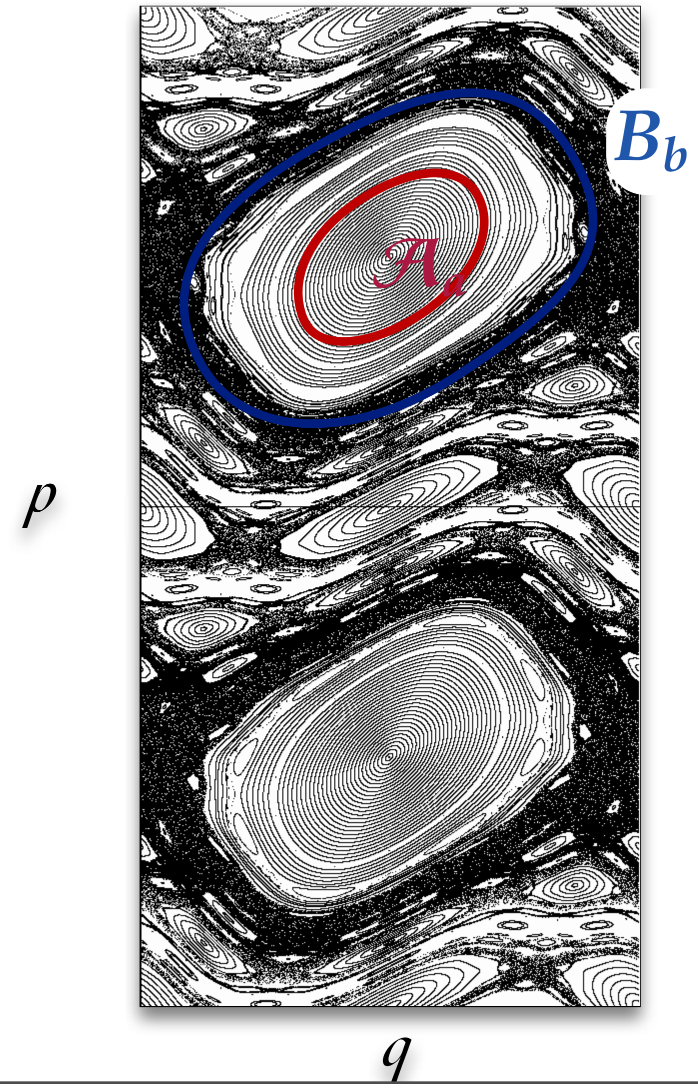
Dynamical tunneling in mixed phase space

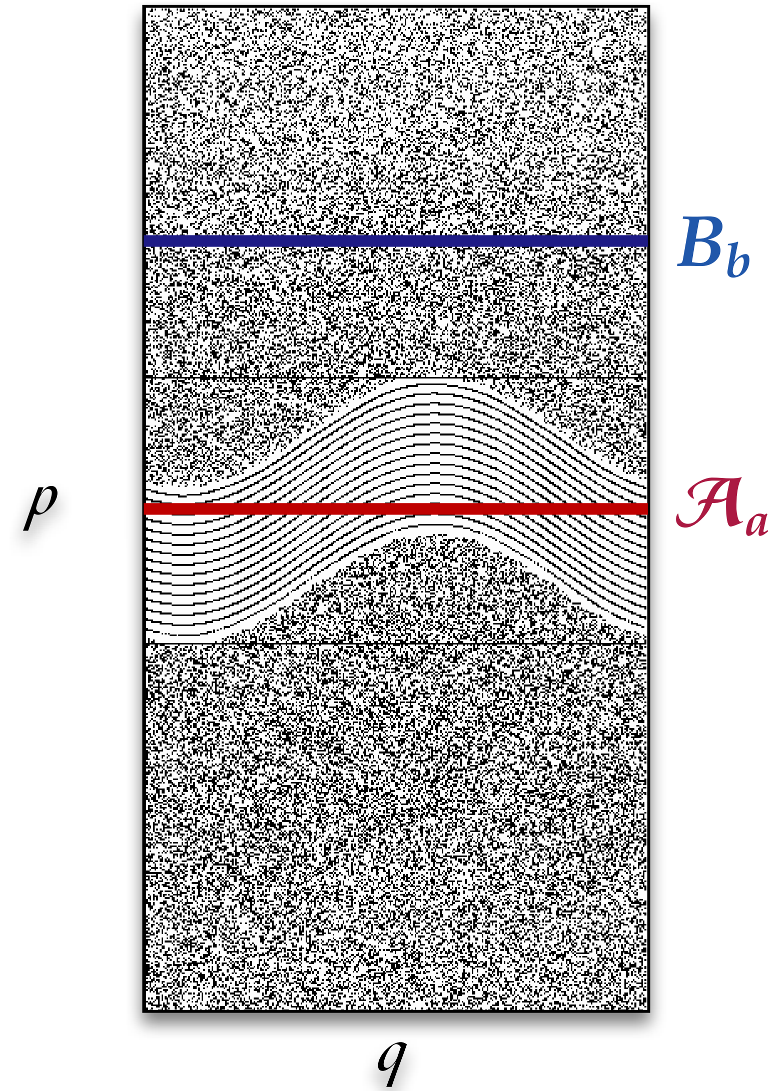
Classical dynamics

$$F : \begin{pmatrix} p' \\ q' \end{pmatrix} = \begin{pmatrix} p - V'(q) \\ q + p' \end{pmatrix}$$

Forbidden process in classical dynamics

$\mathcal{A}_a \cap F^{-n}(\mathcal{B}_b) = \emptyset$ **for** $\forall n$**, if** $\mathcal{A}_a, \mathcal{B}_b (\in \mathbb{R})$ **are dynamically separated.**





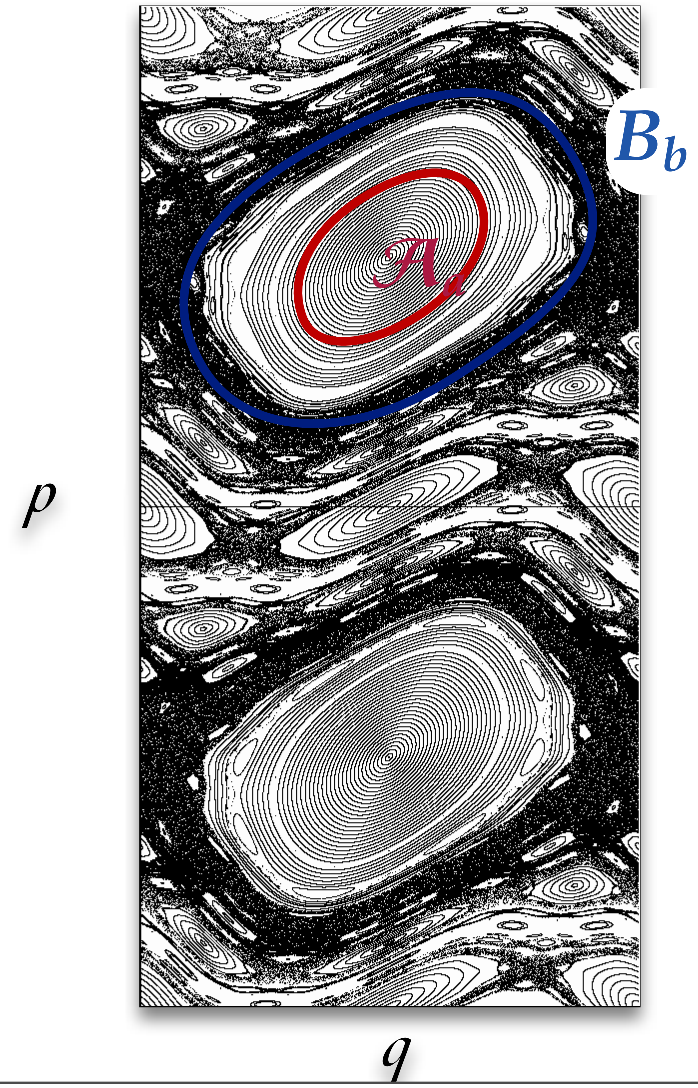
Dynamical tunneling in mixed phase space

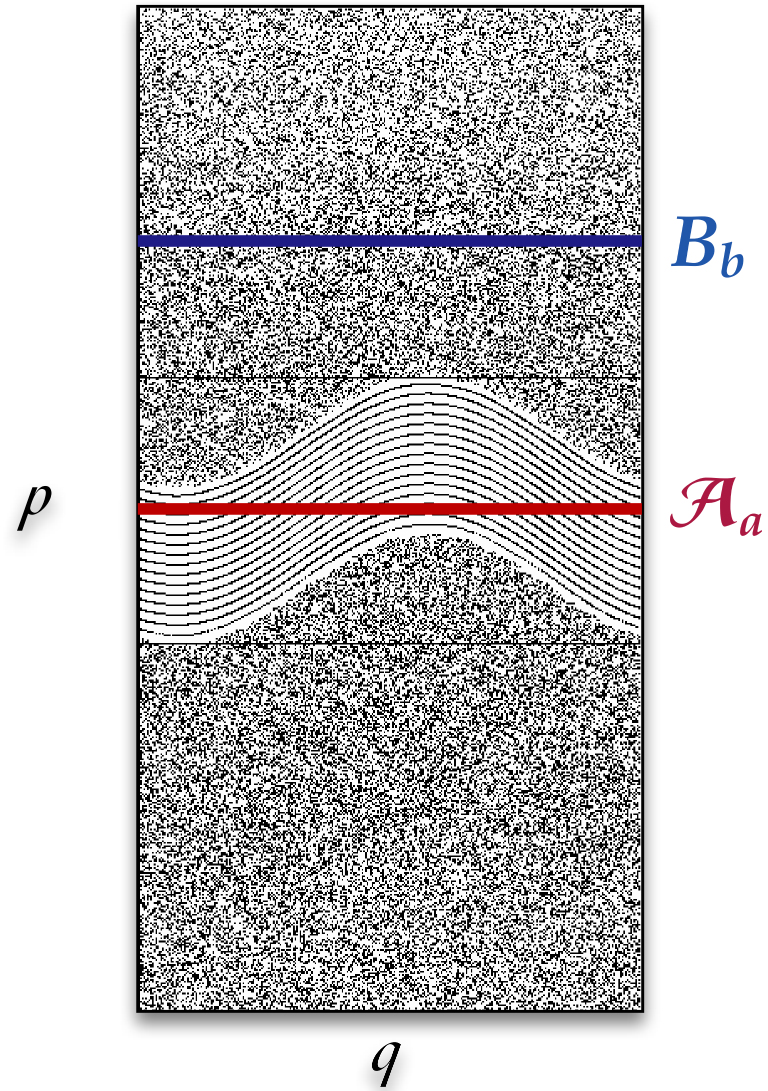
Quantum dynamics

$$K(a,b) = \langle b|\hat{U}^n|a\rangle = \int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty}\prod_j dq_j \prod_j dp_j \exp\left[\frac{i}{\hbar}S(\{q_j\},\{p_j\})\right]$$

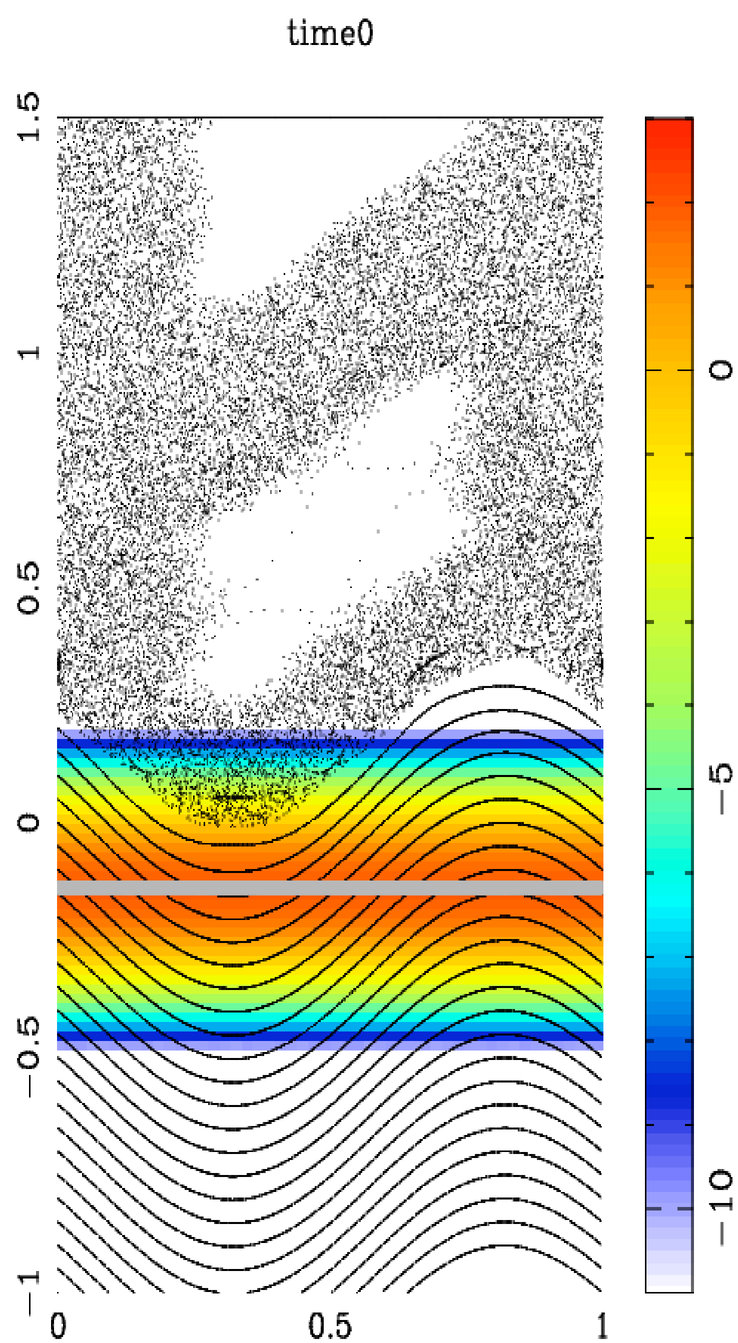
Tunneling process in quantum dynamics

$K(a,b) \neq 0$ even if $\mathcal{A}_a, B_b (\in \mathbb{R})$ are dynamically separated.

