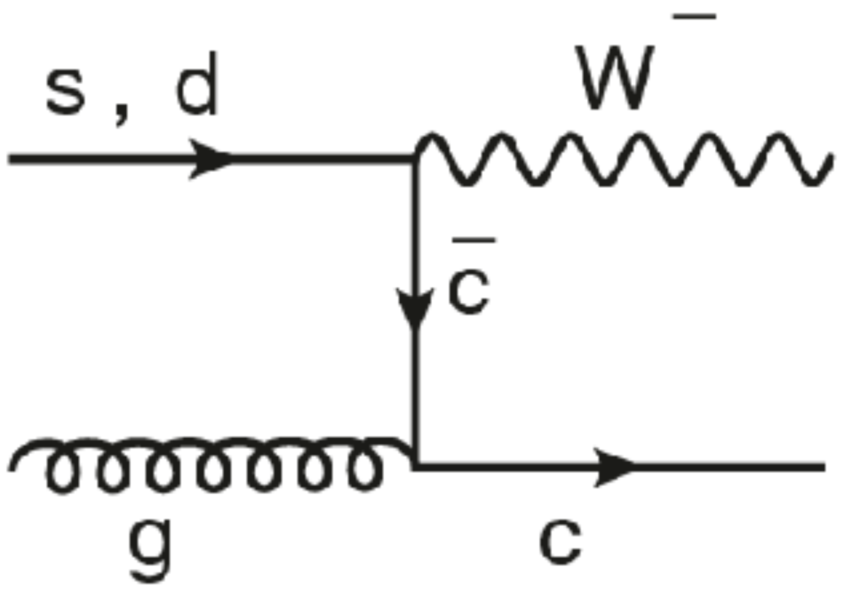


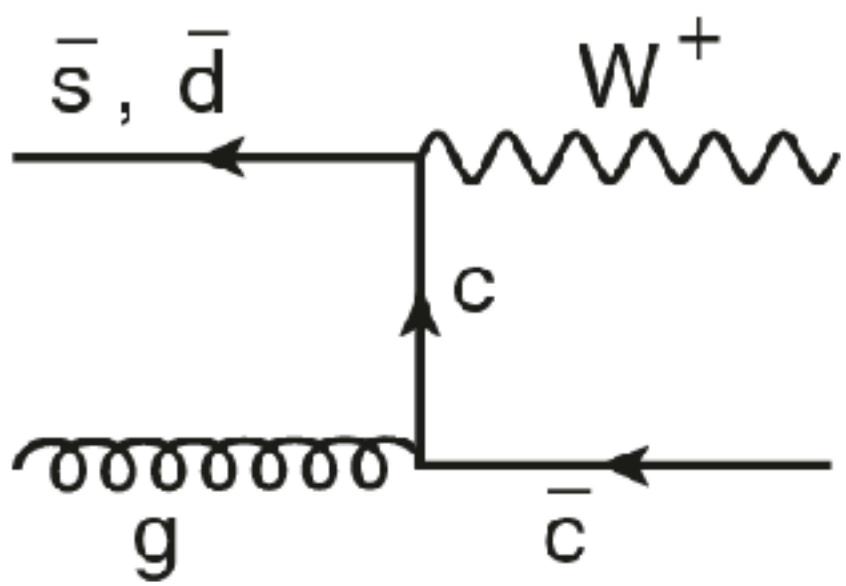


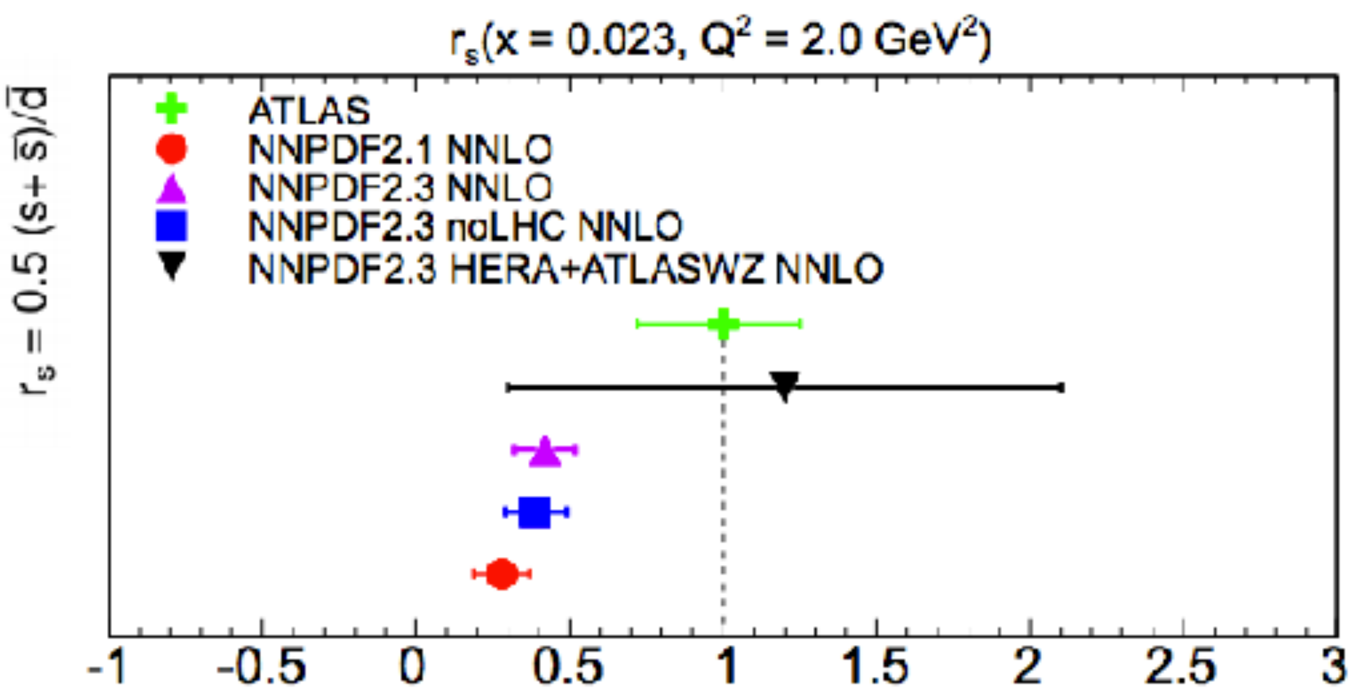
$$x_{1,2} = \frac{M^2}{s} e^{\pm y}$$

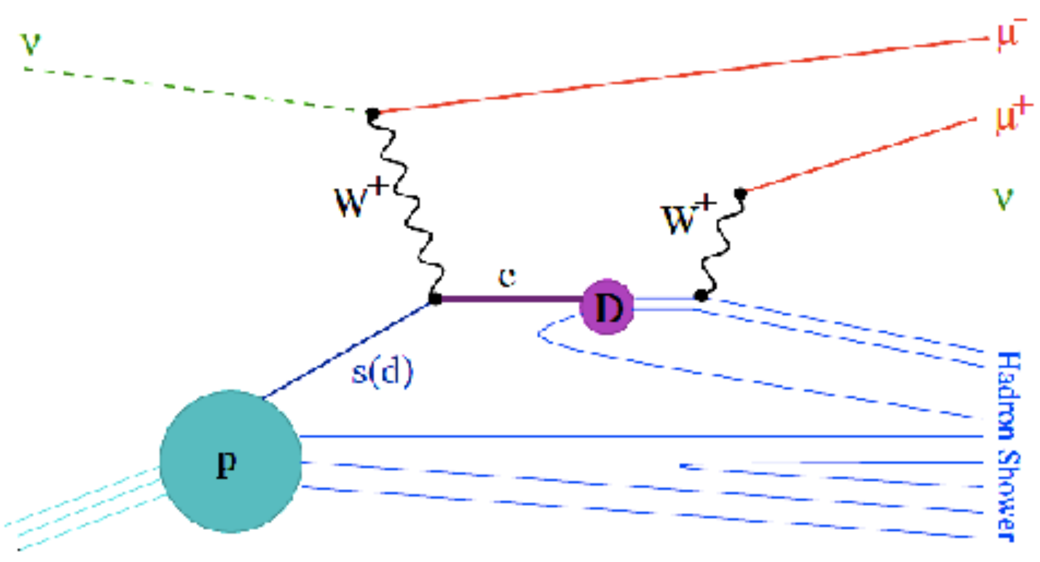
LHCb: forward region

W+charm data


s , d
W −
c̄
g
c


s̄ , d̄
W +
c
g
c̄

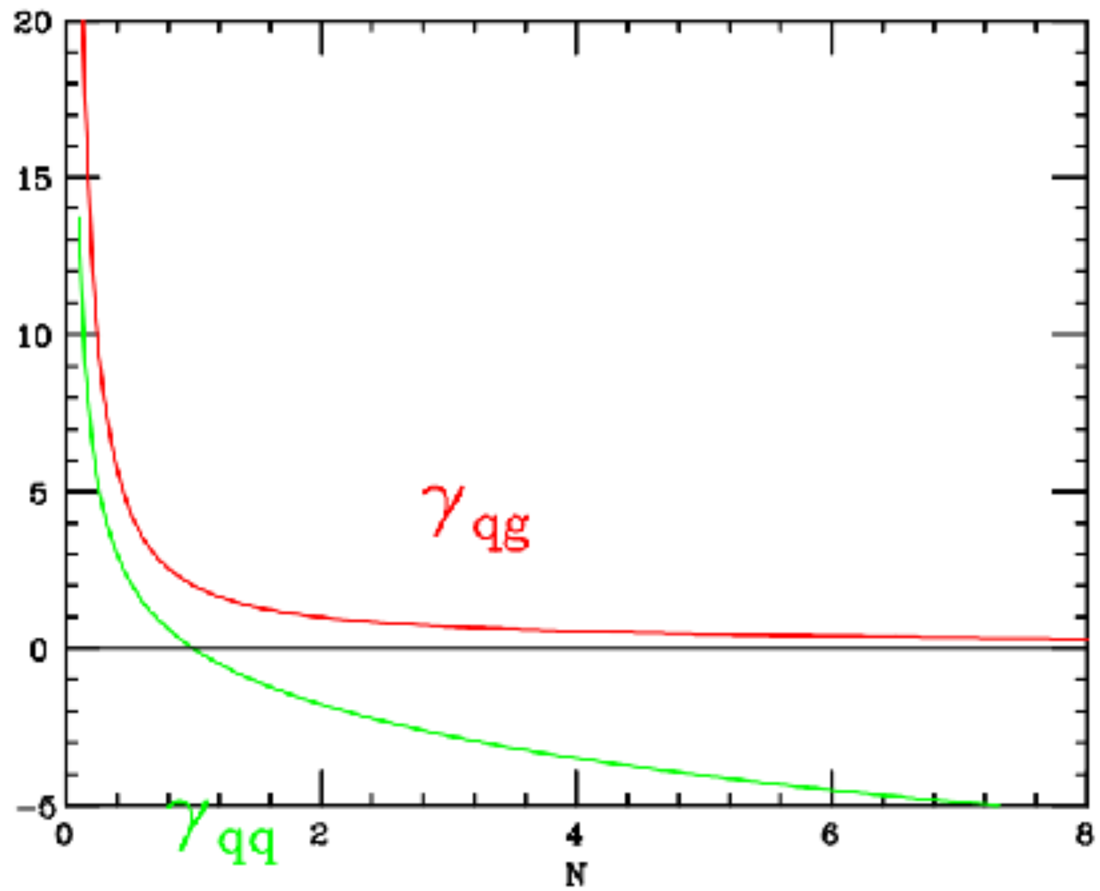

$r_s(x = 0.023, Q^2 = 2.0$ GeV$^2)$
$r_s = 0.5\,(s+\bar{s})/\bar{d}$
ATLAS
NNPDF2.1 NNLO
NNPDF2.3 NNLO
NNPDF2.3 noLHC NNLO
NNPDF2.3 HERA+ATLASWZ NNLO
-1
-0.5
0
0.5
1
1.5
2
2.5
3


ν
μ⁻
μ⁺
ν
W⁺
W⁺
c
D
s(d)
p
Hadron Shower

Gluon: indirect handle

- Gluon is partially determined by scale dependence of DIS structure functions and Drell-Yan/Vector Boson production

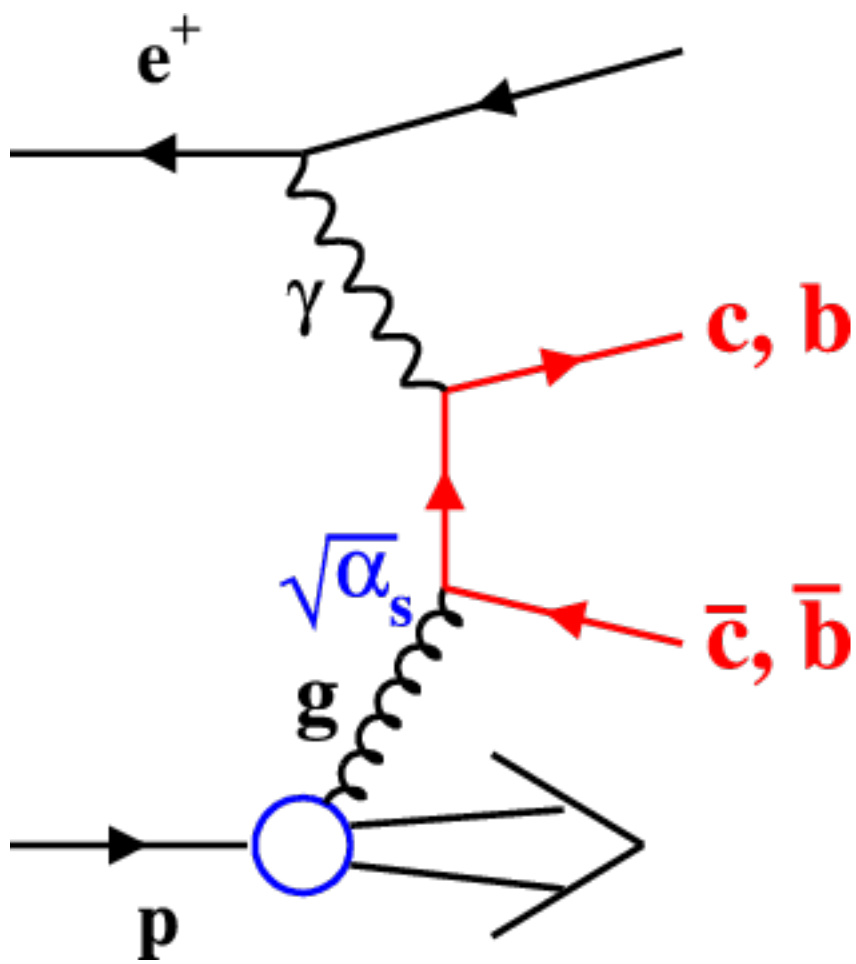
$$\frac{d}{d\log\mu^2}F_2(x,\mu^2) = \frac{\alpha_s(\mu^2)}{2\pi}\left[P_{qq}\otimes F_2(x,\mu^2) + 2n_f P_{qg}\otimes g(x,\mu^2)\right]$$