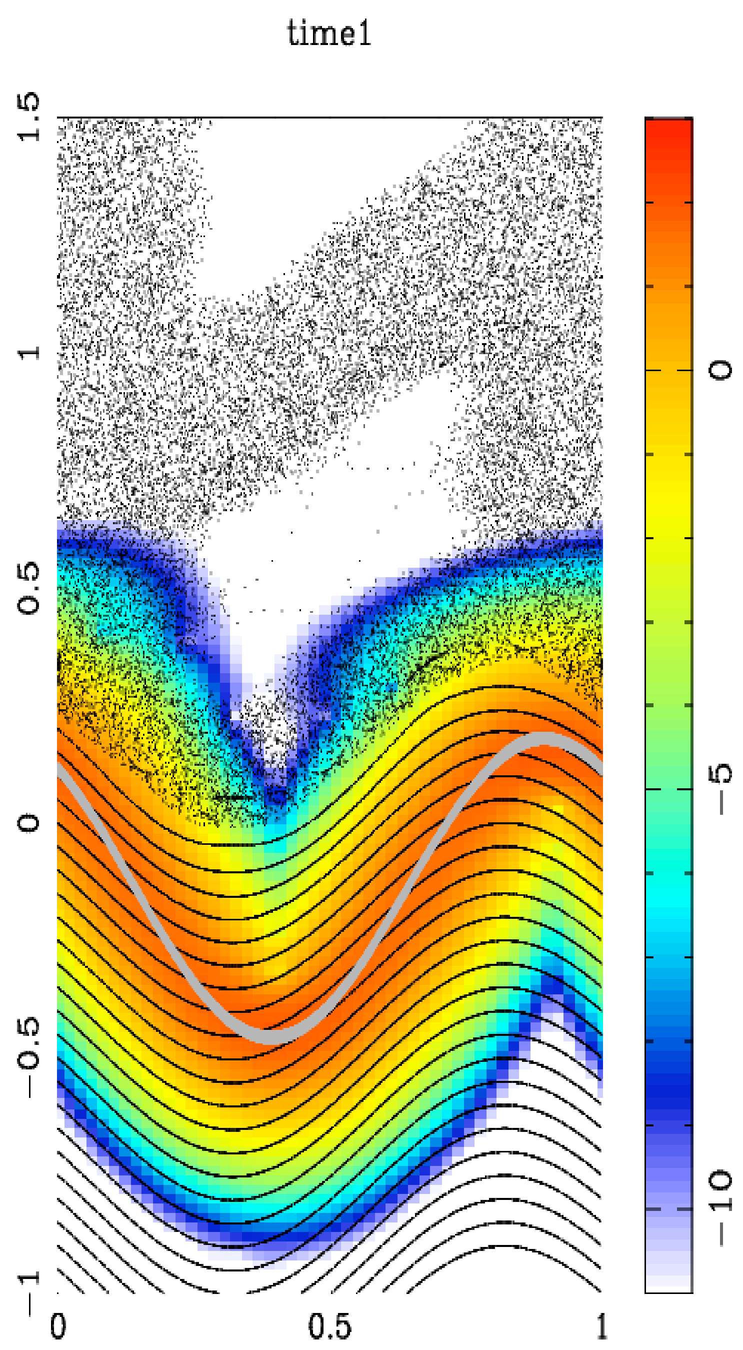




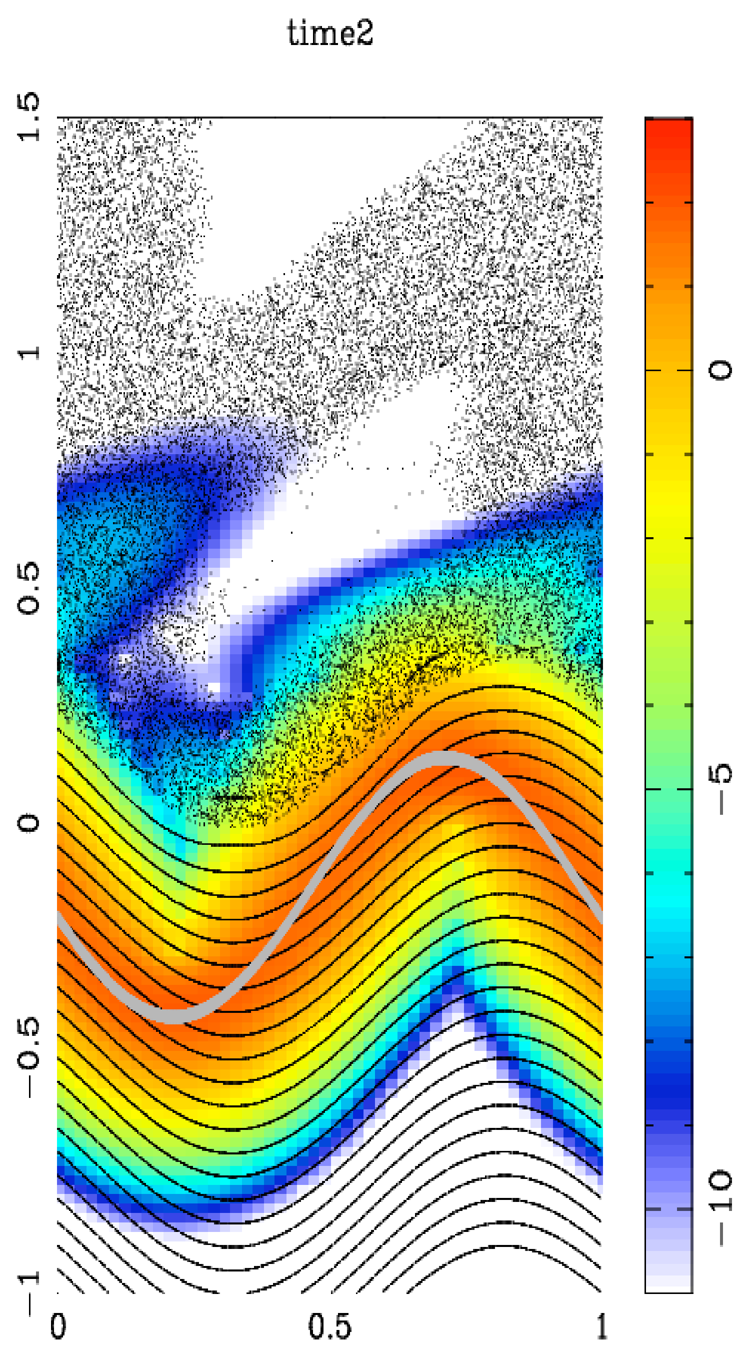
Dynamical tunneling in mixed phase space



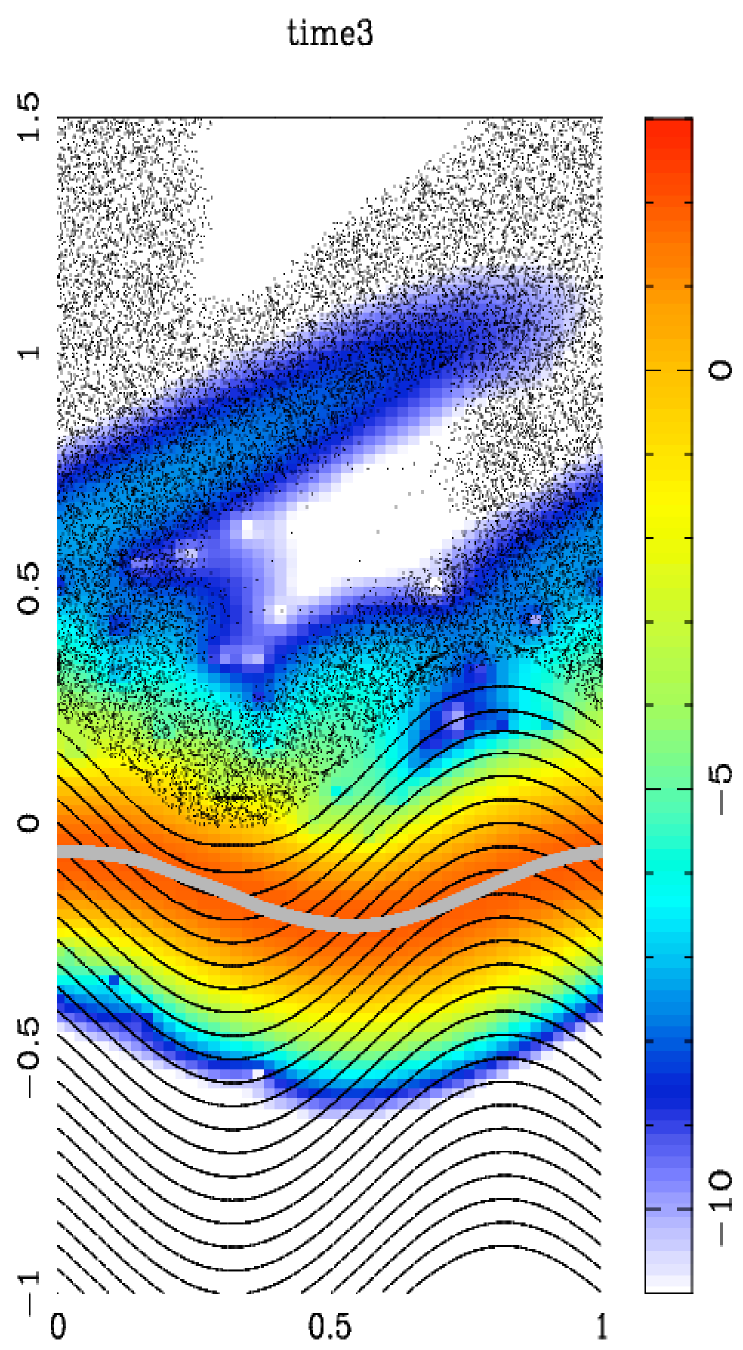
Dynamical tunneling in mixed phase space



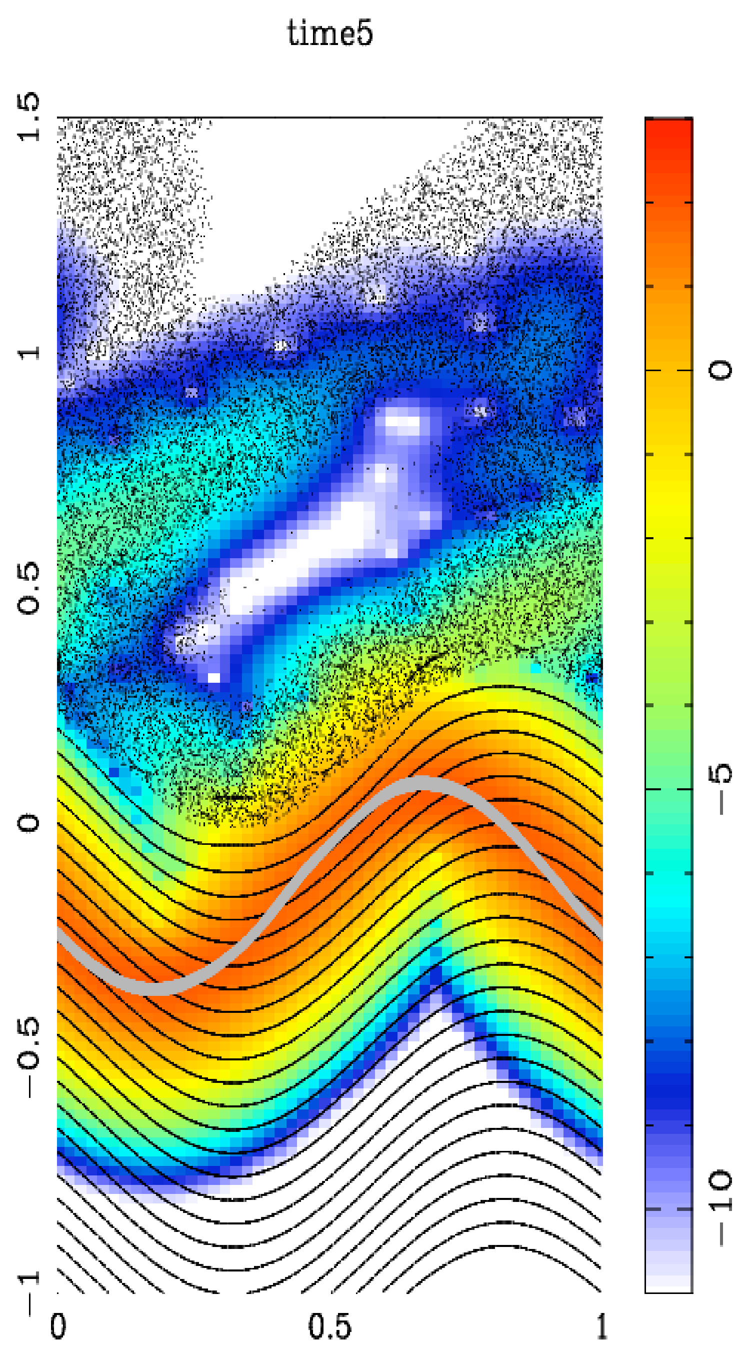
Dynamical tunneling in mixed phase space



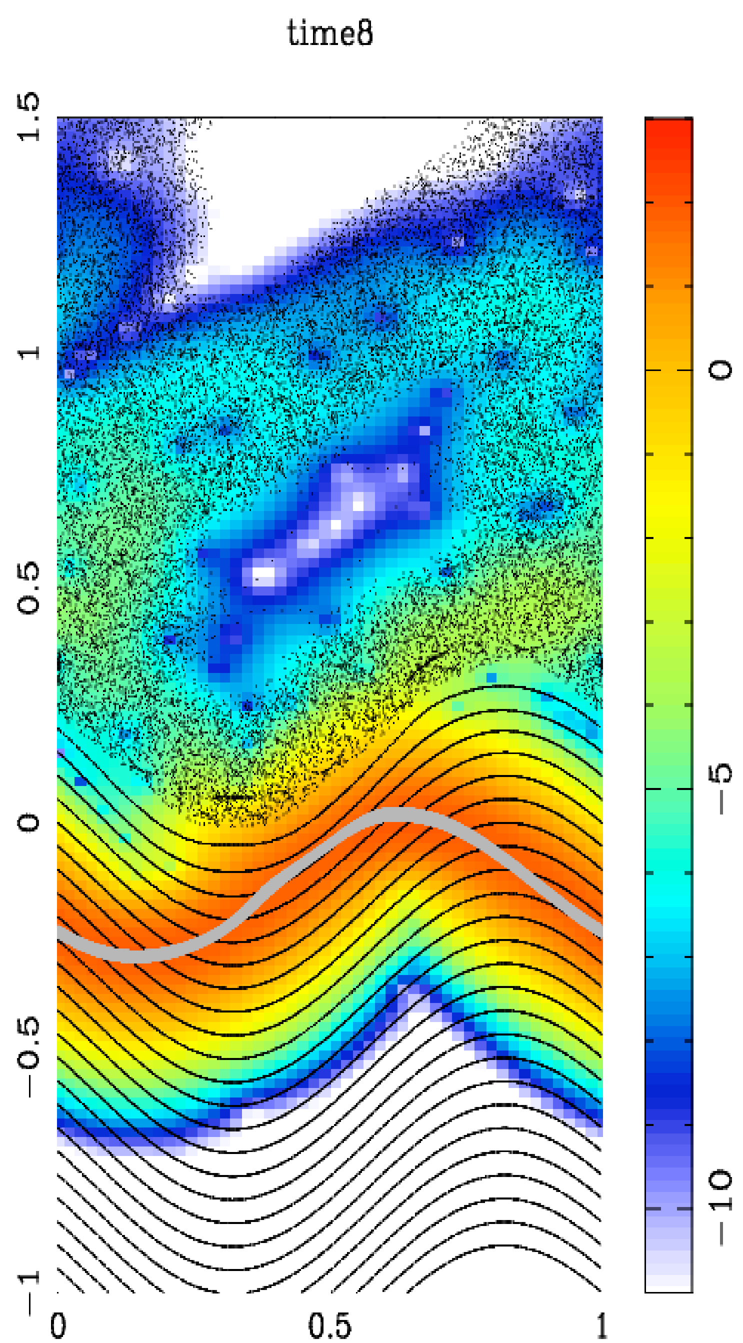
Dynamical tunneling in mixed phase space



Dynamical tunneling in mixed phase space

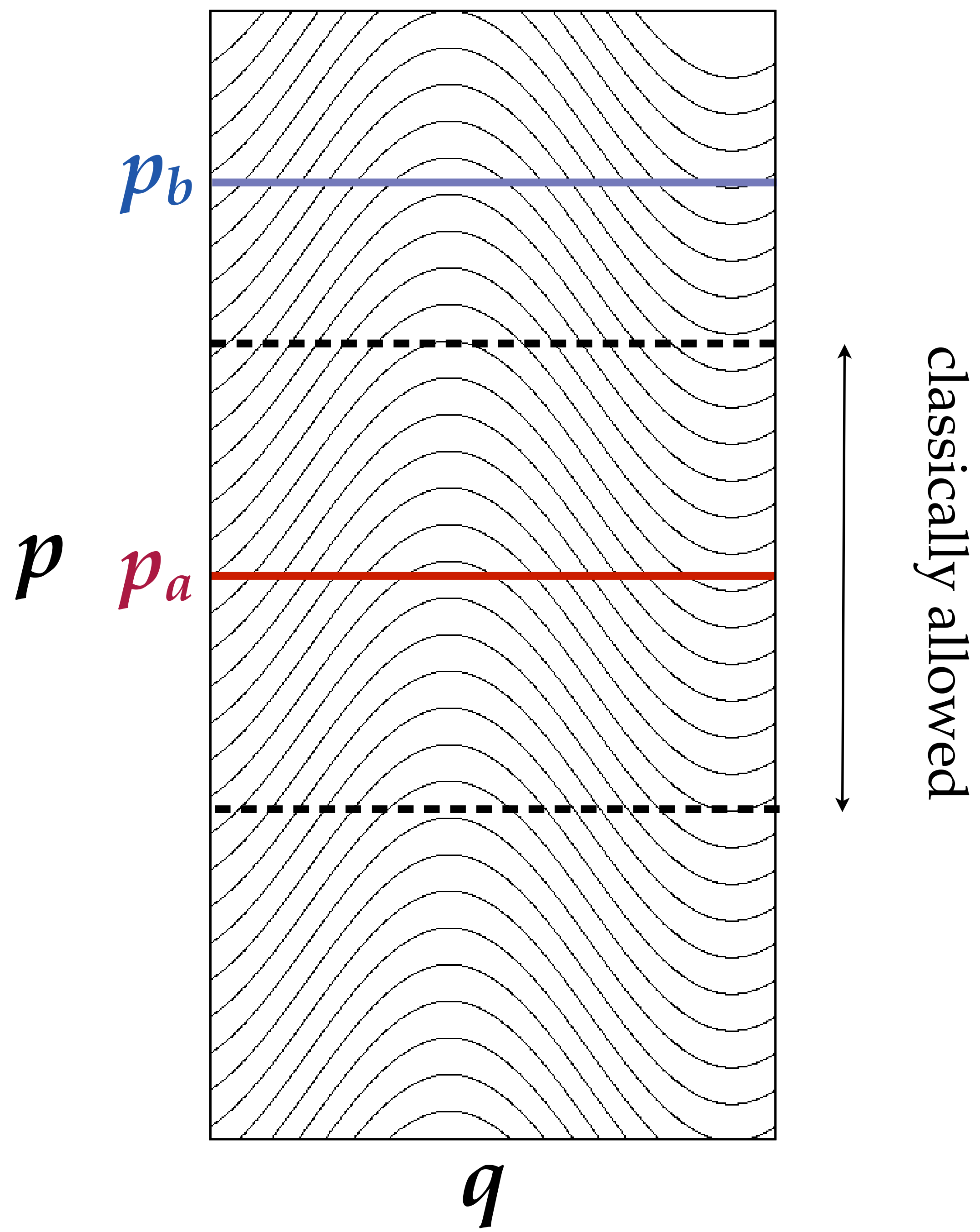


Dynamical tunneling in mixed phase space



A completely integrable model

$$F : \begin{pmatrix} p' \\ q' \end{pmatrix} = \begin{pmatrix} p + K \sin q \\ q + \omega \end{pmatrix}$$



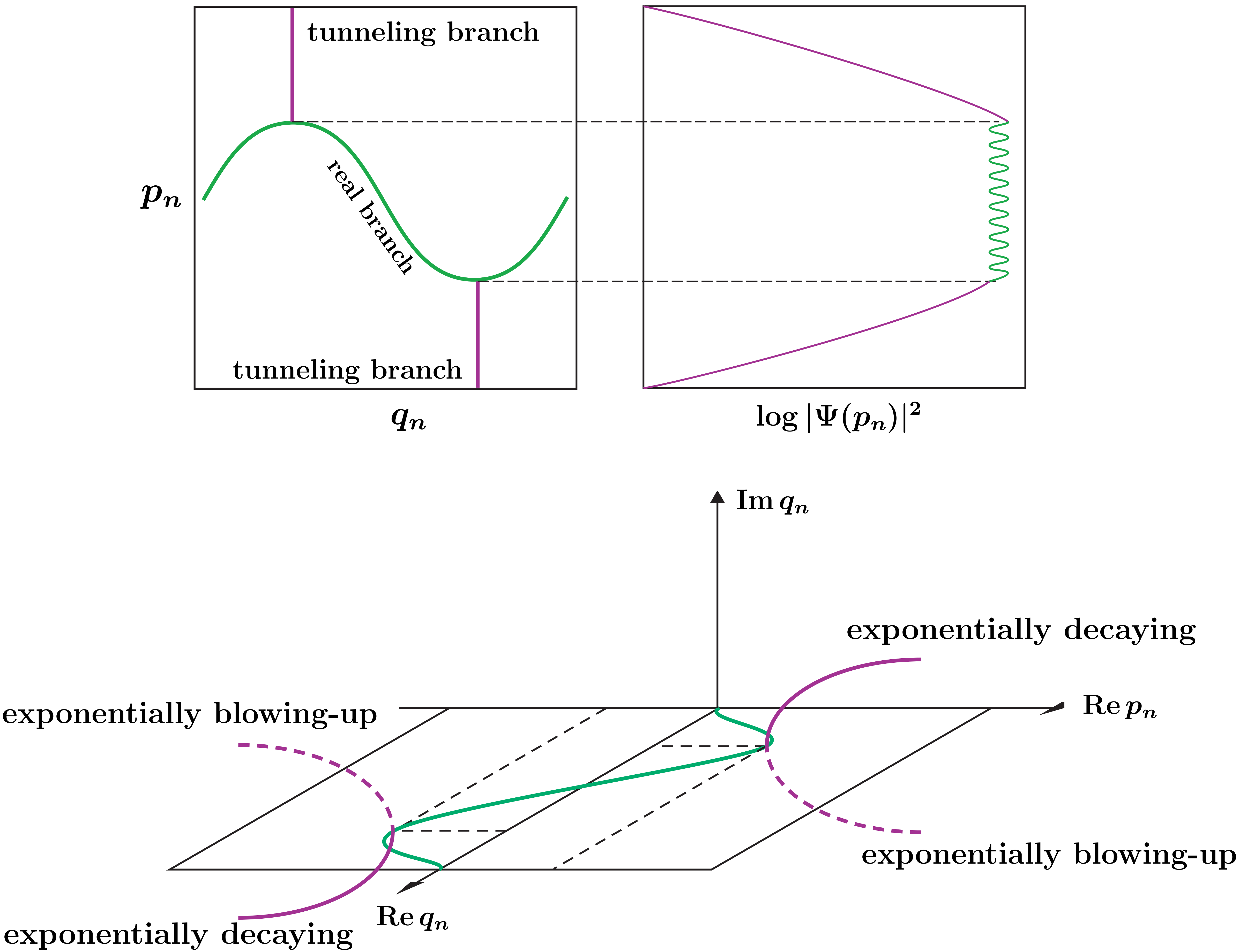
$\mathcal{M}_n^{a,b} = A_a \cap F^{-n}(B_b) = \emptyset$ for $\forall n \in \mathbb{Z}$

if B_b is outside the classically allowed region.