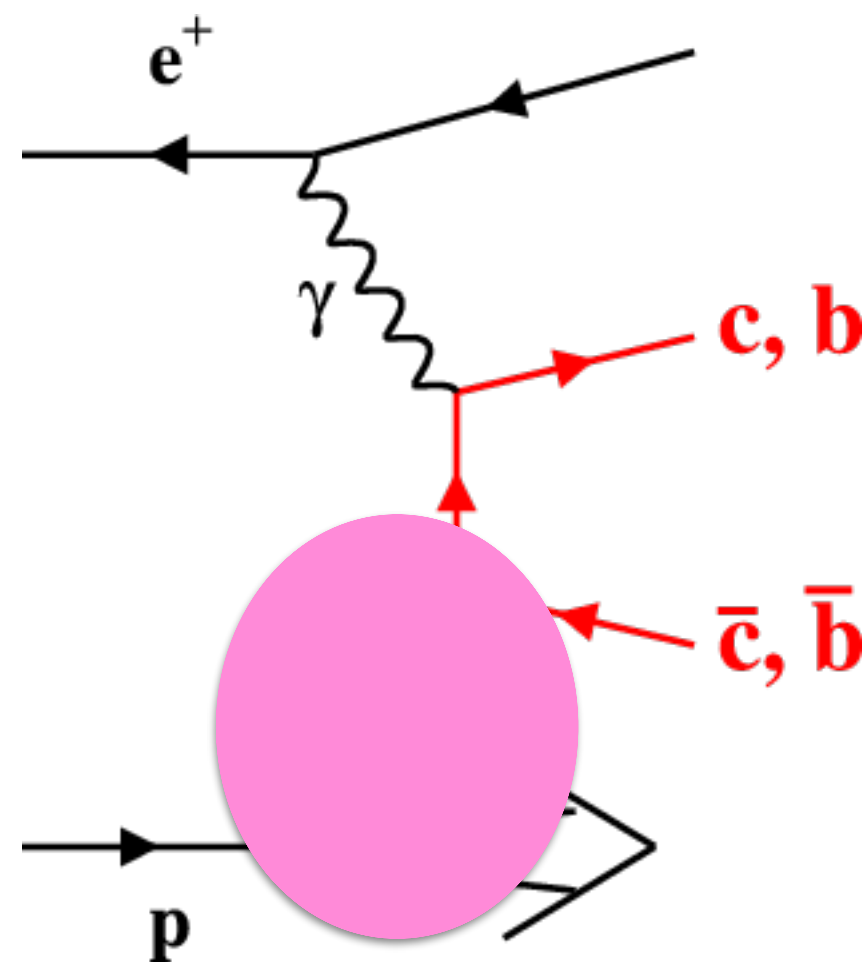
- Mostly determine small-x gluon, large-x gluon hard to determine from DIS+DY only data

Gluon: indirect handle

- Heavy quarks are produced at threshold inside proton
- Heavy quark production process (at ep and pp colliders) probe gluon
- Dependence on heavy flavour scheme adopted in PDF fitting



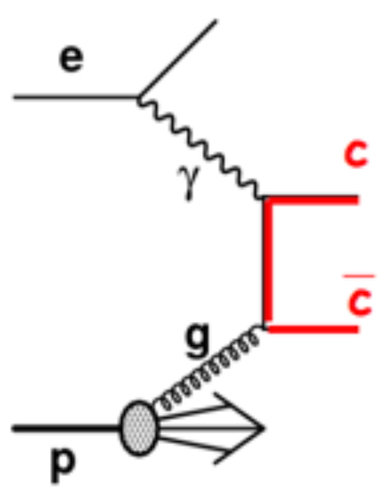
Nf = 3,4

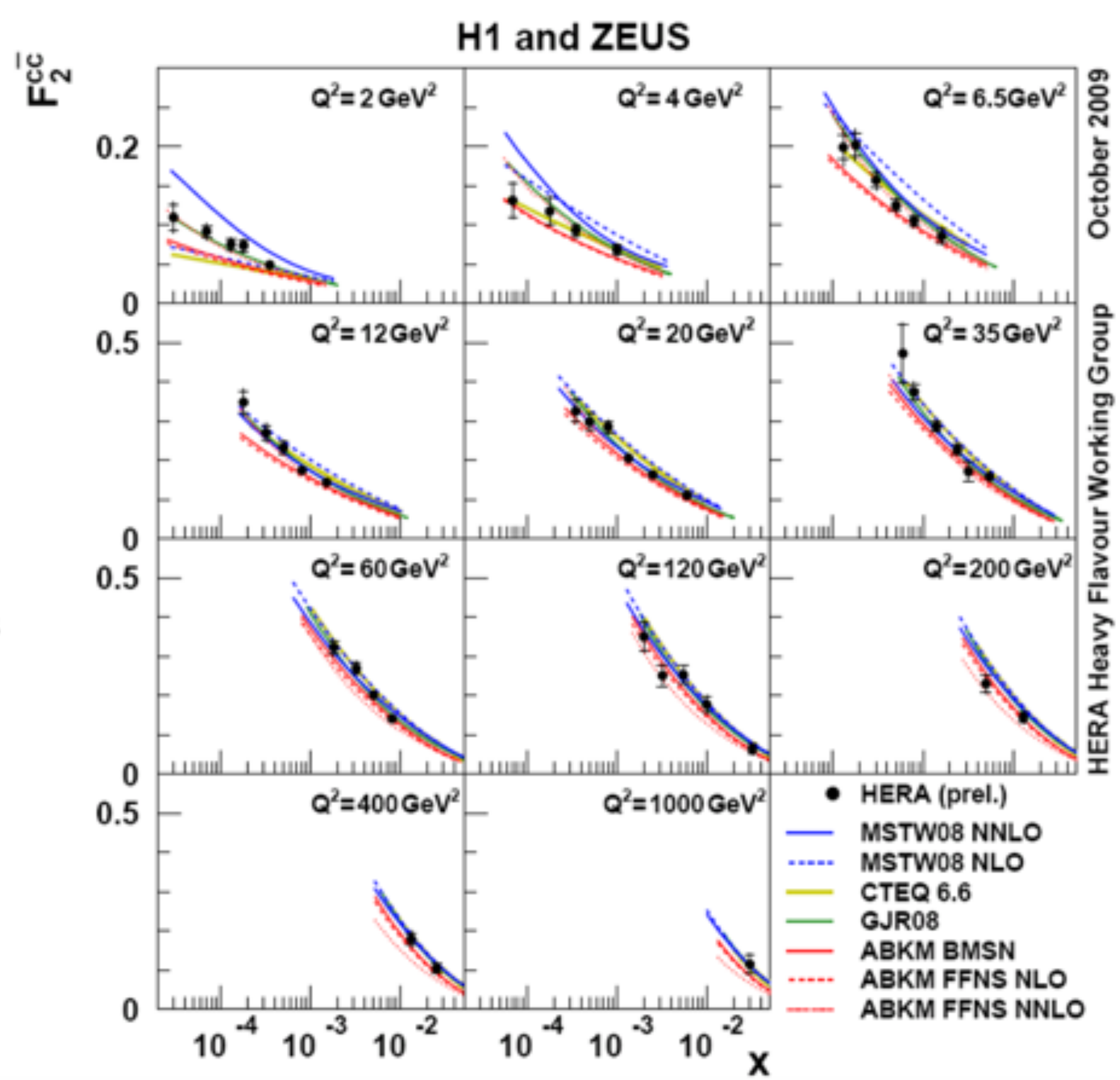


Nf = 5

Gluon: indirect handle

- Heavy quarks are produced at threshold inside proton
- Heavy quark production process (at ep and pp colliders) probe gluon
- Dependence on heavy flavour scheme adopted in PDF fitting





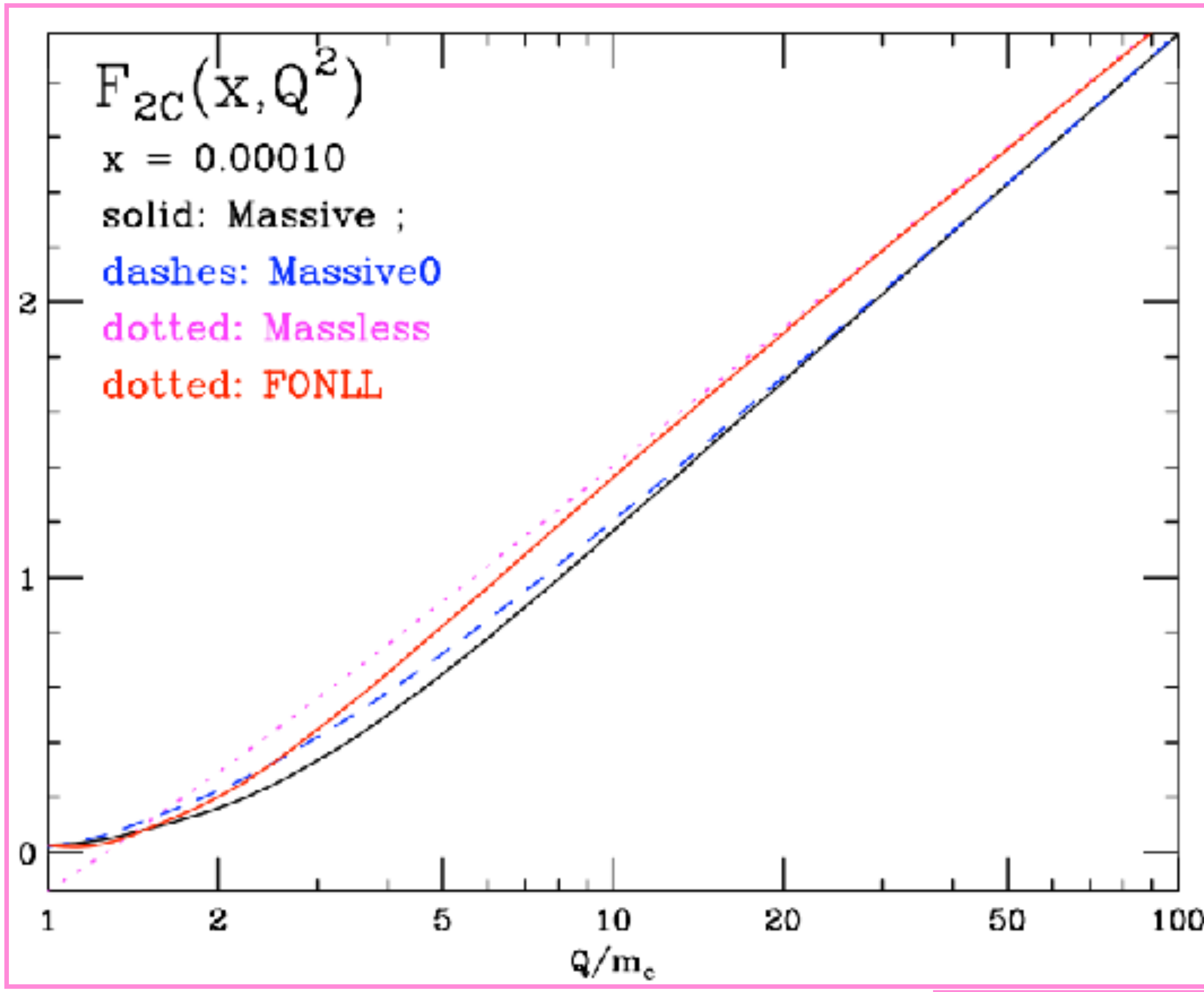
Intermission: heavy flavour schemes

- Charm, Bottom and Top have mass >> Λ_{QCD} - heavy quarks (HQ)
- The presence of a new scale, m_Q, makes pert QCD calculations more challenging
- Two well understood schemes:
    - Assume heavy quark effectively massless for $Q > m_Q$
      HQ becomes active massless parton above threshold
    - Heavy quarks retain their mass for all Q
      HQ is not a parton, it is a final state particle
- However in PDF fits we have all scales. General-Mass Variable-Flavor-Number schemes allow to match between the zero-mass and the massive scheme
- Many schemes available

e.g. FONLL

$$\sigma^{(FONLL)} = \sigma^{(4)} + \sigma^{(5)} - \text{double counting}$$
$$= \mathcal{L}_{ij}(x_1, x_2, \mu^2) \otimes \sum_{p}^{N} \left(\alpha_S^{(5)}(\mu^2)\right)^p$$
$$\times \left\{ \mathcal{B}_{ij}^{(p)}\left(x_1, x_2, \frac{\mu^2}{m_b^2}\right) + \sum_{k=0}^{\infty} \mathcal{A}_{ij}^{(p),(k)}(x_1, x_2)\left(\alpha_S^{(5)}(\mu^2) L\right)^k \right\}$$
$$- \text{double counting}$$

Intermission: heavy flavour schemes



- heavy quarks (HQ)

D calculations more challenging

sless for Q > mQ
above threshold

Q2
article

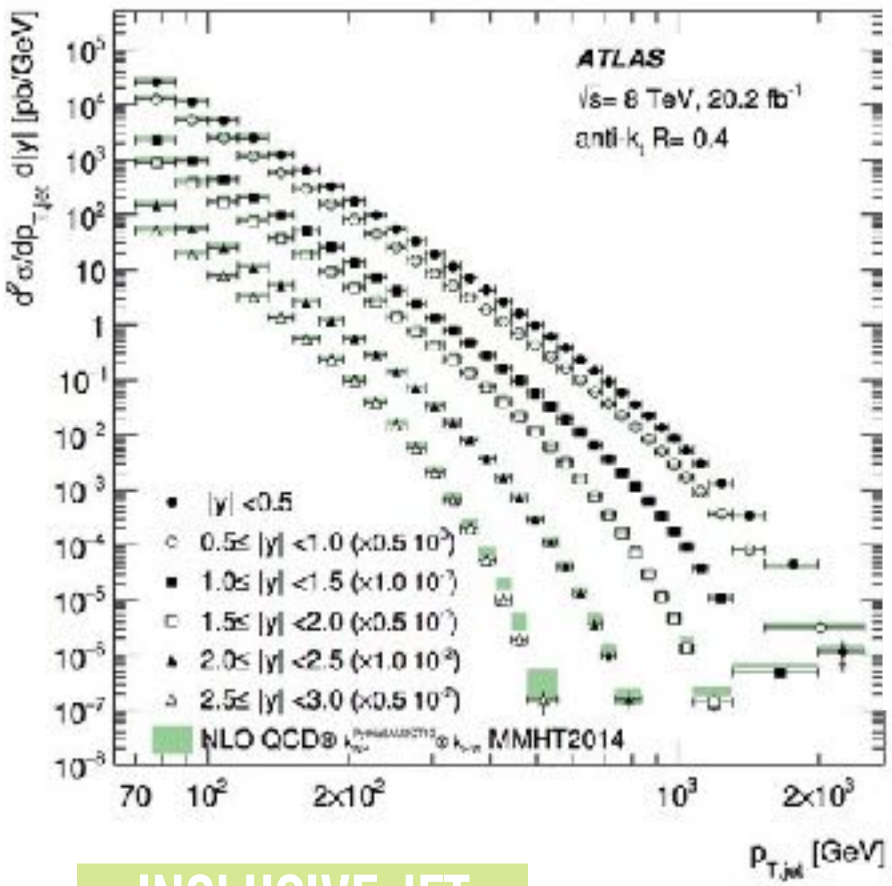
l-Mass Variable-Flavor-Number
and the massive scheme