where

$A_a = \{ (p, q) \in \mathbb{R}^2 \,|\, p = p_a \}$

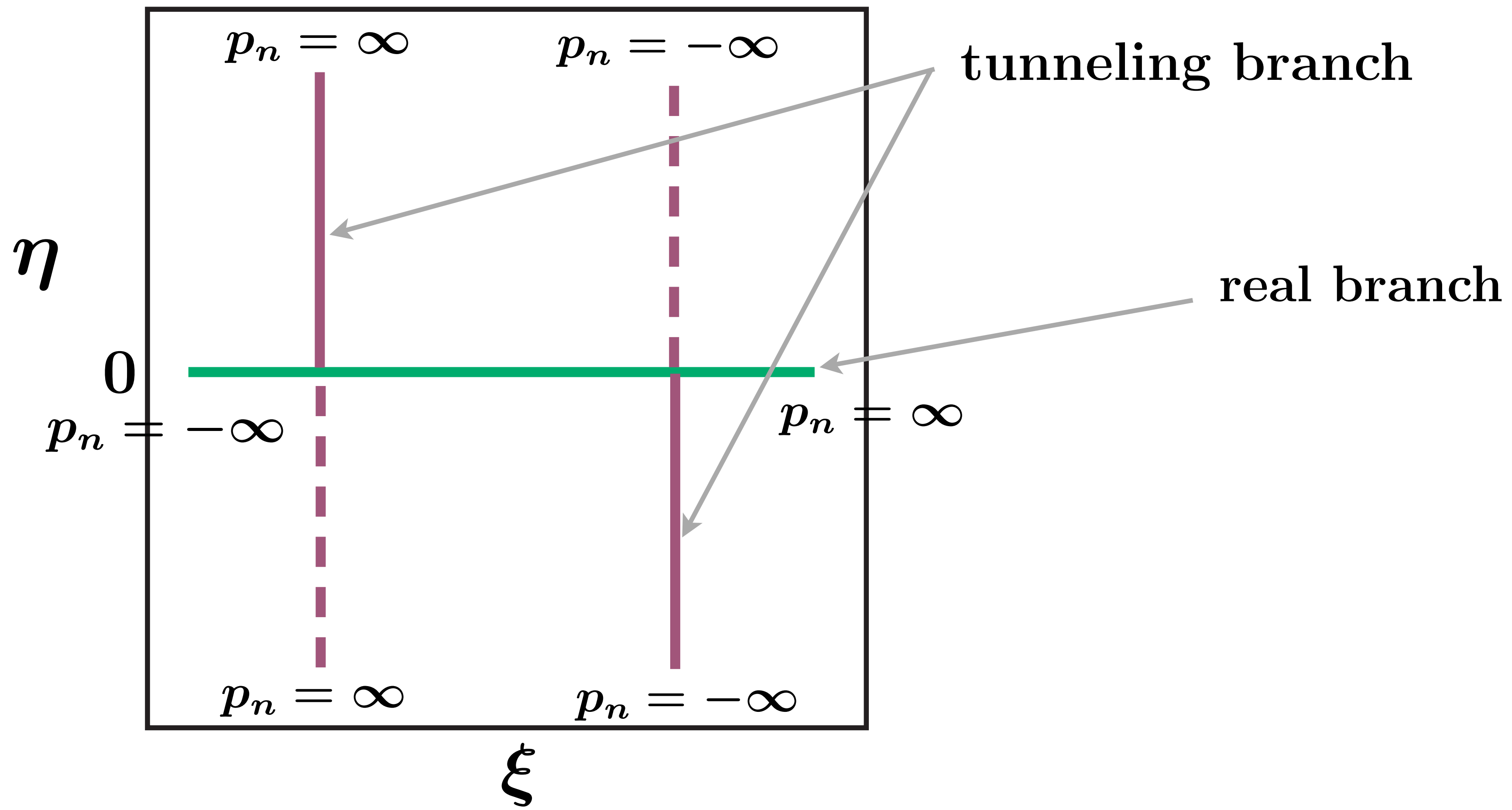
$B_b = \{ (p, q) \in \mathbb{R}^2 \,|\, p = p_b \}$


tunneling branch
real branch
p_n
tunneling branch
q_n
$\log|\Psi(p_n)|^2$
$\mathrm{Im}\, q_n$
exponentially decaying
exponentially blowing-up
$\mathrm{Re}\, p_n$
exponentially blowing-up
$\mathrm{Re}\, q_n$
exponentially decaying

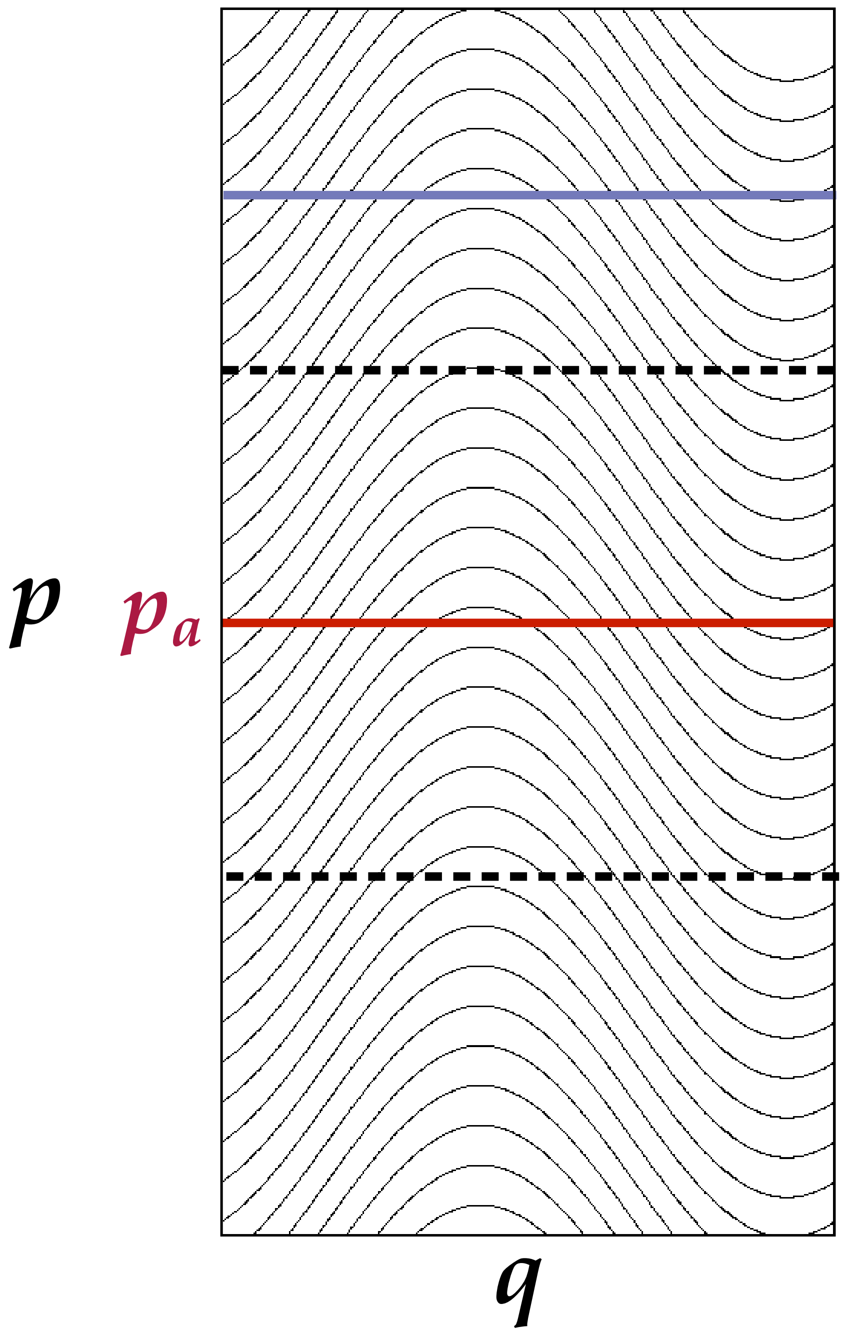
Initial value representation of complex orbits

Set of initial conditions contributing to semiclassical propagator

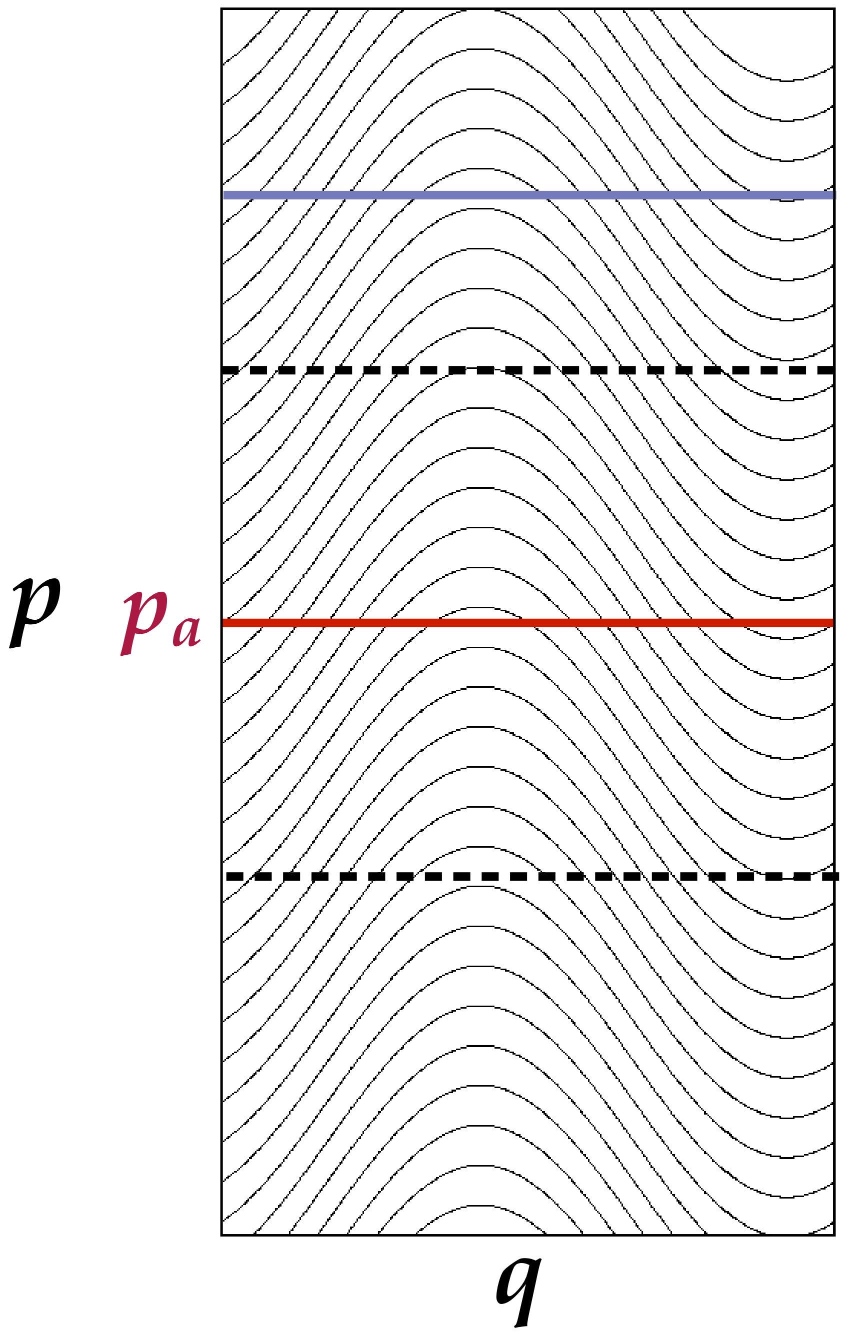
$$\mathcal{M}_n^{\alpha,*} = \{\, q_0 = \xi + i\eta \,|\, p_0 = \alpha \in \mathbb{R},\ -\infty < p_n < \infty \,\}$$