- Many schemes available

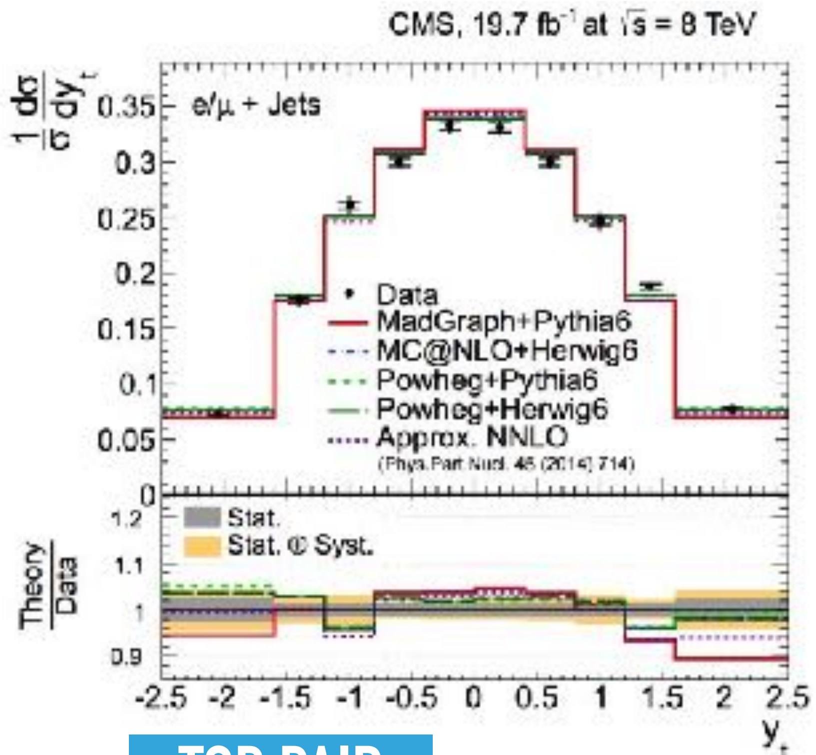
e.g. FONLL

$$\sigma^{(FONLL)} = \sigma^{(4)} + \sigma^{(5)} - \text{double counting}$$
$$= \mathcal{L}_{ij}(x_1, x_2, \mu^2) \otimes \sum_{p}^{N} \left(\alpha_S^{(5)}(\mu^2)\right)^p$$
$$\times \left\{ \mathcal{B}_{ij}^{(p)}\left(x_1, x_2, \frac{\mu^2}{m_b^2}\right) + \sum_{k=0}^{\infty} \mathcal{A}_{ij}^{(p),(k)}(x_1, x_2)\left(\alpha_S^{(5)}(\mu^2) L\right)^k \right\}$$
$$- \text{double counting}$$

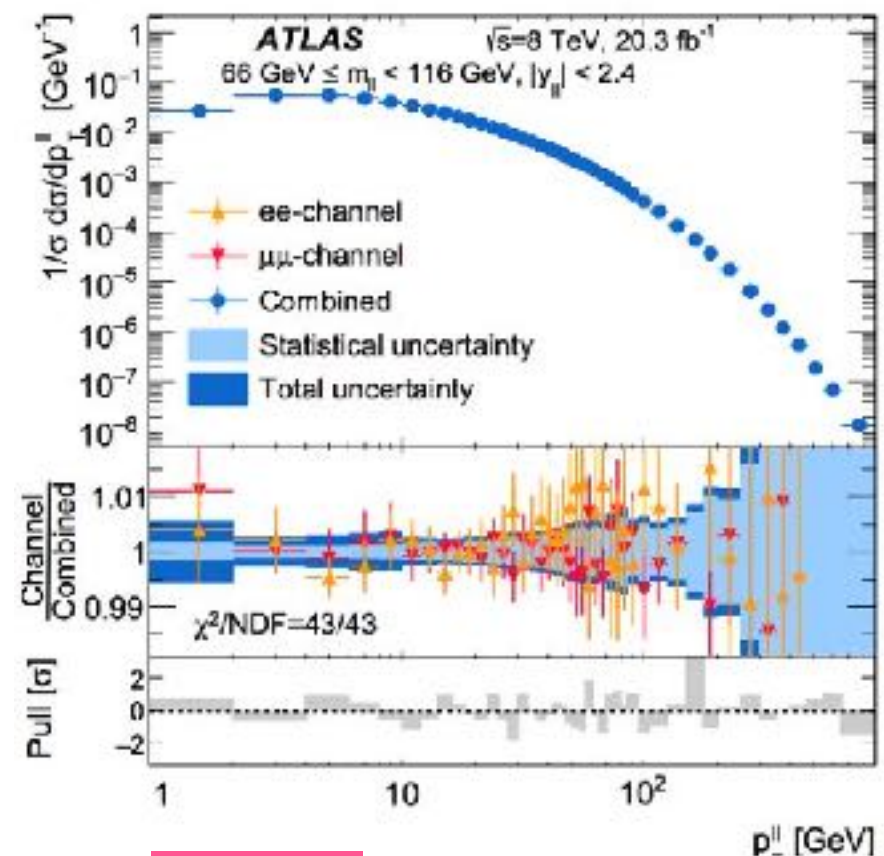
Gluon: direct handle



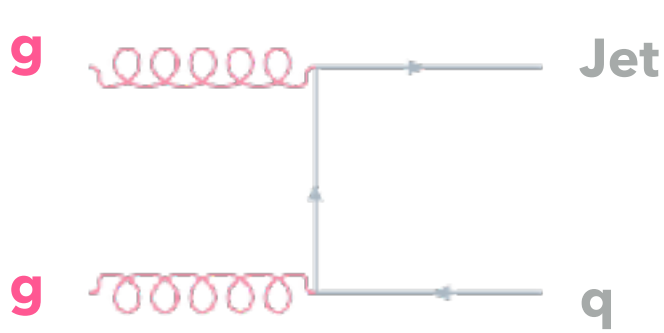
INCLUSIVE JET

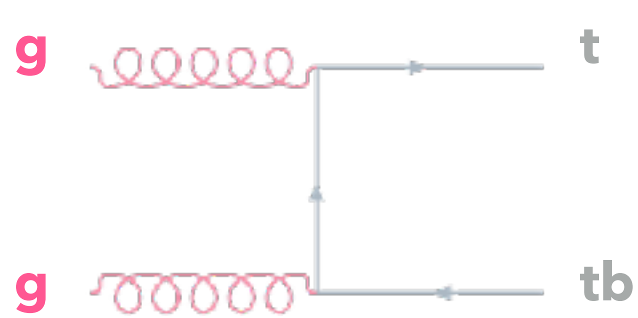


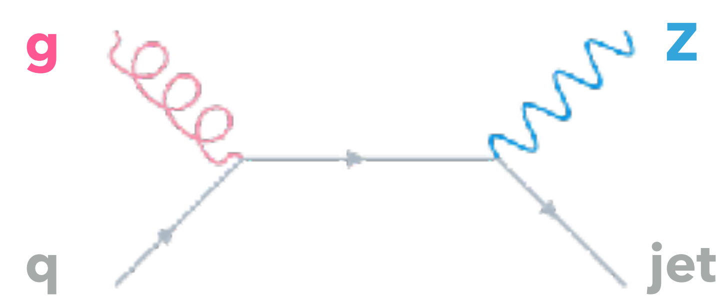
TOP PAIR



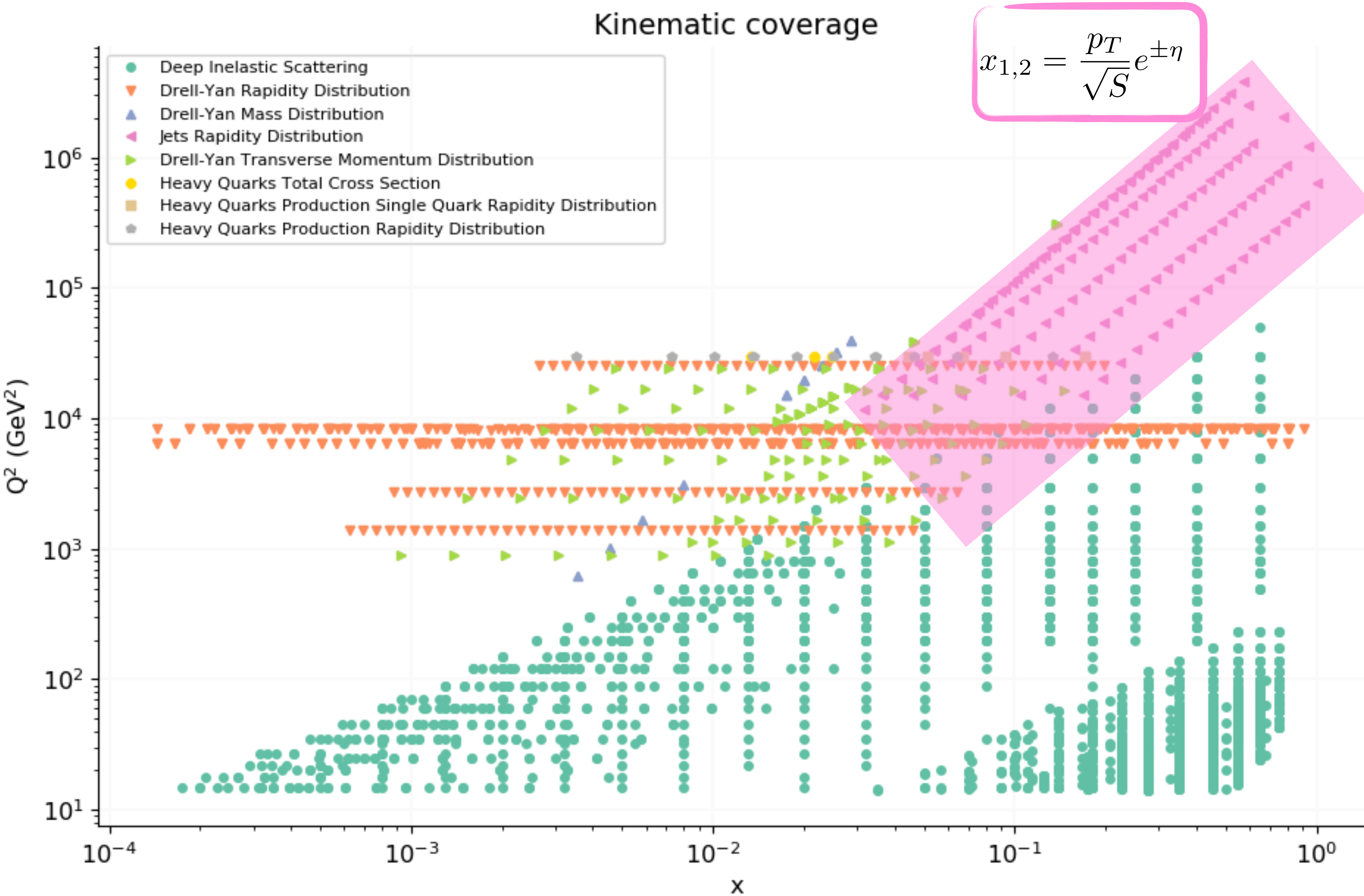
Z P_T





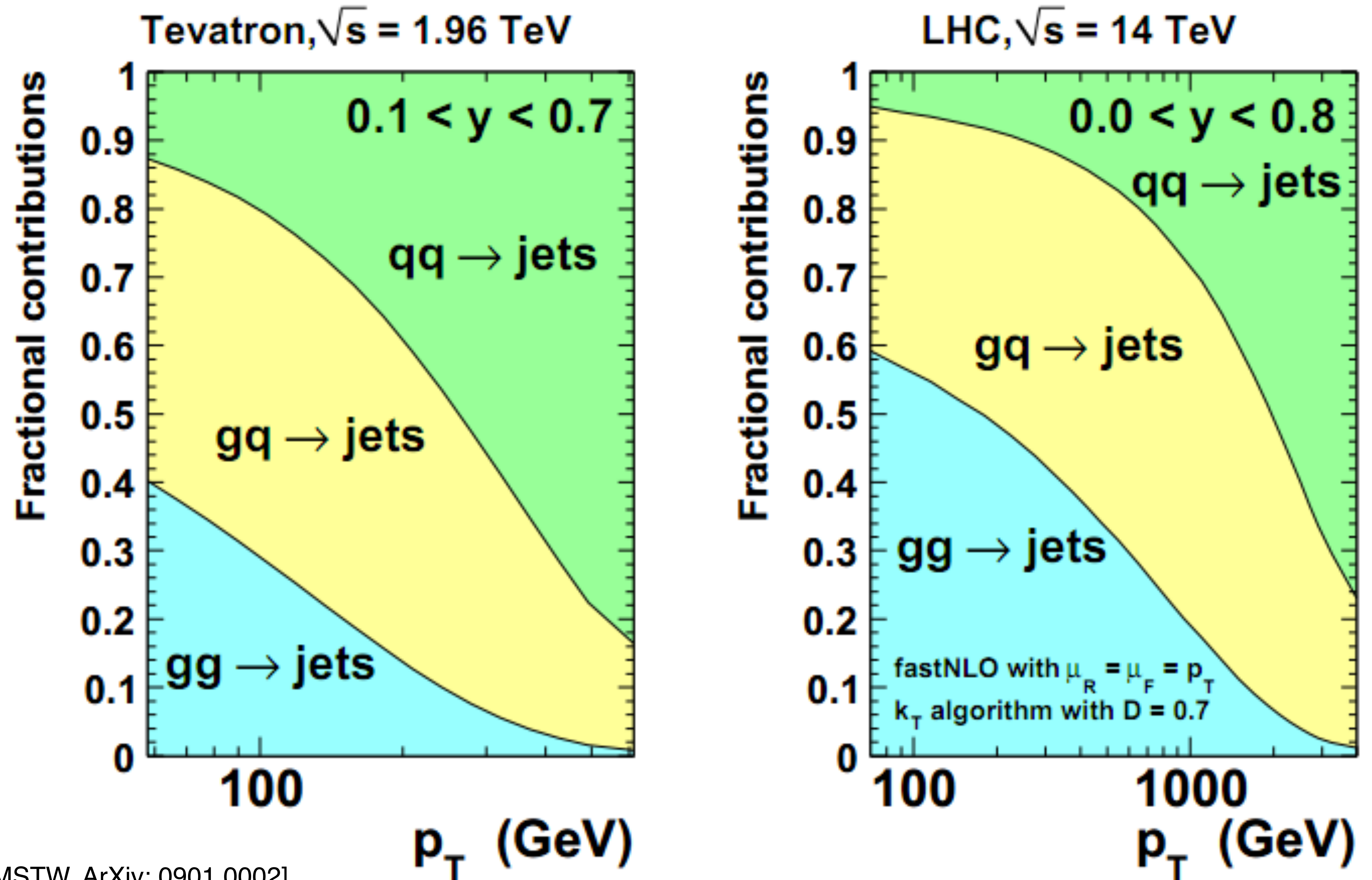


Gluon: jets data



$$x_{1,2} = \frac{p_T}{\sqrt{S}} e^{\pm\eta}$$

Gluon: jets data

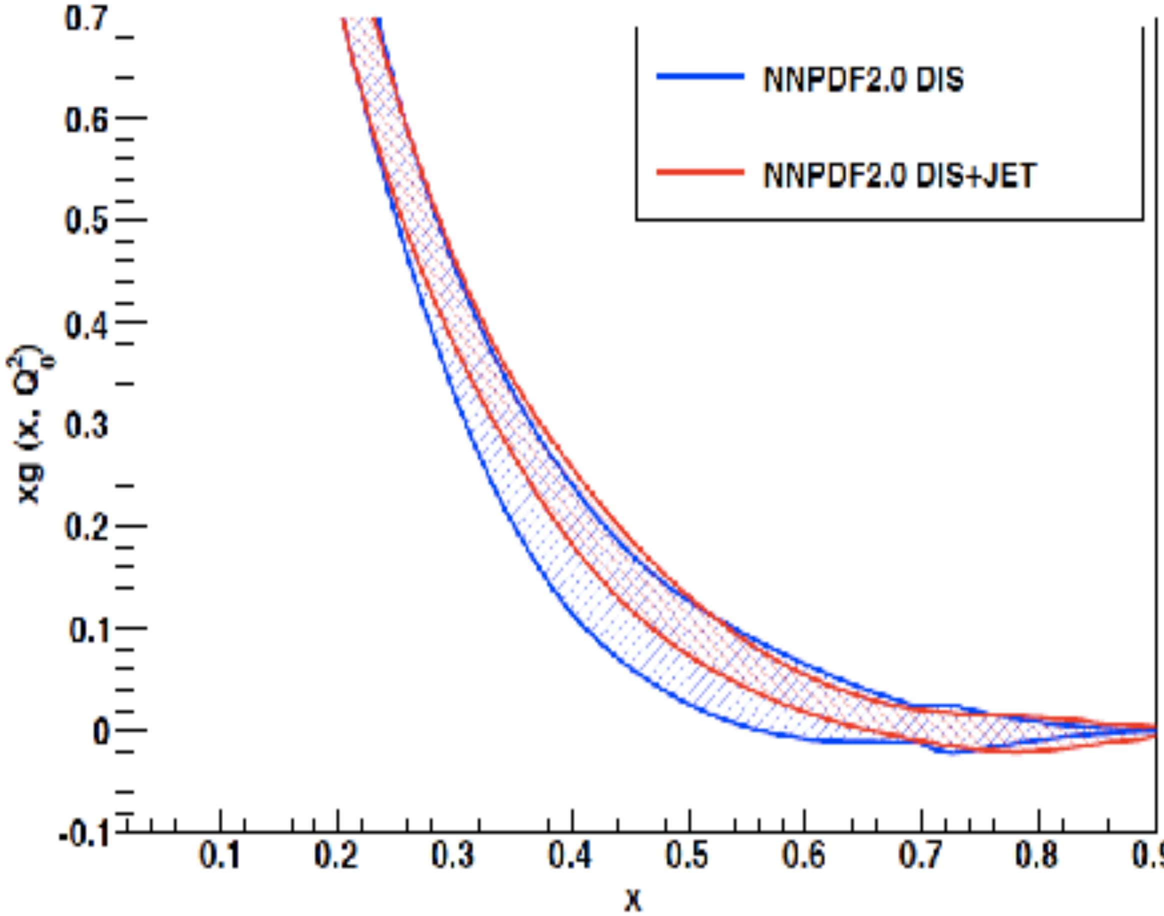


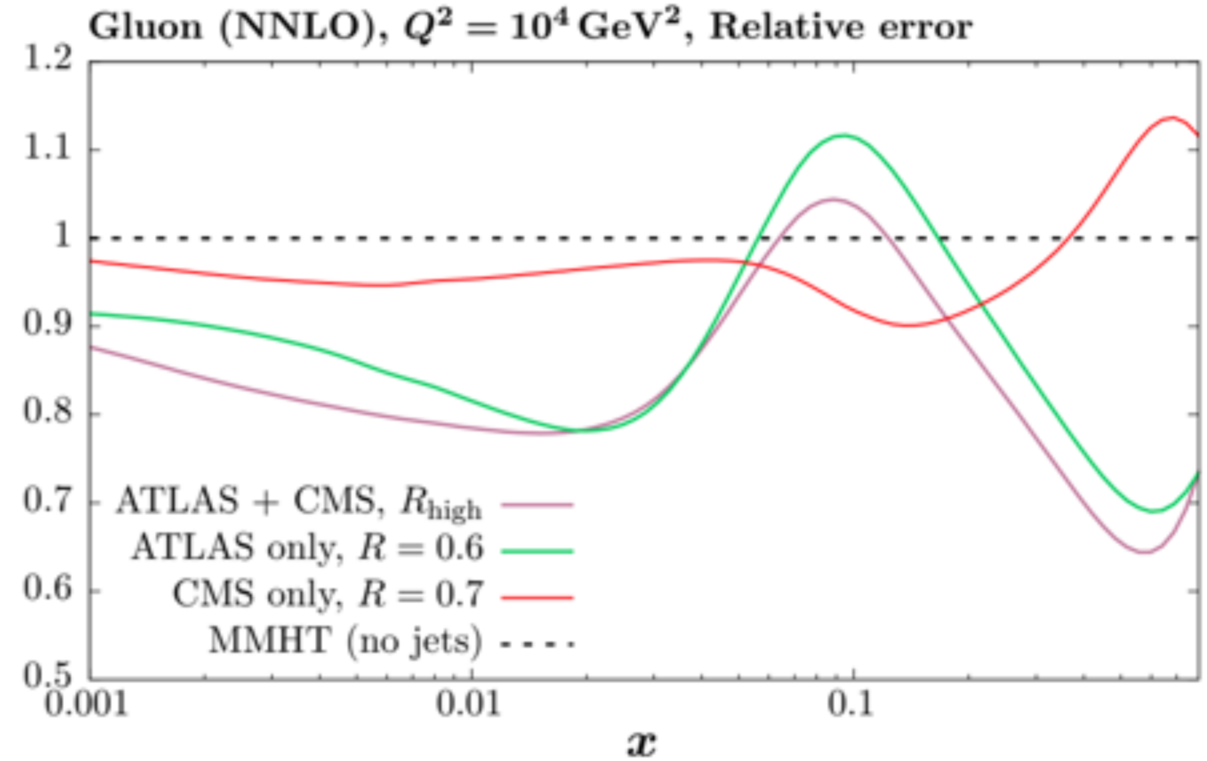
[MSTW, ArXiv: 0901.0002]

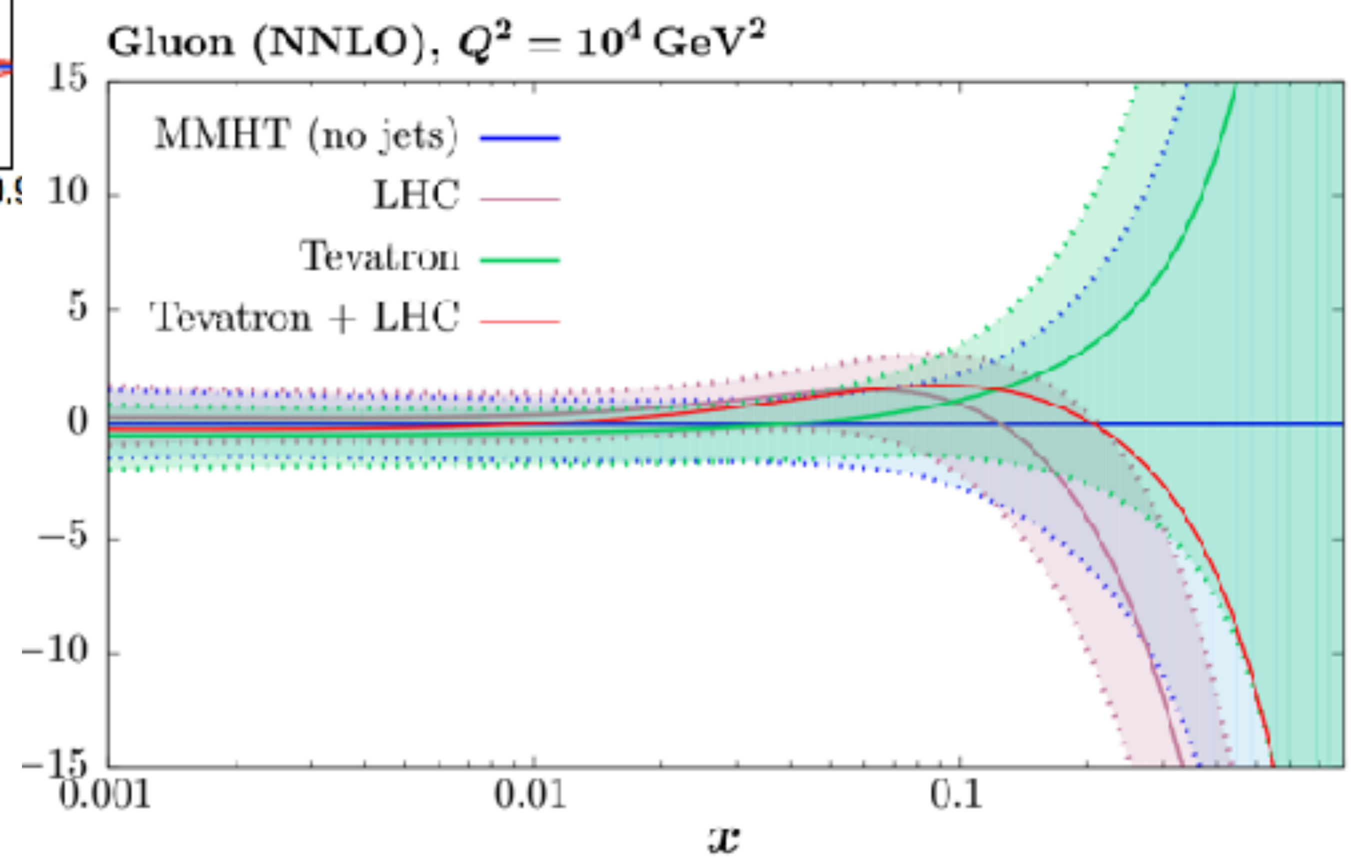
Gluon: jets data

LHC jet data

Tevatron jet data





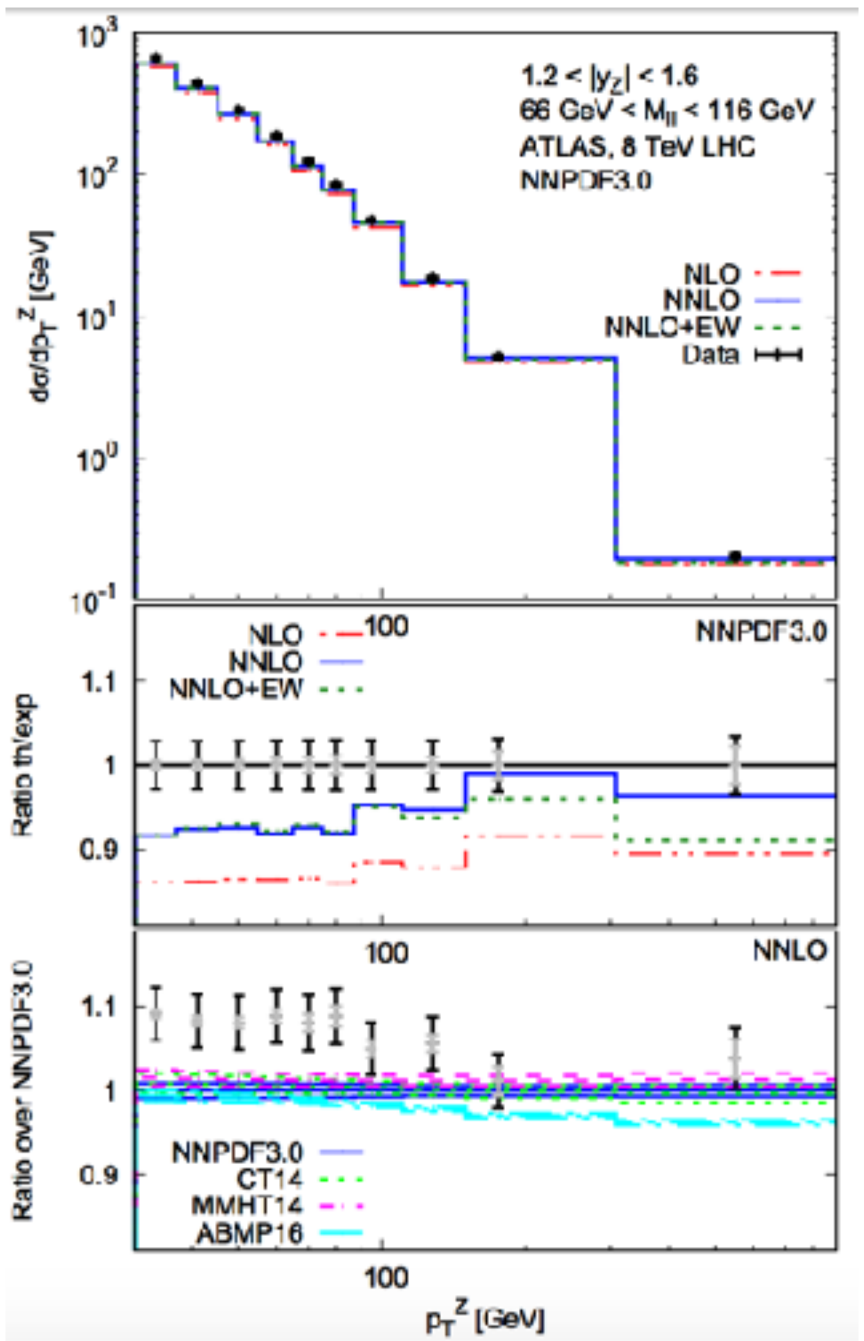


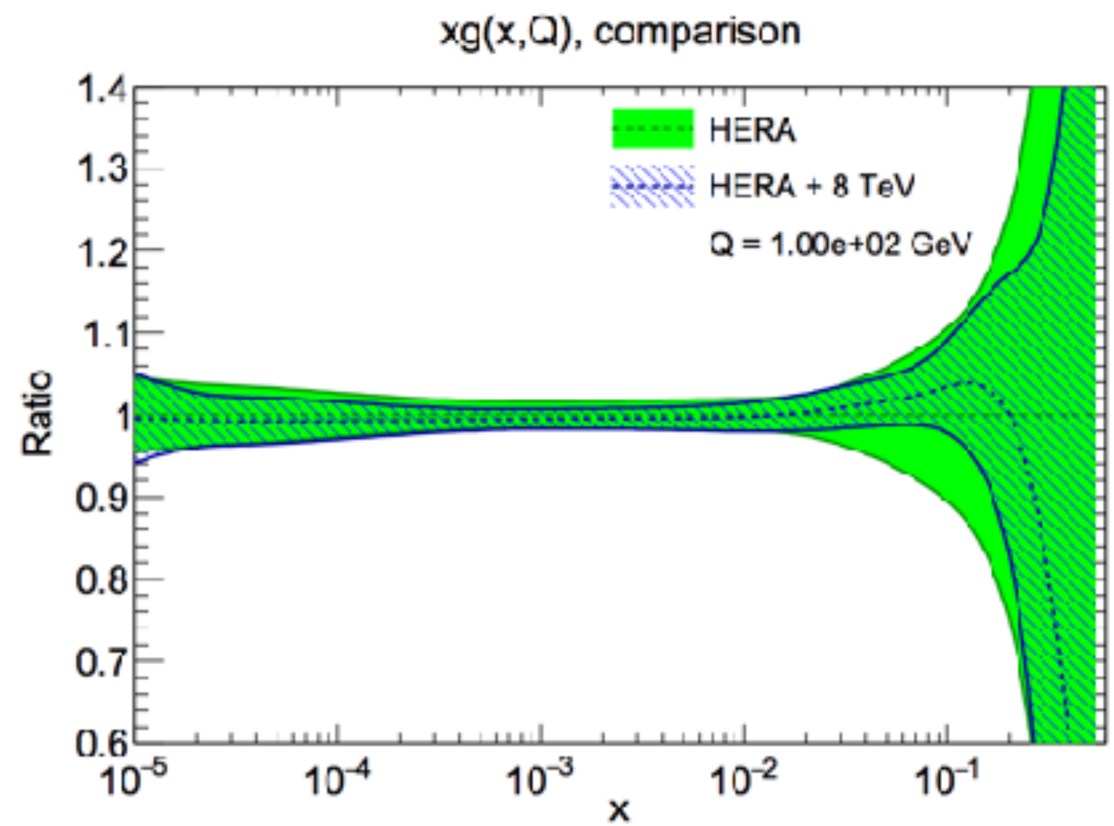
[Ball et al, ArXiv: 1002.4407]