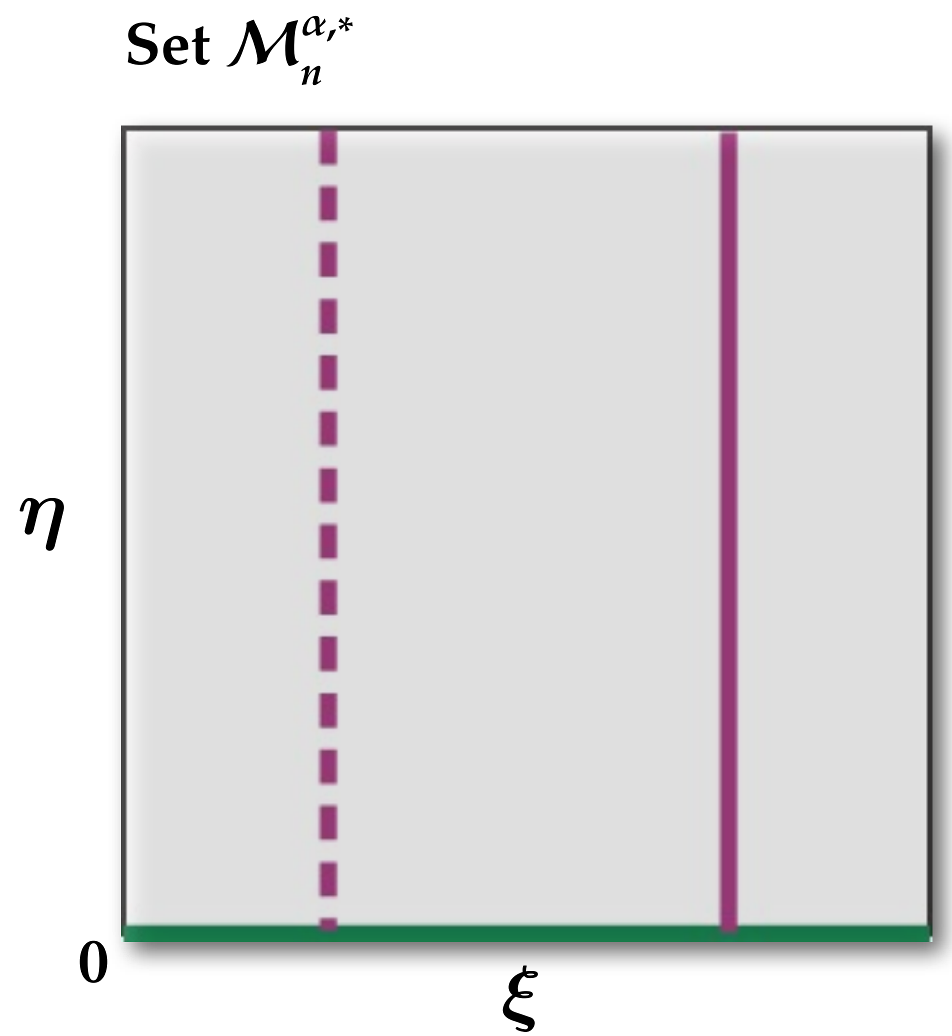


Completely integrable map

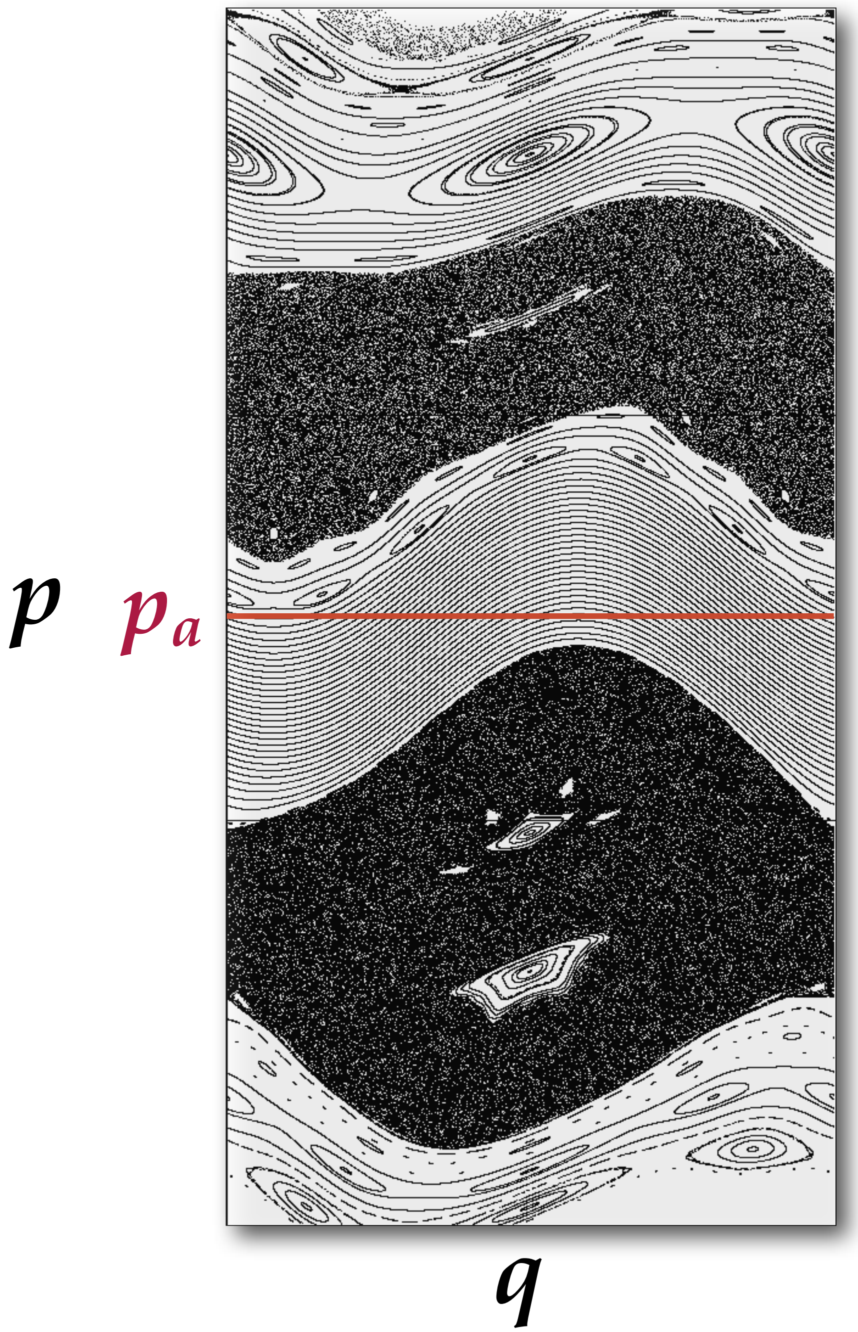


Completely integrable map

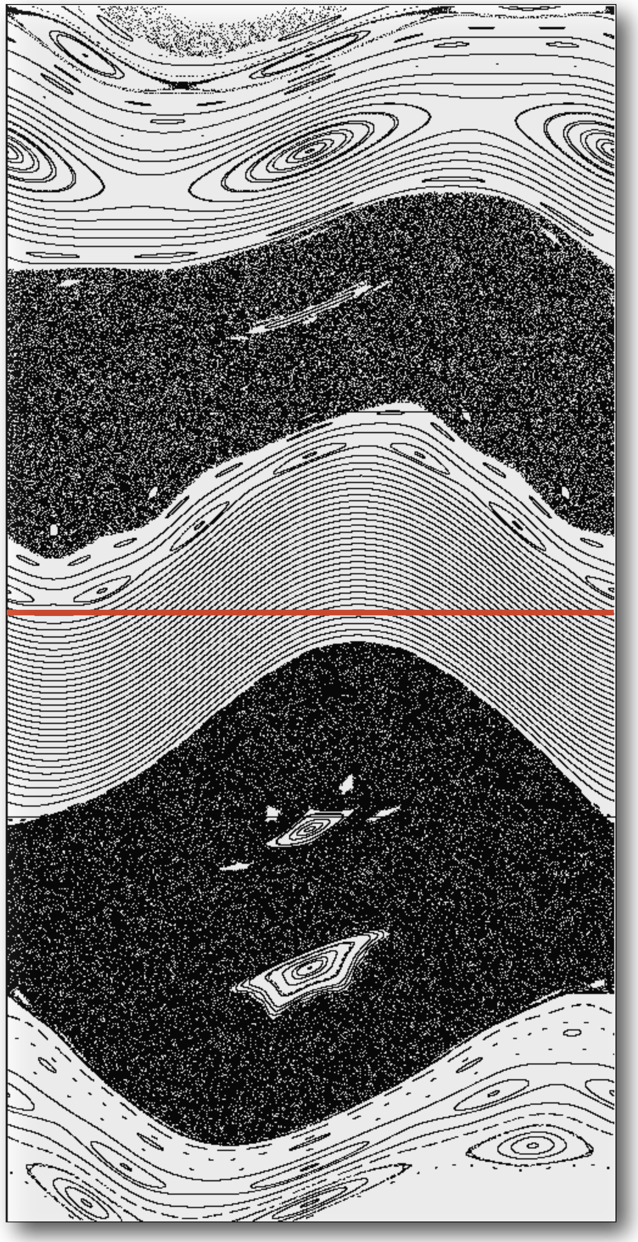


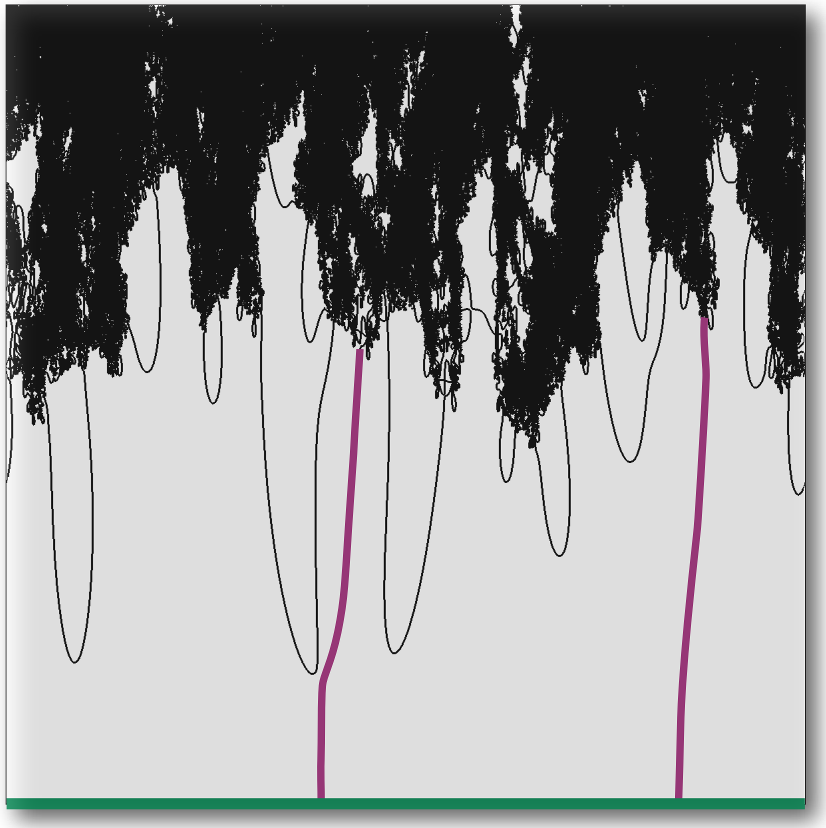


Nonintegrable map



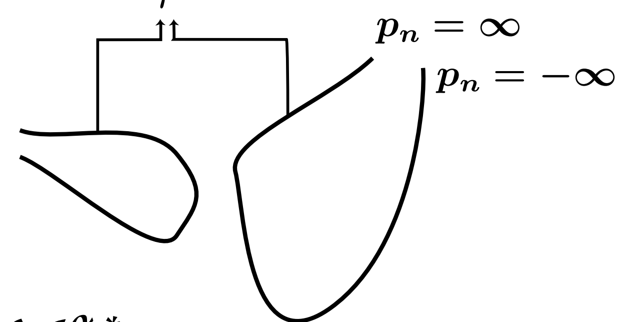
Nonintegrable map

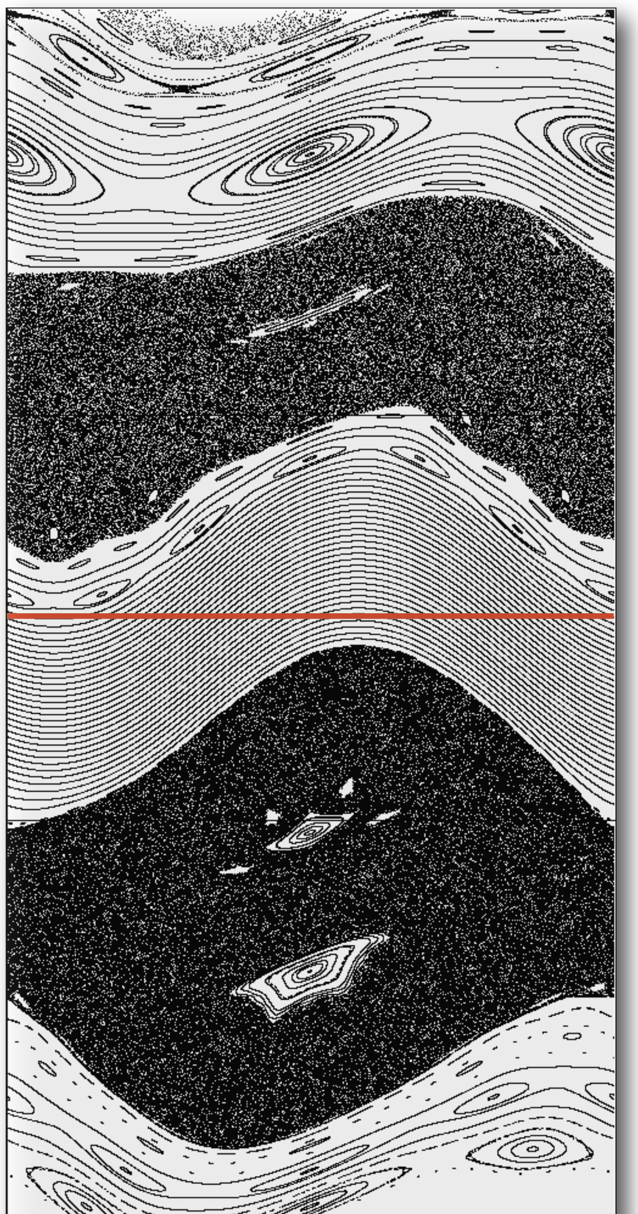




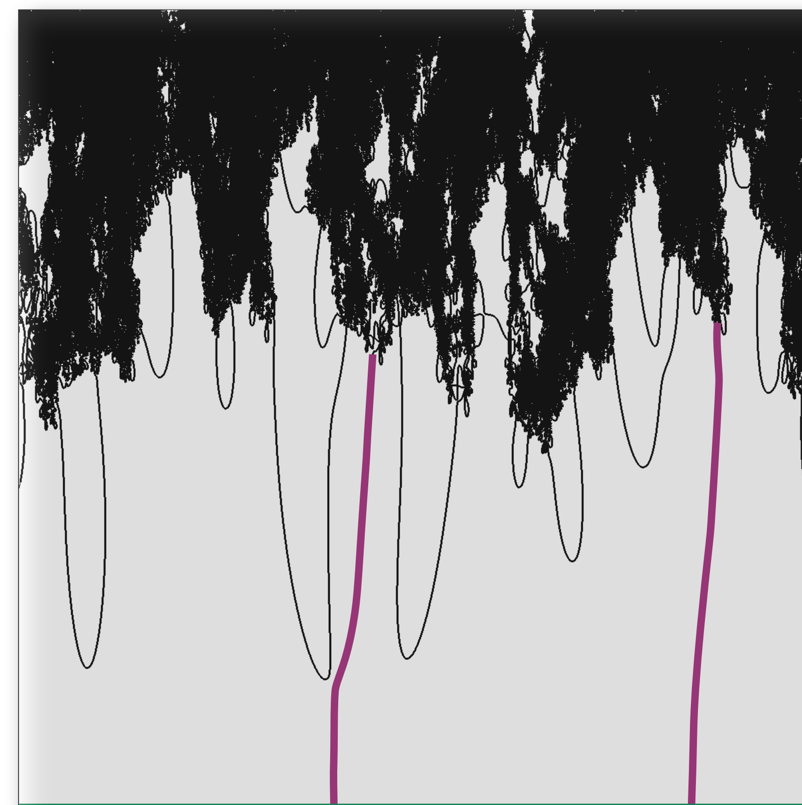
Nonintegrable map

$$K^{sc} = \sum_{\gamma} A_{\gamma} \exp\left[\frac{i}{\hbar} S_{\gamma}\right]$$

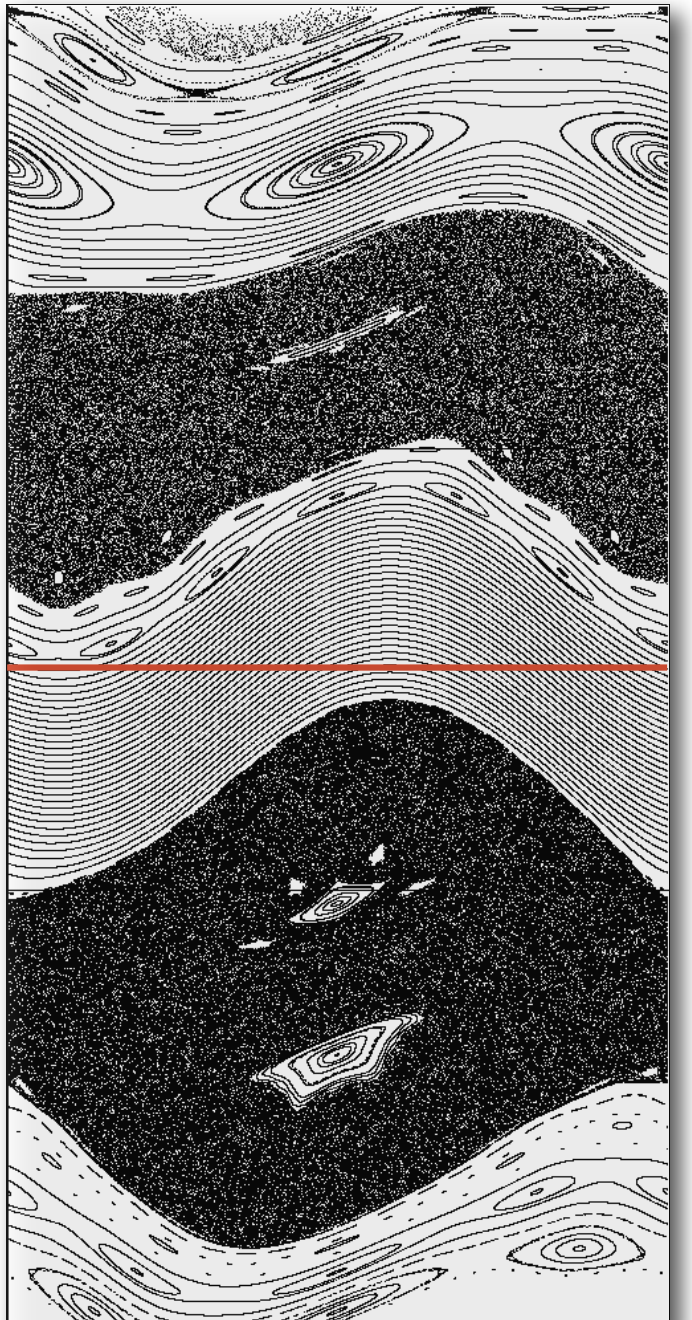


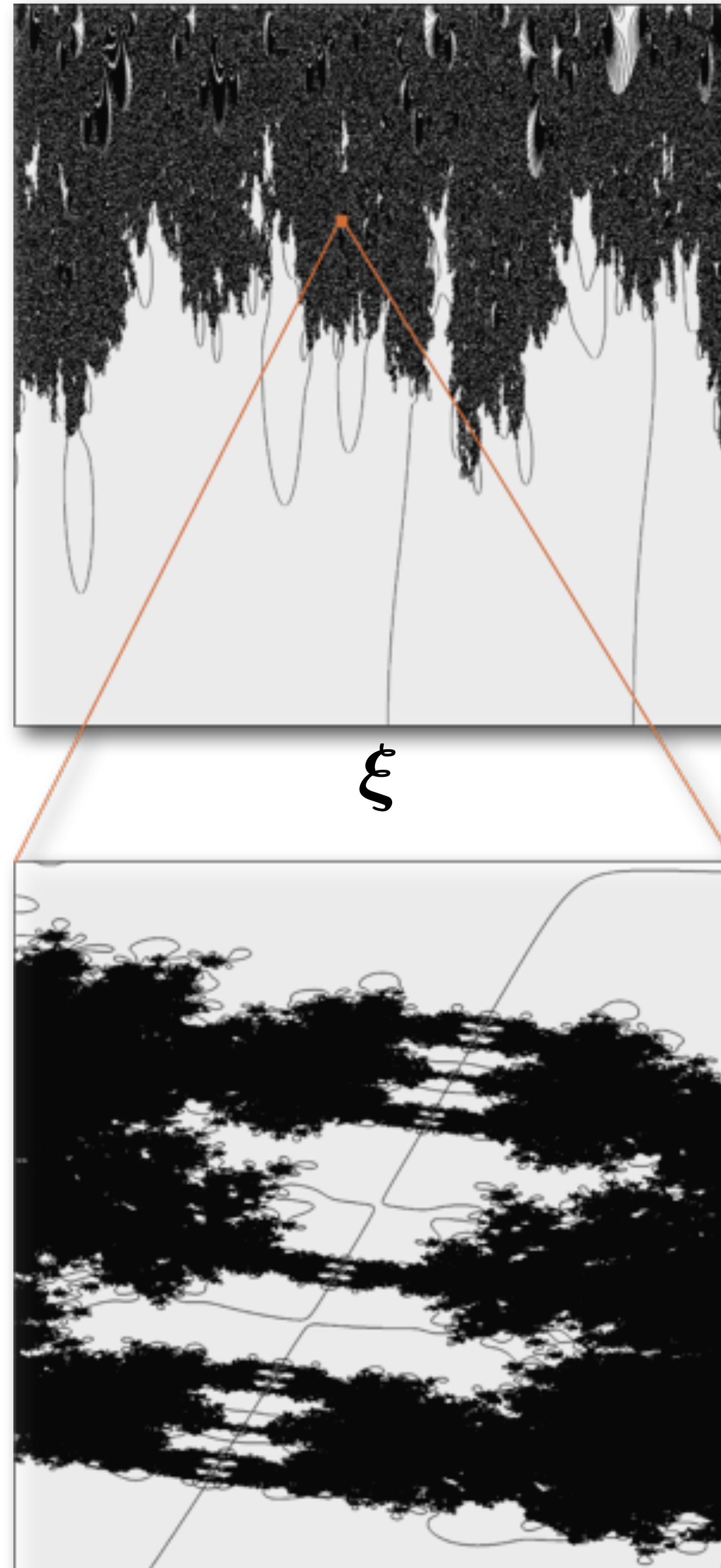


Set $\mathcal{M}_n^{\alpha,*}$

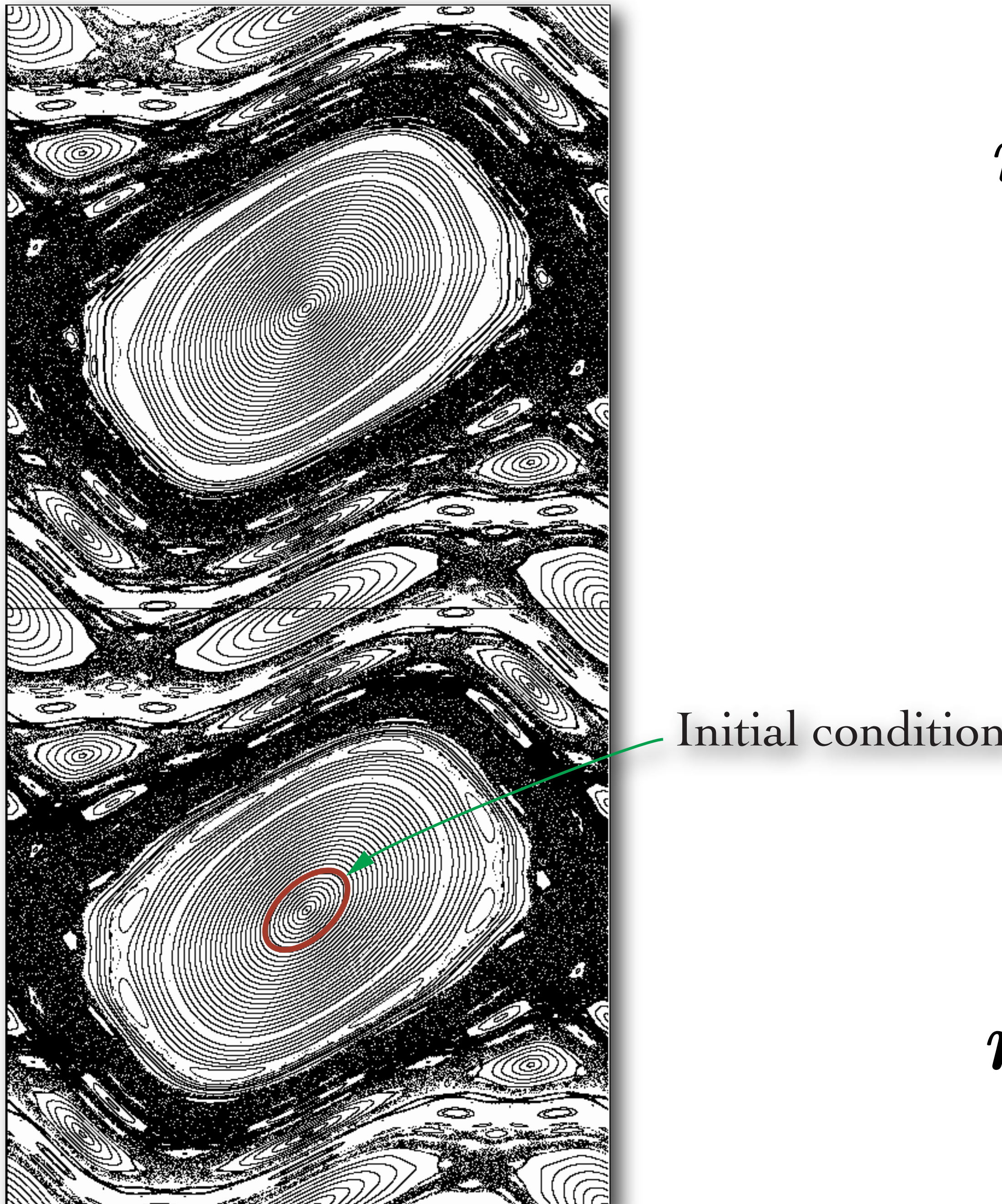


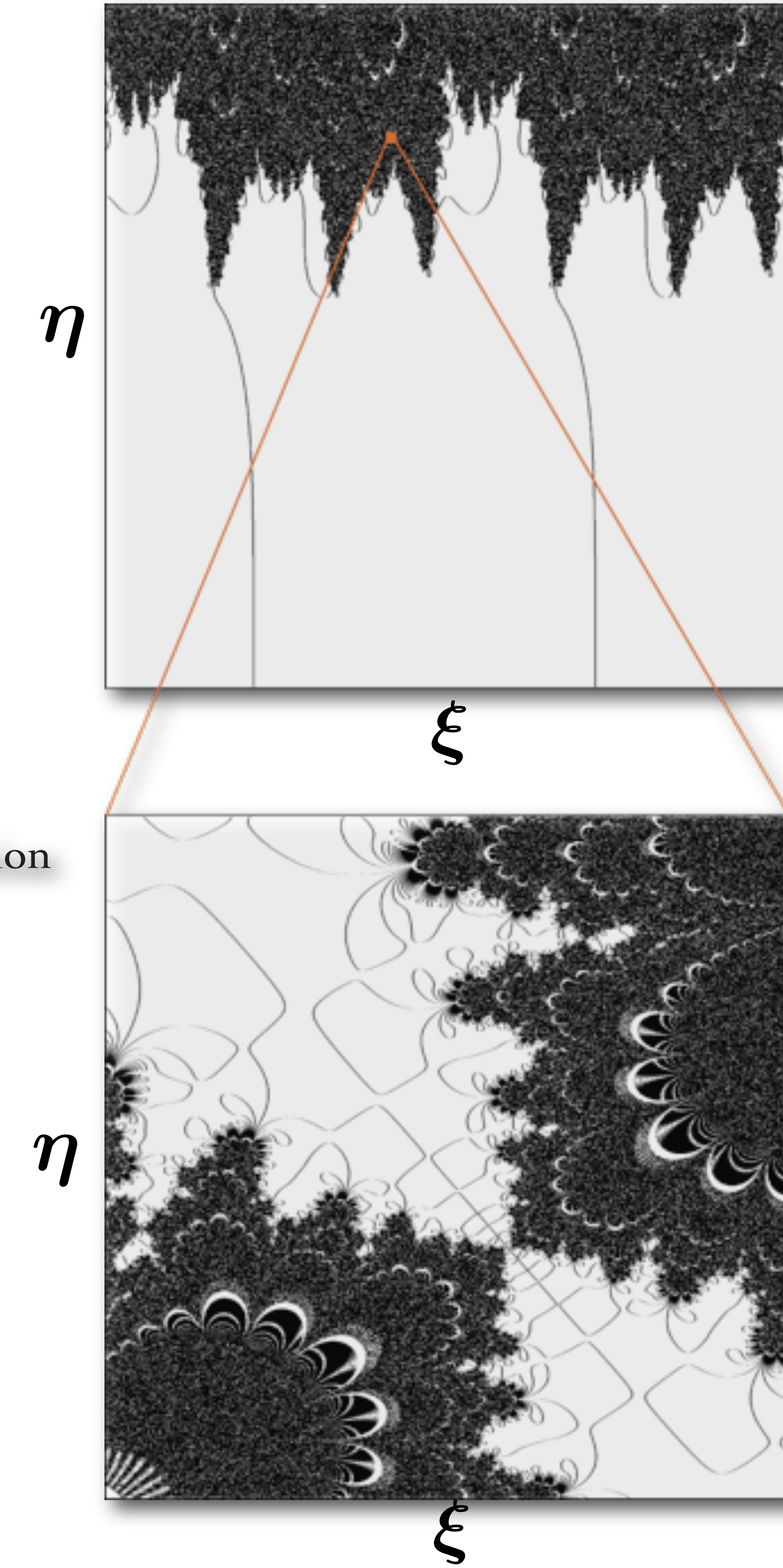
Modfied standard
K = 1.2