[Harland-Lang et al, ArXiv: 1711.05757]

Gluon: Z transverse momentum

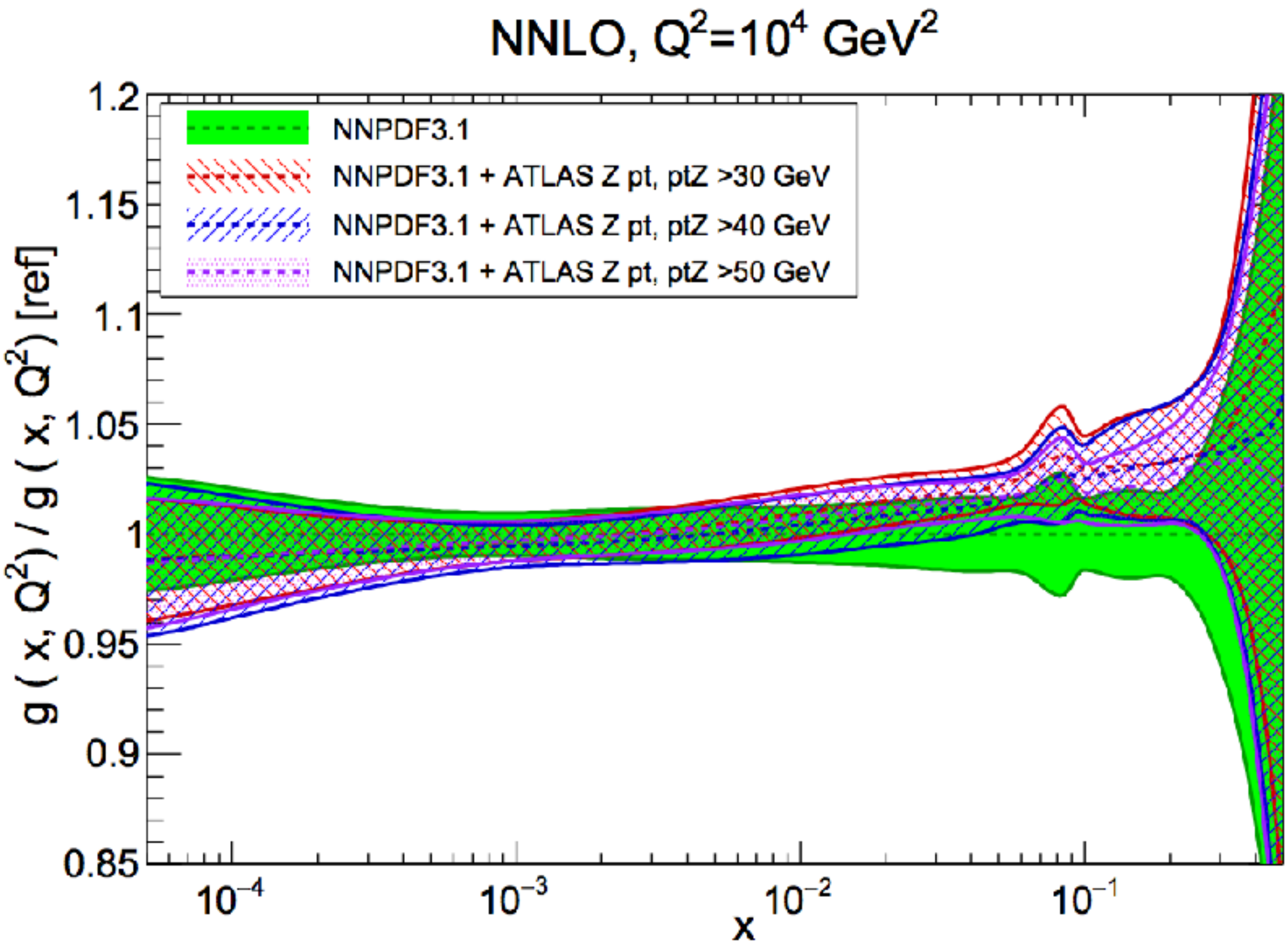
- Experimental precision < 1% up to pT~200 GeV
- Data hugely dominate by correlated systematic uncertainties
- Interesting case-study to probe current theory-experiment frontier



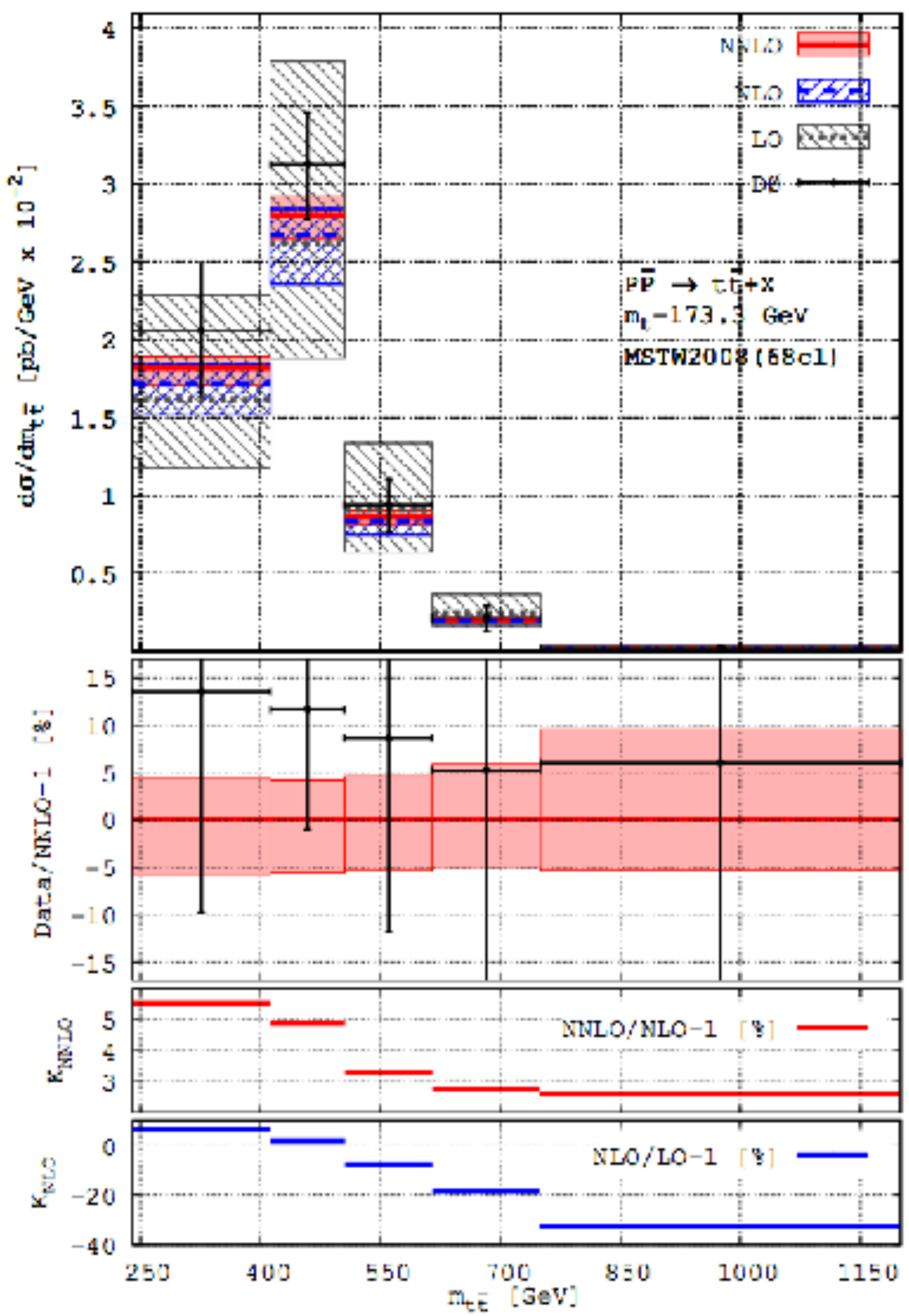


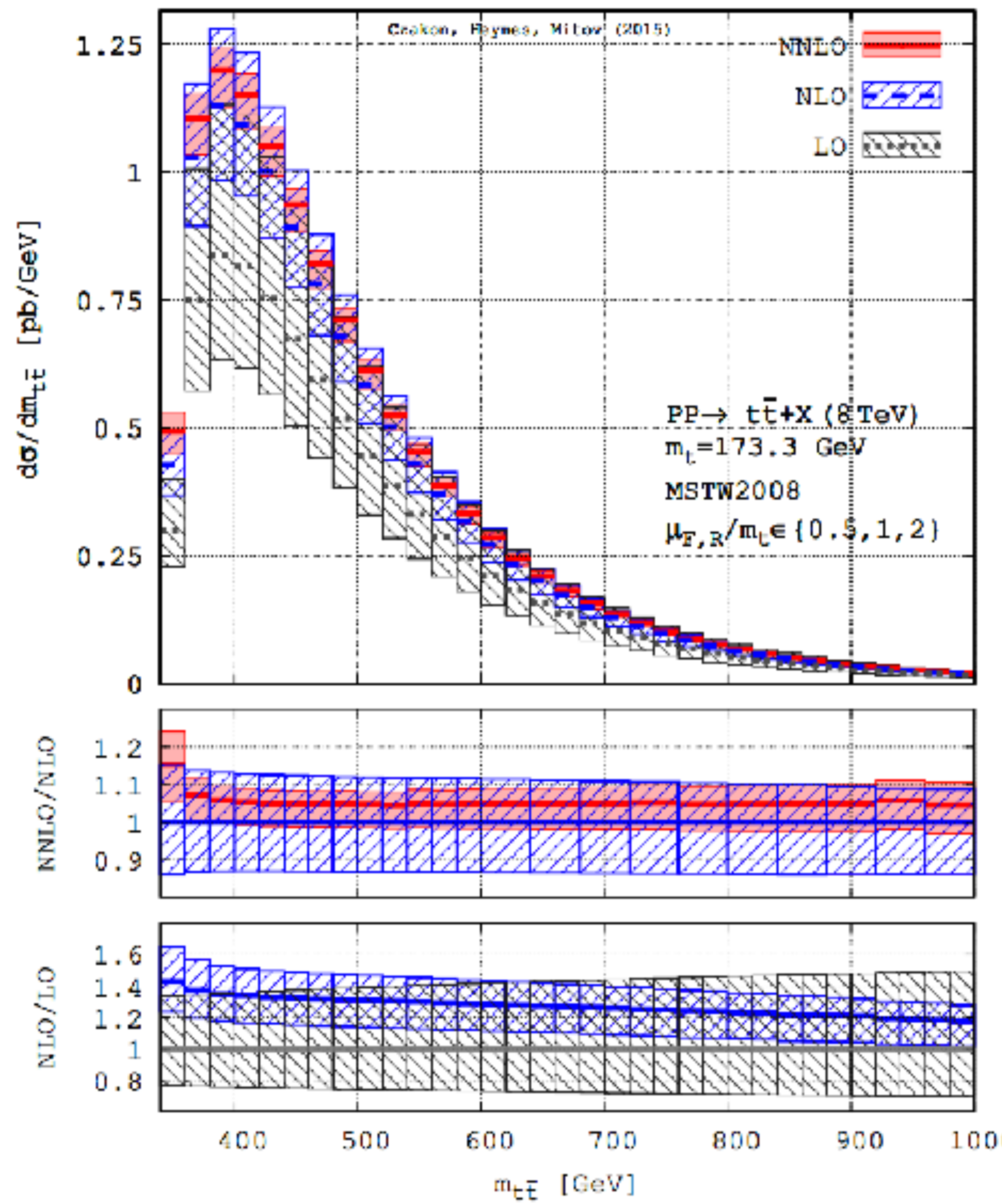
- Data/Theory comparison not so intuitive for correlation-dominated data
- Fluctuation in NNLO predictions (0.5 - 1%) had to be accounted for as extra nuisance parameter to get a good fit of such precise data