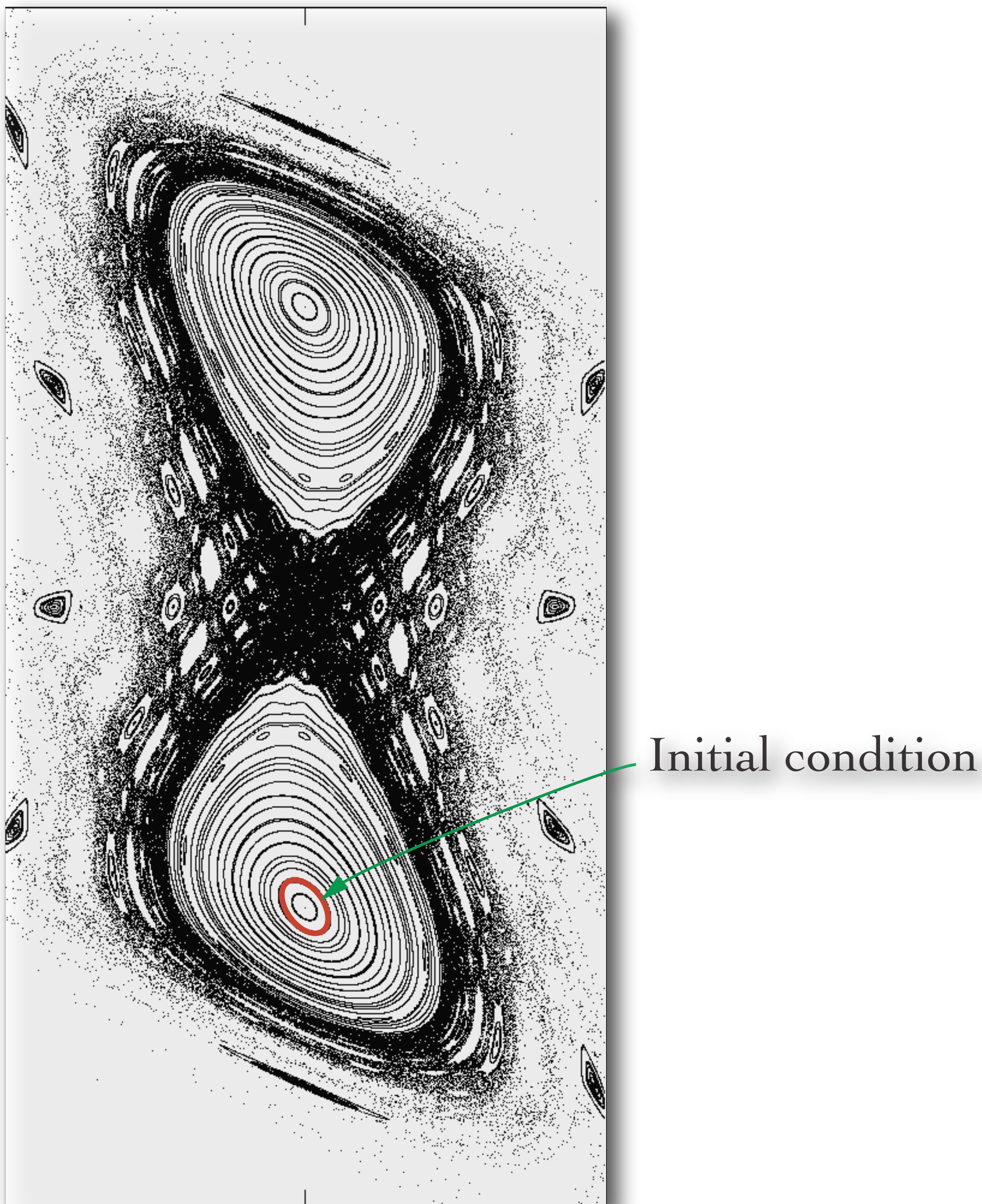


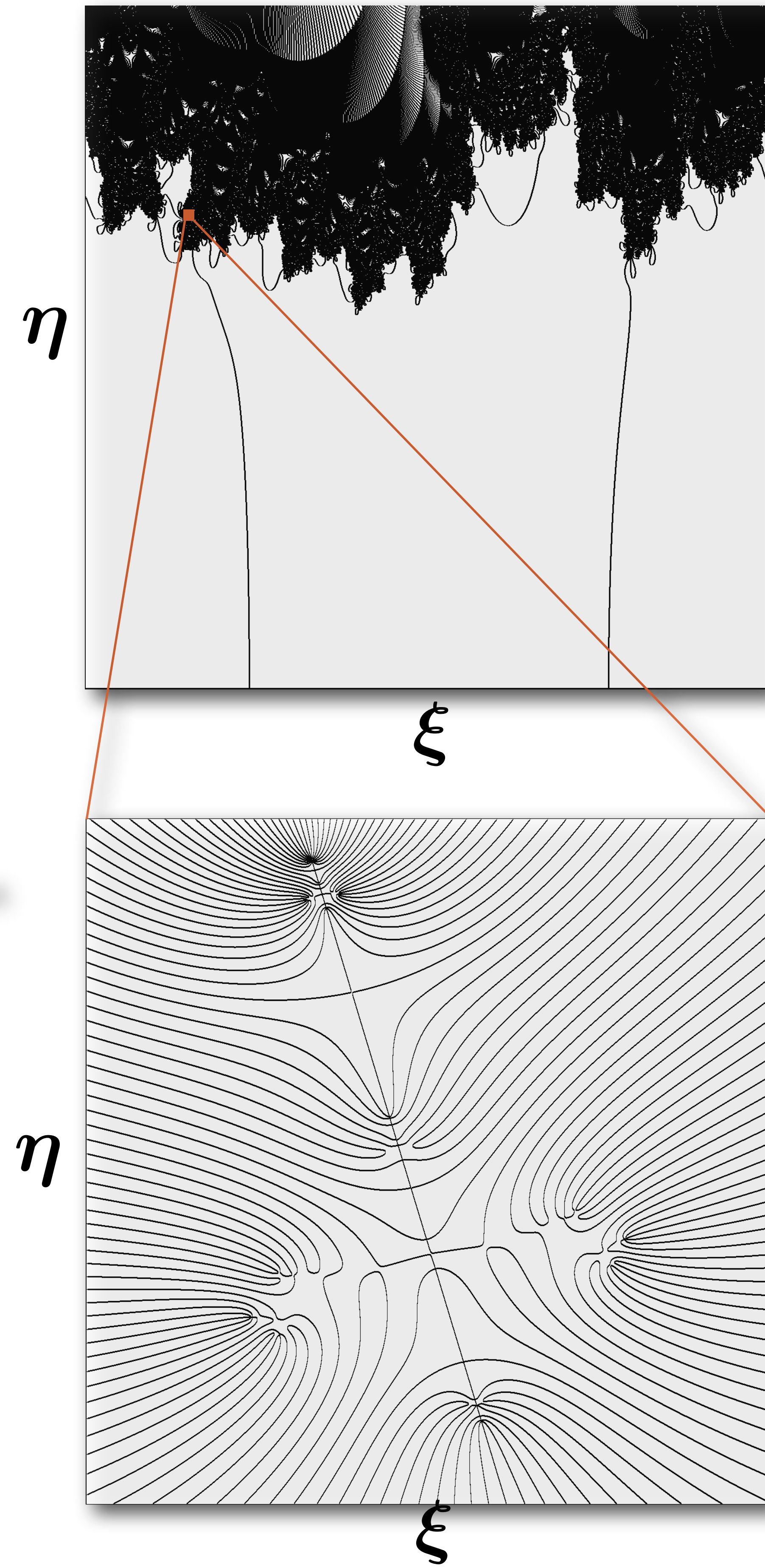
Standard map
K = 1.0

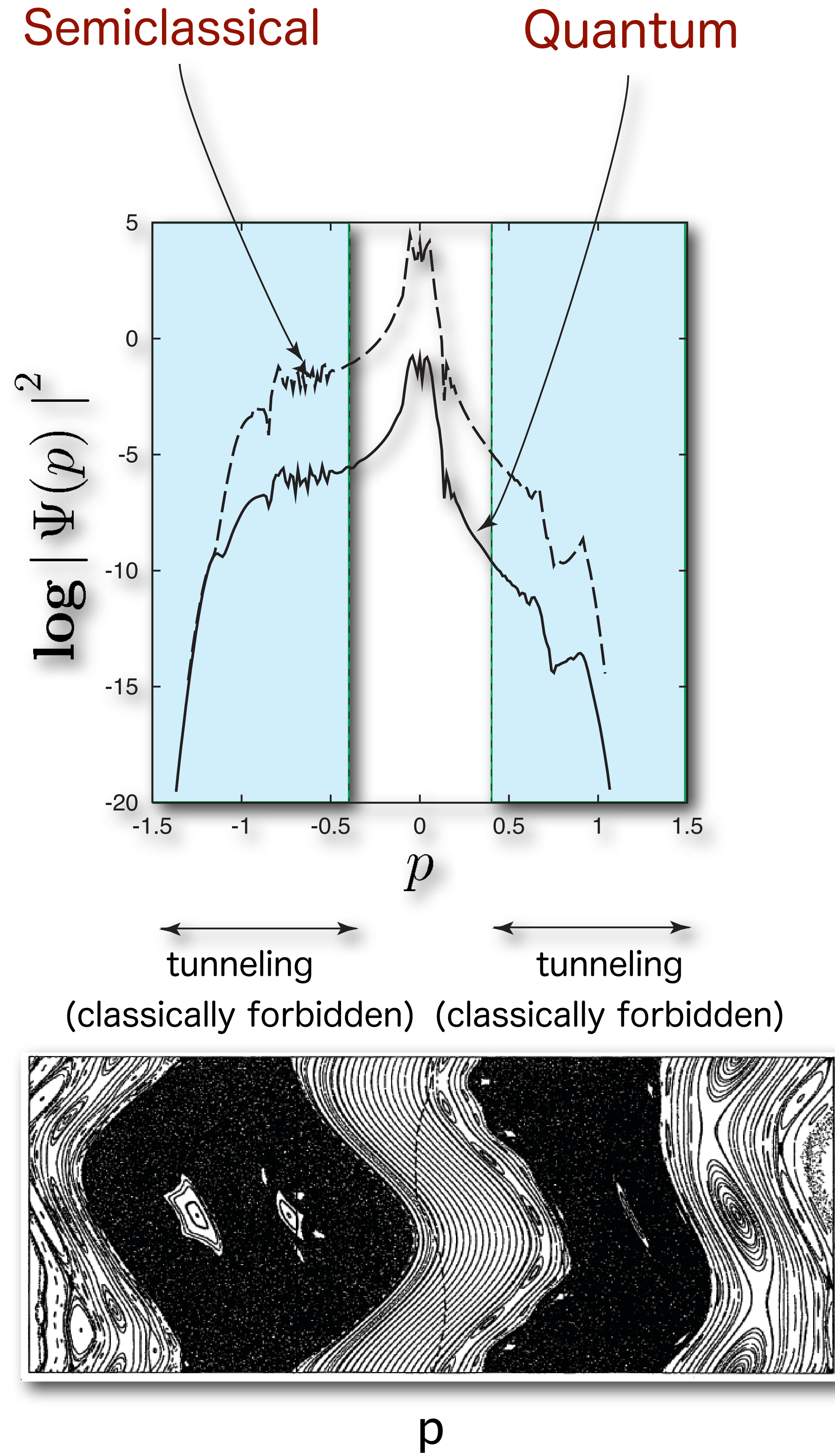




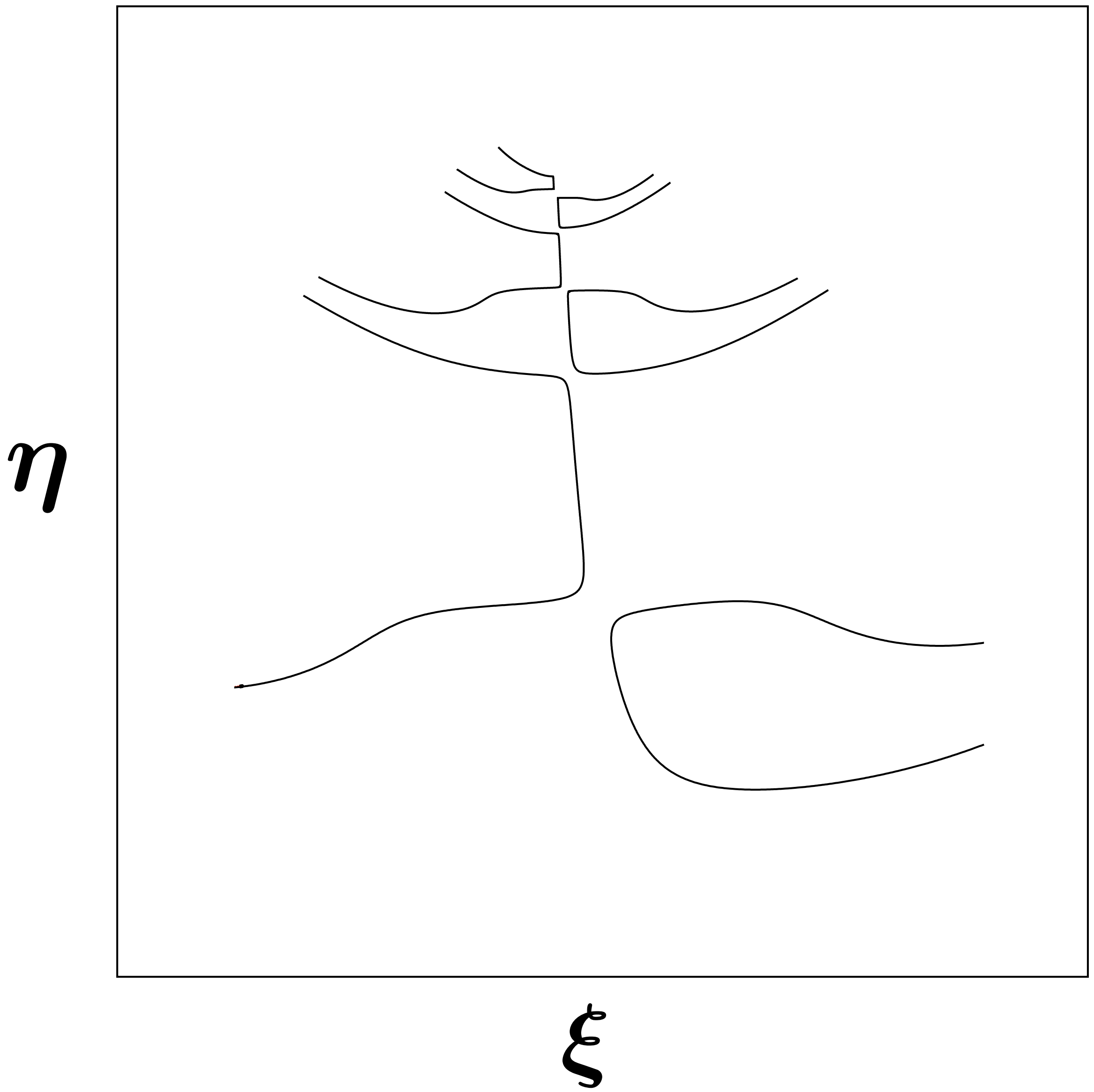
4-th order polynomial potential



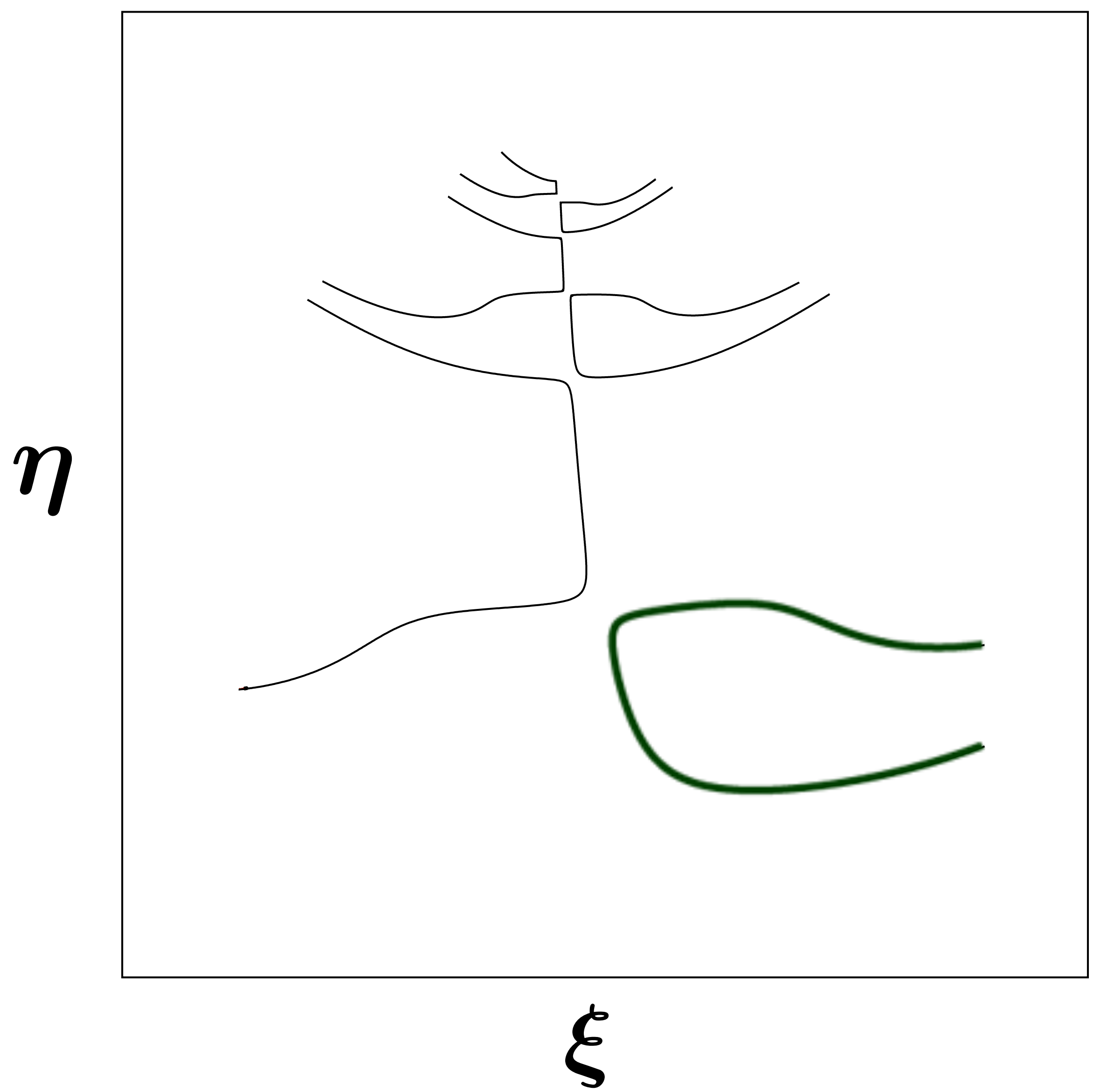