Gluon: Z transverse momentum



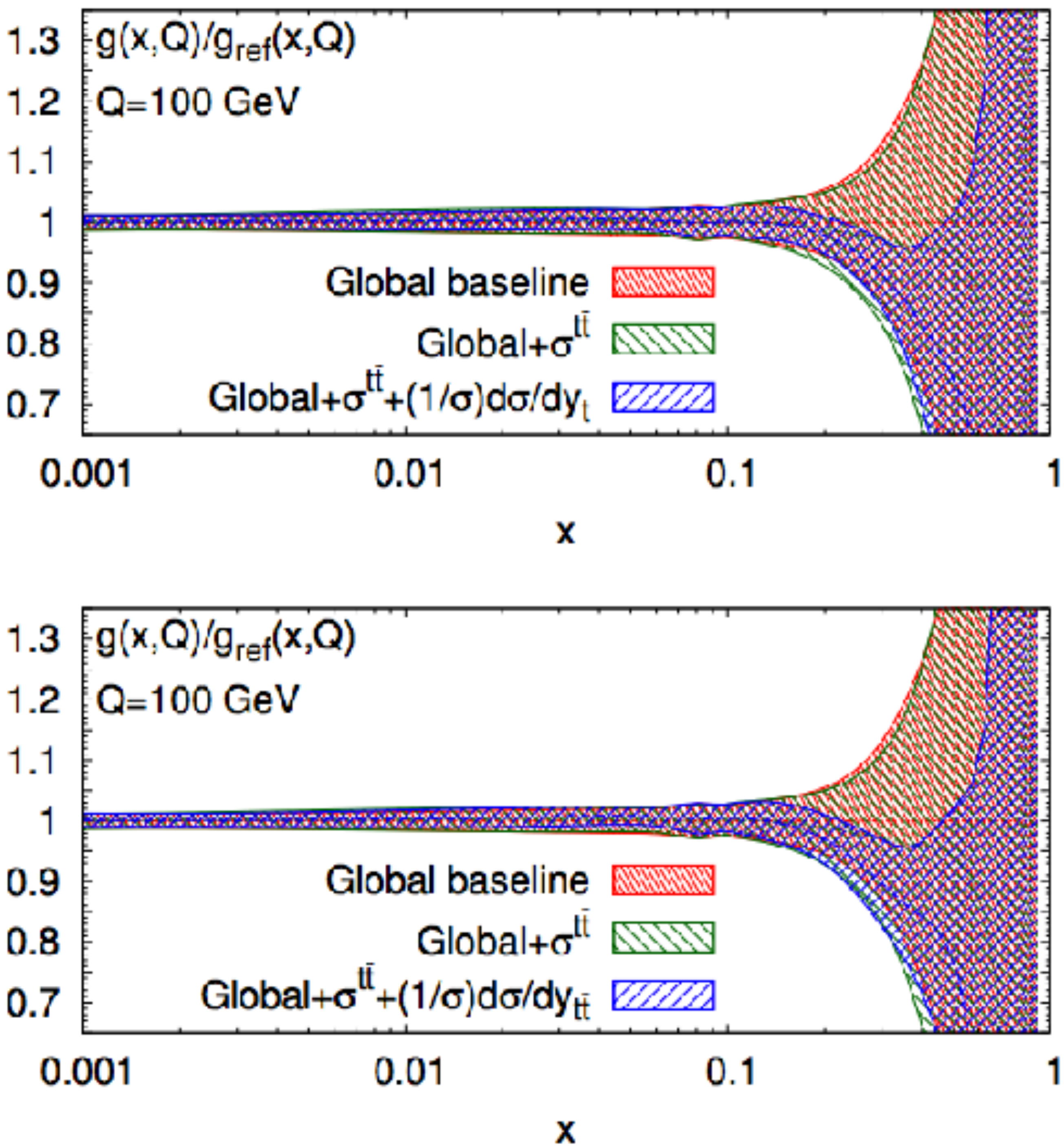
Gluon: top pair production

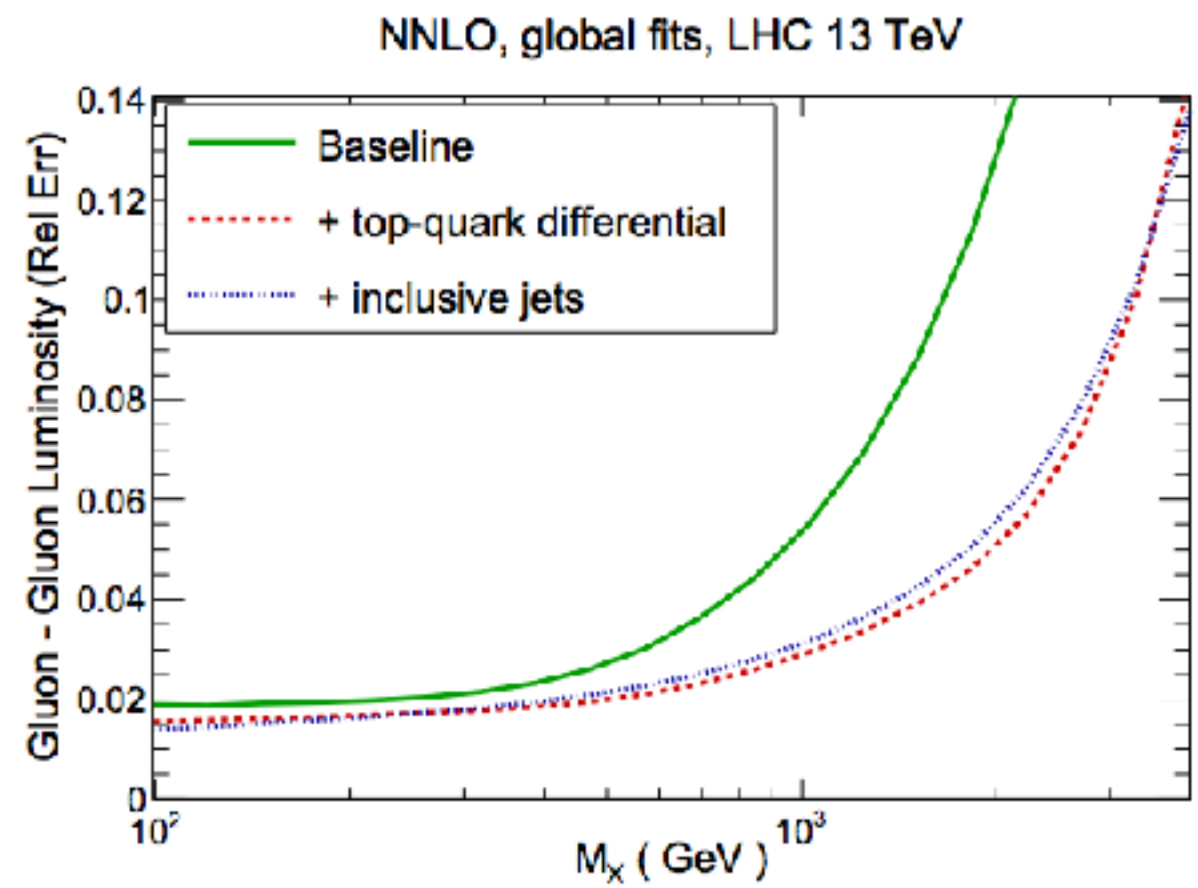




Czakon, Fiedler, Mitov [PRL 116(2016) 082003]

Gluon: top pair production



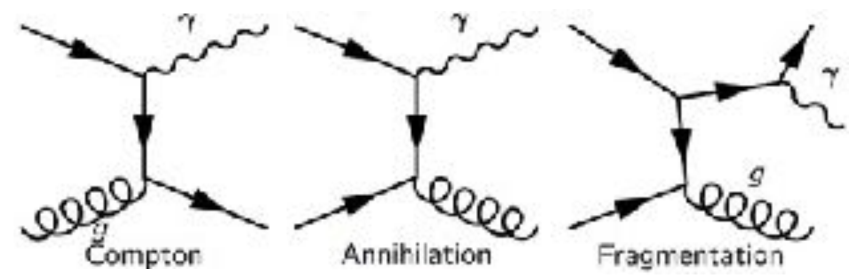


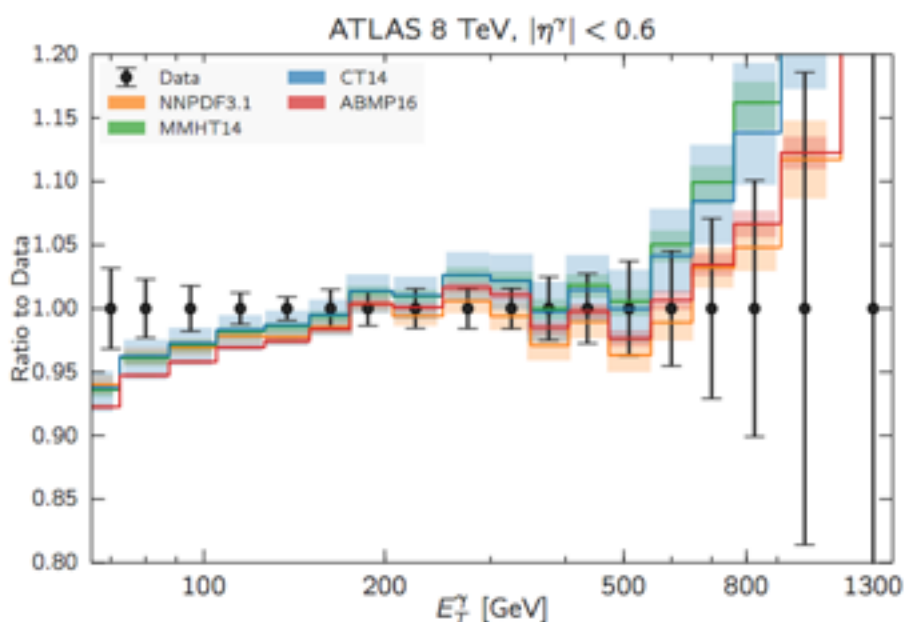
Czakon, Hartland, Mitov, Nocera and Rojo, arXiv: 1611.08609

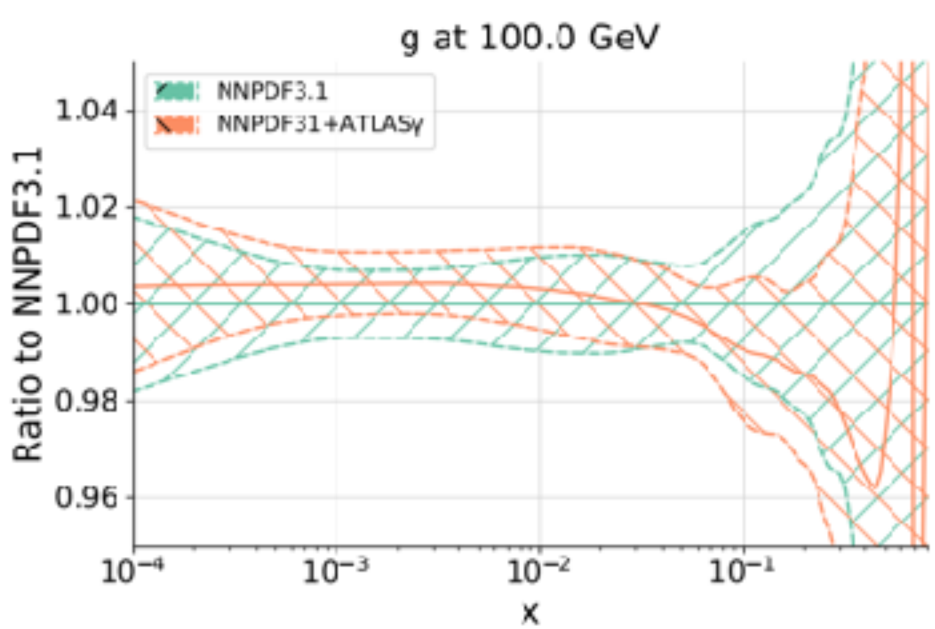
- Most constraining is inclusion of y_t list from ATLAS and y_{tt} from CMS jointly with total xsec
- Competitive reduction of gluon uncertainty with jets measurement
- Slight tension between ATLAS and CMS in NNPDF3.1 ($\chi^2_{ATLAS} \sim 1.6$, $\chi^2_{CMS} \sim 0.9$)

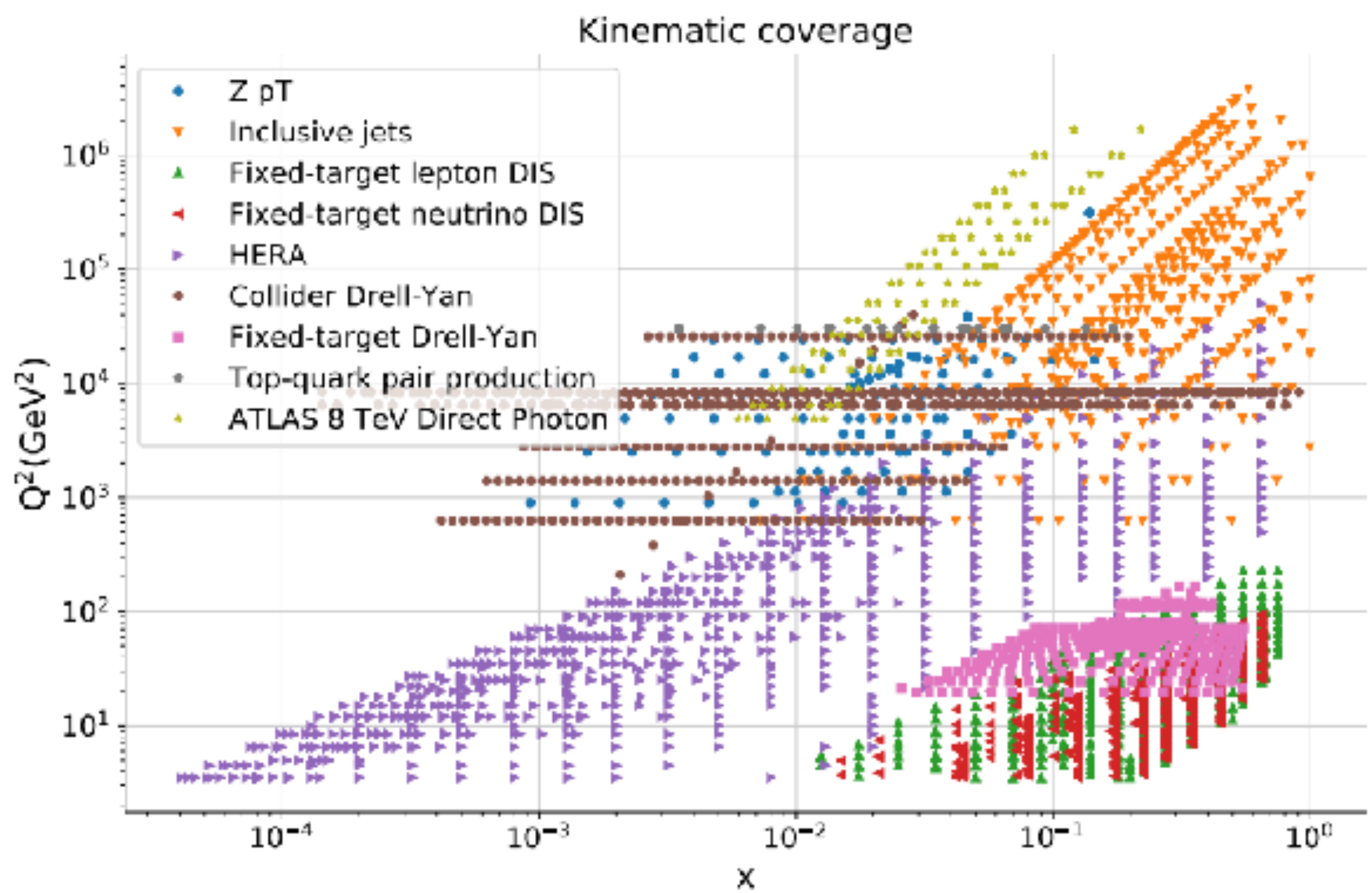
Gluon: direct photon production

- Prompt photon production directly sensitive to the gluon-quark luminosity via Compton scattering
- Isolated prompt photon data known at NNLO [Campbell et al 1612.04333] and accurately measured by ATLAS









Campbell et al 1802.03021

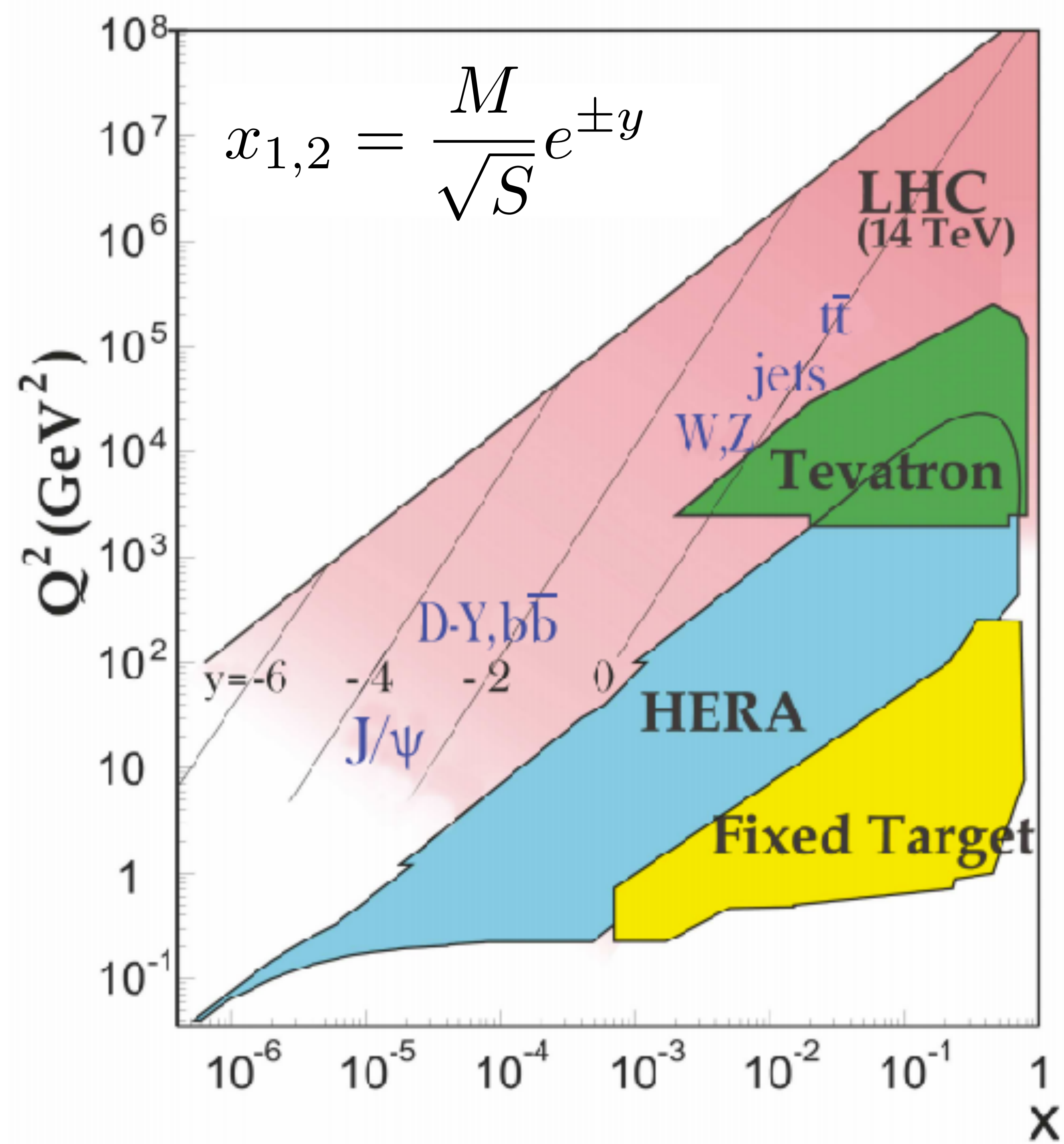
To summarise

GLUON

- Inclusive jets and dijets **(medium/large x)**
- Isolated photon and γ+jets **(medium/large x)**
- Top pair production **(large x)**
- High p_T V(+jets) distribution **(medium x)**

QUARKS

- High p_T V(+jets) ratios **(medium x)**
- W and Z production **(medium x)**
- Low and high mass Drell-Yan **(small and large x)**
- Wc **(strangeness at medium x)**



PDG 2016

Parton Luminosities

- A quick and easy way to assess the mass and the collider dependence of production cross sections at hadron-hadron colliders is to use Parton Luminosities
- At leading order in QCD (parton model)

$$\hat{\sigma}_{ab\to X} = C_X \delta(x_a x_b S - M^2)$$

$$\sigma_{pp\to X} = \int_0^1 dx_a dx_b f_a(x_a, M^2) f_b(x_b, M^2) \hat{\sigma}_{ab\to X}$$

- Thus

$$\sigma_{pp\to X} = C_X \int_0^1 dx_a dx_b f_a(x_a, M^2) f_b(x_b, M^2) \delta(x_a x_b S - M^2)$$

$$= \frac{C_X}{S} \frac{\partial \mathcal{L}_{ab}}{\partial \tau}$$

with

$$\tau = \frac{M^2}{\tau}$$

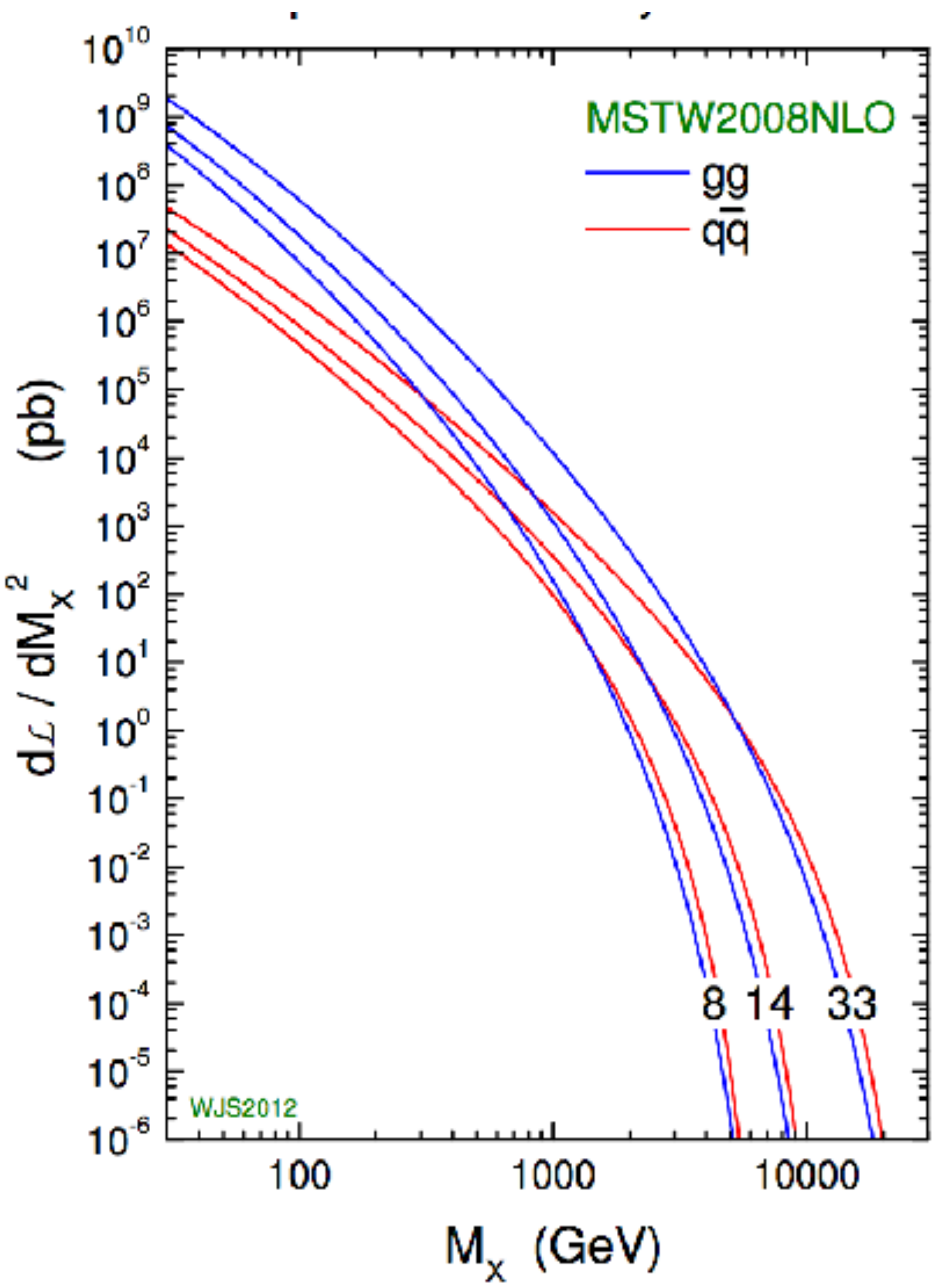
- Define

$$\Phi_{ab}(M^2) = \frac{\partial \mathcal{L}_{ab}}{\partial \tau}$$

$$= \int_0^1 dx_a dx_b f_a(x_a, M^2) f_b(x_b, M^2) \delta(x_a x_b - \tau)$$

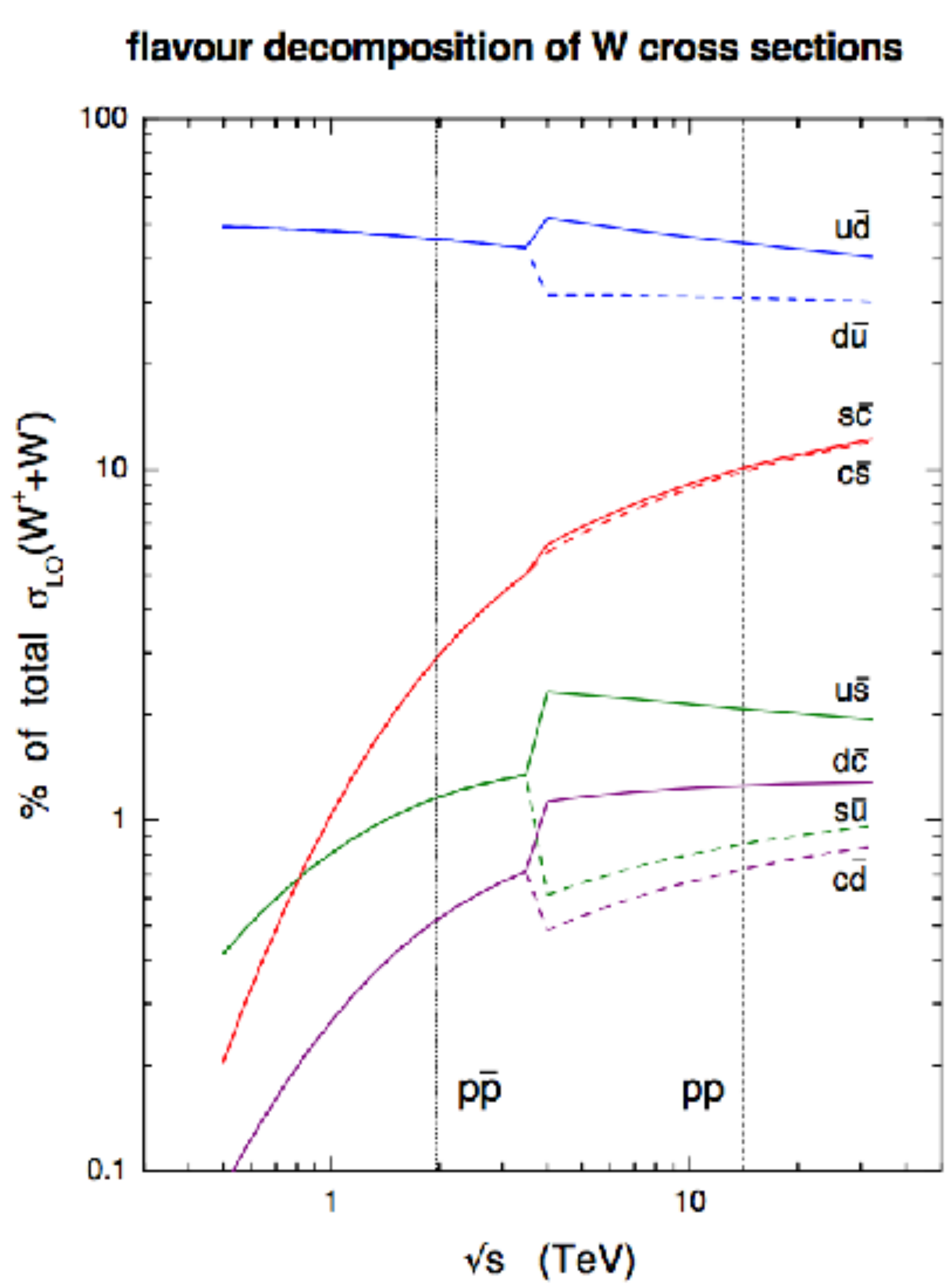
$$= \frac{1}{S} \int_\tau^1 \frac{dy}{y} f_a(y, M^2) f_b\left(\frac{\tau}{y}, M^2\right)$$

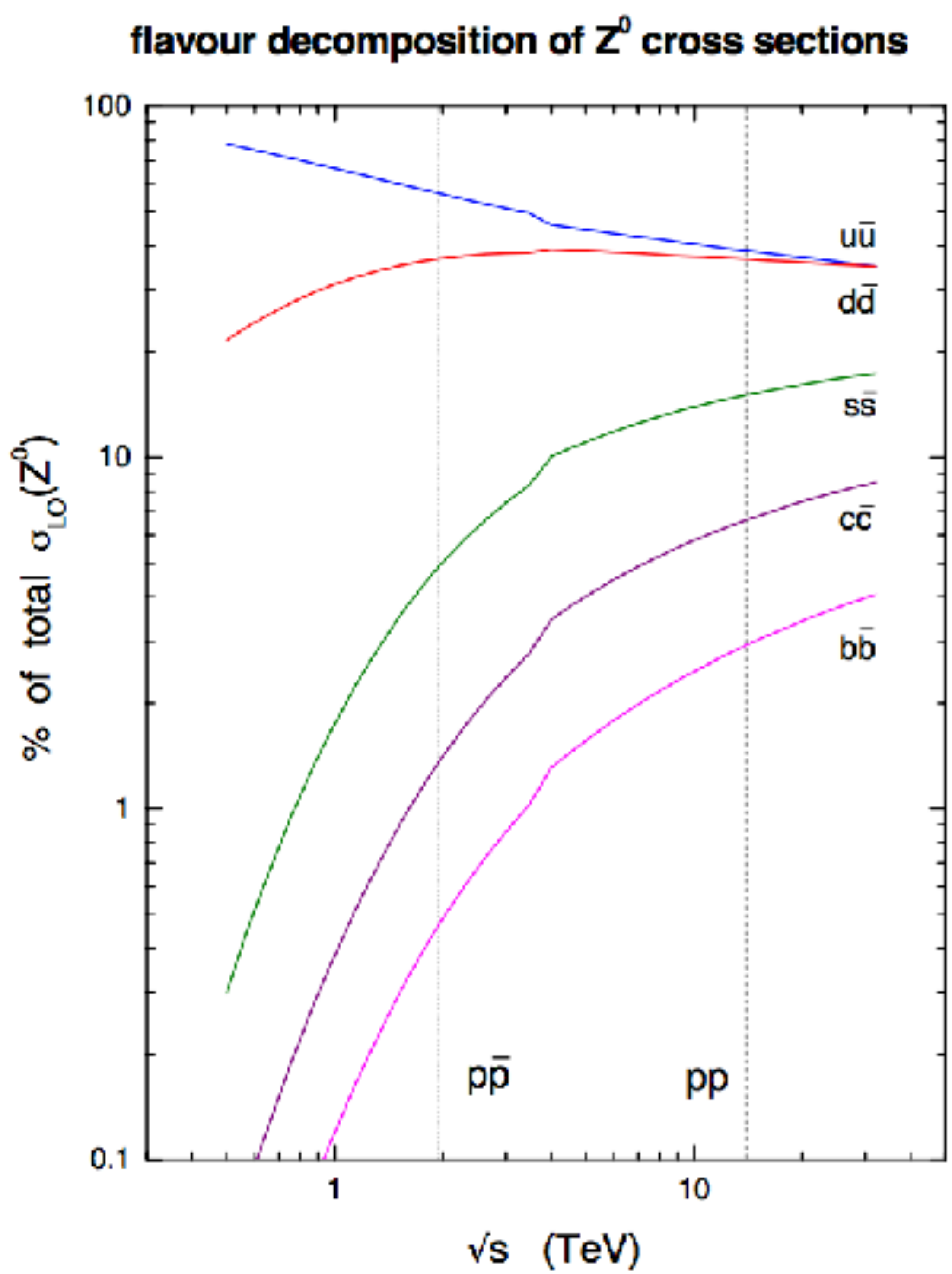
Parton Luminosities



$$
\begin{aligned}
\Phi_{ab}(M^2) &= \frac{\partial \mathcal{L}_{ab}}{\partial \tau} \\
&= \int_0^1 dx_a dx_b f_a(x_a, M^2) f_b(x_b, M^2) \delta(x_a x_b - \tau) \\
&= \frac{1}{S} \int_\tau^1 \frac{dy}{y} f_a(y, M^2) f_b\left(\frac{\tau}{y}, M^2\right)
\end{aligned}
$$

Parton Luminosities



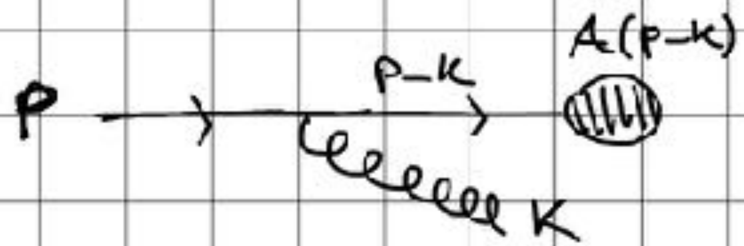


Extra material

QCD and improved parton model

REAL EMISSION

Detailed version



Sudakov parametrization

$$k = (1-z)\,p + k_T + \xi\,\eta$$

where η such that
$p \cdot k_T = 0$
$\eta \cdot k_T = 0$
$\eta^2 = 0$
$2p \cdot \eta = 1$

For example

$$p = p^0(1, 0, 0, 1)$$

$$\eta = \frac{1}{4p^0}(1, 0, 0, -1)$$

$$k_T = (0, k_T^1, k_T^2, 0)$$

From $k^2 = 0$ (on-shell condition)

$\Rightarrow\ k_T^2 + 2p\cdot\eta\,\xi(1-z) = 0 \qquad \Rightarrow\ \xi = -\frac{k_T^2}{1-z}$

From $(p-k)^2 < 0$

$\Rightarrow\ z < 1$

From $k_3 = 0 \quad \Rightarrow\ |k_T^2| < 4p_0^2(1-z)$

QCD and improved parton model

$$\frac{d^3k}{(2\pi)^3 2k^0} = \frac{1}{16\pi^2} \frac{d|k_T^2|\, dz}{(1-z)}$$ (integrating over $d\varphi$)

Singular part of amplitude:

$$\mathcal{M}^{(1)}_{q_{sing}}(p,k) = g_s\, A_i(p-k) \frac{\not p - \not k}{(p-k)^2} \not\epsilon(k)\, t^A_{ij}\, u_j(p)$$

$\not p - \not k \to z\not p - \not k_T - \xi \not n$, with $\xi \not n \to O(k_T^2)$

$\frac{1}{(p-k)^2} \to \frac{1-z}{k_T^2}$

$t^A_{ij} \to$ SU(3) gen. fund. representation

$$= g_s \frac{(1-z)}{k_T^2} A_i(p-k)\, (z\not p - \not k_T)\, \not\epsilon(k)\, t^A_{ij}\, u_j(p)$$

But $\not p u(p) = 0$, $k_T \cdot \epsilon(k) = 0$ $\Rightarrow$ $(1-z)\not p \not\epsilon(k) u(p) = -2\epsilon_\mu(k) k_T^\mu\, u(p)$

$$\Rightarrow \mathcal{M}^{(1)}_{q,sing}(p,k) = -\frac{g_s}{k_T^2} A_i(p-k)\left[2z\, k_T\cdot\epsilon(k) + (1-z)\not k_T \not\epsilon(k)\right] t^A_{ij}\, u_j(p)$$

QCD and improved parton model

$$|M^{(1)}_{qg^*q}|^2 = -\frac{2 g_s^2 C_F}{k_T^2}\,\frac{(1+z)^2}{z}\,|M^{(0)}(zp)|^2$$

where we have used $\sum \epsilon_\mu(k)\,\epsilon^*_\nu(k) = -g^T_{\mu\nu} = -g_{\mu\nu} + \frac{k_\mu n_\nu + k_\nu n_\mu}{k\cdot n}$

$$t^A_{ij}\, t^A_{jk} = \delta_{ik}\, C_F \qquad C_F = \frac{N_c^2-1}{2N_c} = \frac{4}{3}$$

$$\Rightarrow \hat{\sigma}_q^{(1)}(p) = \frac{1}{16\pi^2}\int_0^1 \frac{dz}{1-z}\int_0^{|k_T^2|_{max}} d|k_T^2|\,\frac{1}{p\cdot p'}\,|M_q^{(1)}(p,k)|^2$$

$\alpha_s = \frac{g_s^2}{4\pi}$

$$= \frac{\alpha_s C_F}{2\pi}\int_0^1 \frac{dz}{1-z}\int_0^{|k_T^2|_{max}} d|k_T^2|\,\frac{1+z^2}{|k_T^2|}\,\frac{1}{z(p\cdot p')}\,|M_q^{(0)}(zp)|^2 + \text{regular terms}$$

$$= \frac{\alpha_s C_F}{2\pi}\int_0^1 \frac{dz}{1-z}\int_0^{|k_T^2|_{max}} d|k_T^2|\,\frac{1+z^2}{|k_T^2|}\,\hat{\sigma}_q^{(0)}(zp)$$

divergences! SOFT $z \to 1$ regulator $\epsilon \to 0$

COLLINEAR $|k_T^2| \to 0$ regulator $\lambda \to 0$

QCD and improved parton model

$$\hat{\sigma}_q^{(1)}(p) = \frac{\alpha_s C_F}{2\pi} \int_0^{1-\varepsilon} \frac{dz}{1-z} \int_{\lambda^2}^{|k_T^2|_{max}} \frac{d|k_T^2|}{|k_T^2|} (1+z^2)\, \hat{\sigma}_q^{(0)}(zp)$$

Adding virtual corrections $\quad - \hat{\sigma}_q^{(0)}(p) \frac{\alpha_s C_F}{2\pi} \int_0^{1-\varepsilon} \frac{dz}{1-z} \int_{\lambda^2}^{|k_T^2|_{max}} \frac{d|k_T^2|}{|k_T^2|} (1+z^2)$

the soft singularity cancels

$\Rightarrow \varepsilon \to 0$

$$\hat{\sigma}_q^{(1)}(p) = \frac{\alpha_s C_F}{2\pi} \int_0^{1} \frac{dz}{1-z} \int_{\lambda^2}^{|k_T^2|_{max}} \frac{d|k_T^2|}{|k_T^2|} (1+z^2) \left[\hat{\sigma}_q^{(0)}(zp) - \hat{\sigma}_q^{(0)}(p) \right]$$

Still left with COLLINEAR divergence!

Introduce μ_F to split integration

$$\int_{\lambda^2}^{|k_T^2|_{max}} \frac{d|k_T^2|}{|k_T^2|} \to \underbrace{\int_{\lambda^2}^{\mu_F^2} \frac{d|k_T^2|}{|k_T^2|}}_{\text{singular}} + \underbrace{\int_{\mu_F^2}^{|k_T^2|_{max}} \frac{d|k_T^2|}{|k_T^2|}}_{\text{finite}}$$

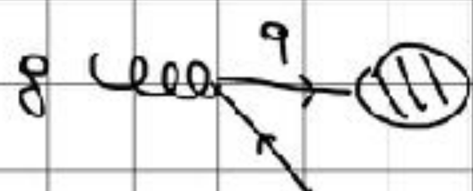
QCD and improved parton model

$$\Rightarrow \quad \hat{\sigma}_q(p) = \hat{\sigma}_q^{(0)}(p) + \hat{\sigma}_q^{(1)}(p)$$

universal function $P(q \to q)$

$$= \hat{\sigma}_q^{(0)}(p) + \frac{\alpha_s}{2\pi}\int_0^1 dz\, P_{qq}(z)\, \hat{\sigma}_q^{(0)}(zp) \log\frac{\mu_F^2}{\lambda^2} + \hat{\sigma}^{(1)}_{q,reg}(p, \mu_F^2)$$

Of course quark can come from gluon



Doing the whole calculation, we get

$$\hat{\sigma}_g(p) = \hat{\sigma}_g^{(1)}(p)$$

$$= \frac{\alpha_s}{2\pi}\int_0^1 dz\, P_{qg}(z)\, \hat{\sigma}_q^{(0)}(zp) \log\frac{\mu_F^2}{\lambda^2} + \hat{\sigma}^{(1)}_{g,reg}(p, \mu_F^2)$$

↳ In the parton model formula

$$\sigma(p) = \int_0^1 dy \left[f_q(y)\, \hat{\sigma}_q(yp) + f_g(y)\, \hat{\sigma}_g(yp) \right]$$

QCD and improved parton model

$$\sigma(p) = \int_0^1 dy \left[f_q(y)\, \hat{\sigma}_q^{(0)}(yp) \right.$$

$$+ \frac{\alpha_s}{2\pi} \int_0^1 dy\, f_q(y) \int_0^1 dz\, \hat{\sigma}_q^{(0)}(yz\, p)\, P_{qq}(z) \log \frac{\mu_F^2}{\lambda^2}$$

$$+ \frac{\alpha_s}{2\pi} \int_0^1 dy\, f_g(y) \int_0^1 dz\, \hat{\sigma}_g^{(0)}(yzp)\, P_{qg}(z) \log \frac{\mu_F^2}{\lambda^2}$$

$$+ \int_0^1 dy\, f_q(y)\, \hat{\sigma}^{(1)}_{q,reg}(yp, \mu_F^2) + \int_0^1 dy\, f_g(y)\, \hat{\sigma}^{(1)}_{g,reg}(yp, \mu_F^2)$$

terms $\propto \log \frac{\mu_F^2}{\lambda^2}$ can be reabsorbed into redefinition of f_q $\quad x = yz$

$$f_q(x, \mu_F^2) = \int_x^1 \frac{dy}{y} \left\{ f_q(y) \left[\delta\left(1 - \frac{x}{y}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{y}\right) \log \frac{\mu_F^2}{\lambda^2} \right] + f_g(y) \left[\frac{\alpha_s}{2\pi} P_{qg}\left(\frac{x}{y}\right) \log \frac{\mu_F^2}{\lambda^2} \right] \right\}$$

QCD and improved parton model

So that

$$\sigma(P) = \int_0^1 dx \; f_q(x, \mu_F^2)\, \hat{\sigma}_q(xP, \mu_F^2) + f_g(x)\, \hat{\sigma}_g(xP, \mu_F^2)$$

both depend on arbitrary FACTORIZATION scale

Note that however the dependence of $f_{q,g}(x, \mu_F^2)$ is totally fixed by perturbation theory

$$\mu^2 \frac{\partial f_q}{\partial \mu^2}(x, \mu^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[P_{qq}\left(\frac{x}{y}\right) f_q(y, \mu^2) + P_{qg}\left(\frac{x}{y}\right) f_g(y, \mu^2) \right]$$

Exercise III: PDF evolution

- Consider Z production at Tevatron and at LHC. How to determine the contribution of the different parton channels to the total cross section?

- You might want to use the Parton Luminosity definition

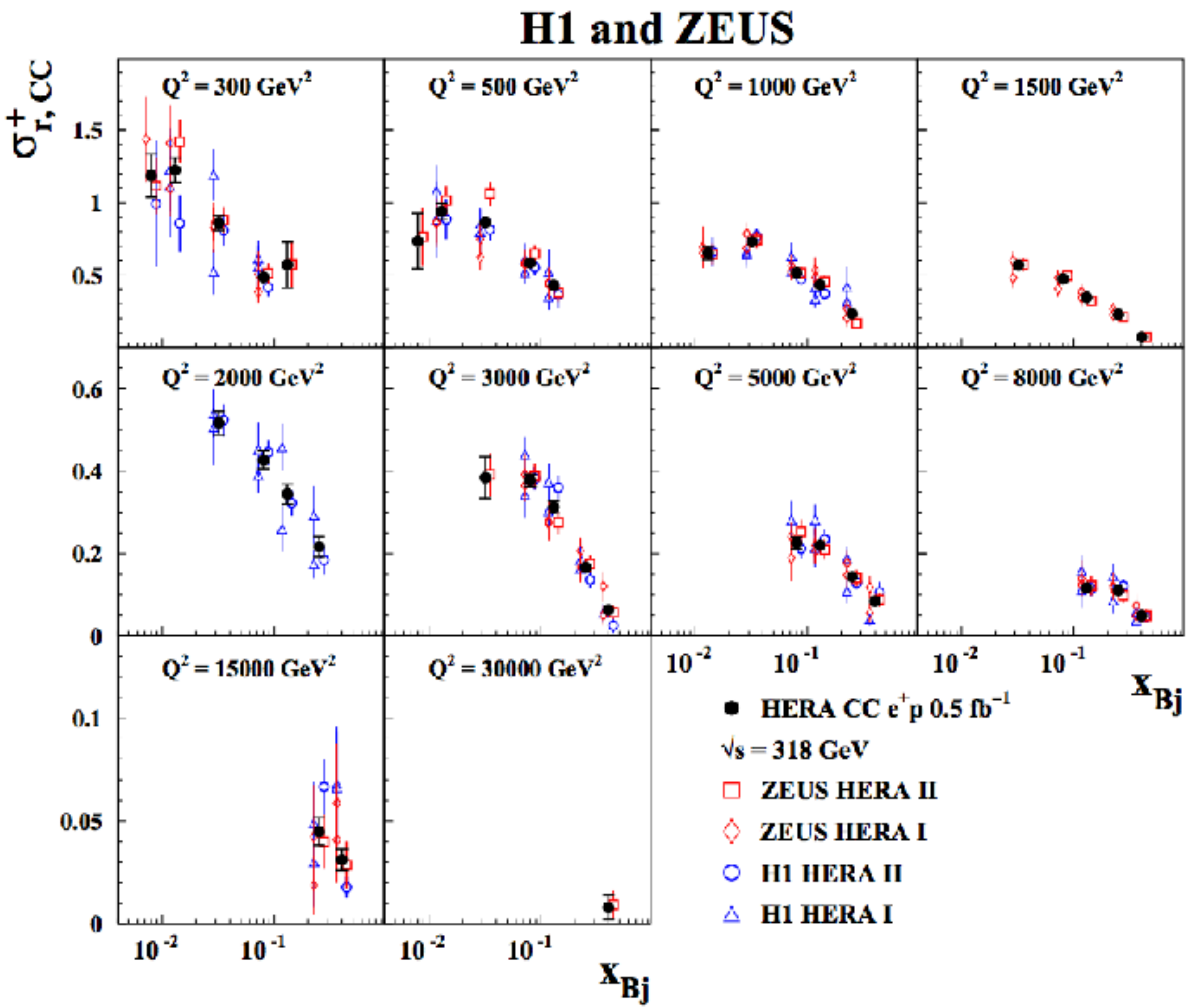
$$\Phi_{ij} = \frac{1}{S_{\rm had}} \int_\tau^1 \frac{dy}{y} f_i(y, M_X^2)\, f_j\left(\frac{\tau}{y}, M_X^2\right) \qquad \tau = \frac{M_X^2}{S_{\rm had}}$$

And plot them by using APFELweb or (older) hepdata

https://apfel.mi.infn.it/

http://hepdata.cedar.ac.uk/pdf/pdf3.html

HERA data



arXiv: 1506.06042