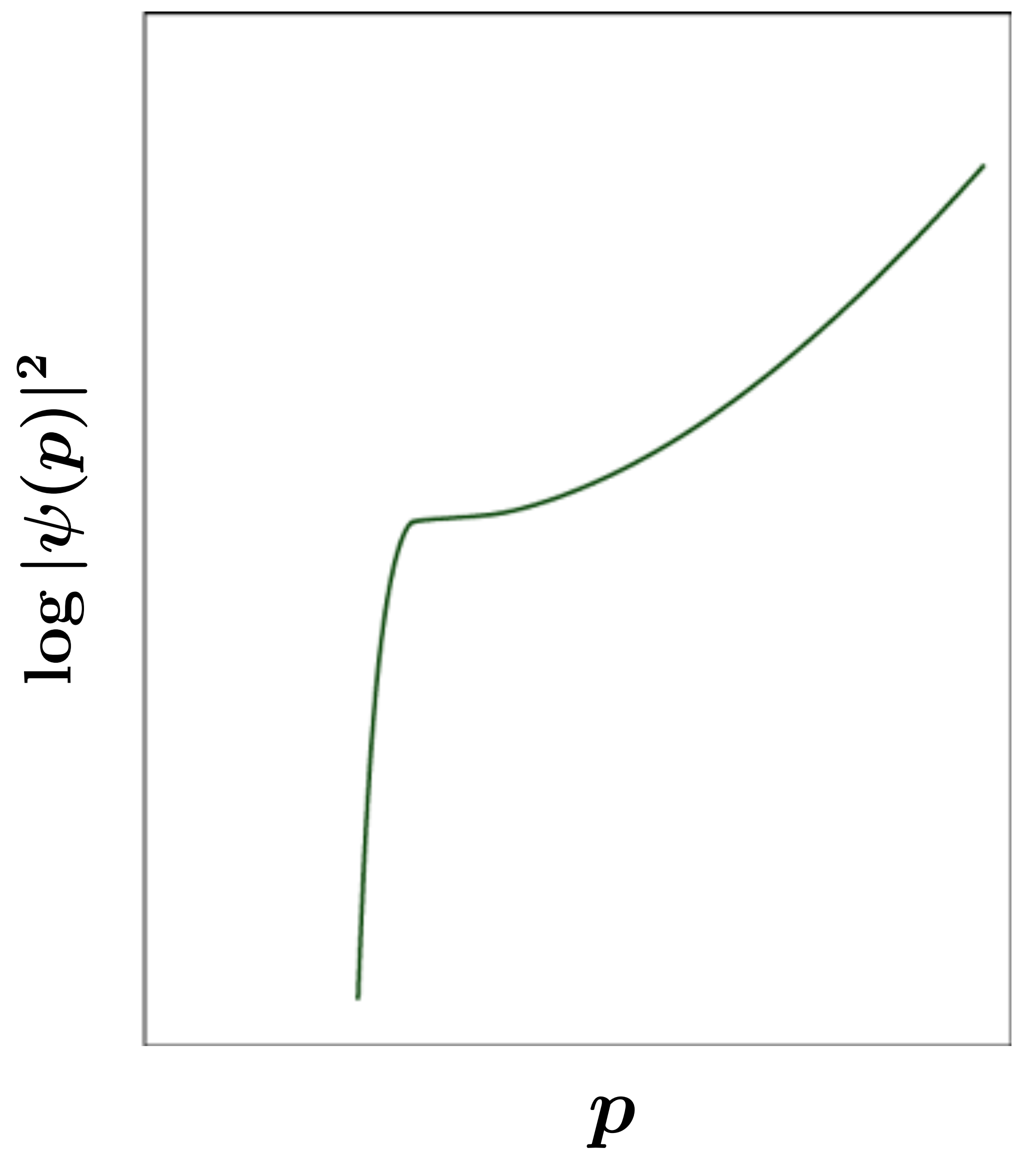

Semiclassical
Quantum
5
0
-5
-10
-15
-20
-1.5
-1
-0.5
0
0.5
1
1.5
$\log|\Psi(p)|^2$
p
tunneling
(classically forbidden)
tunneling
(classically forbidden)
p

Semiclassical contributions in chained branches

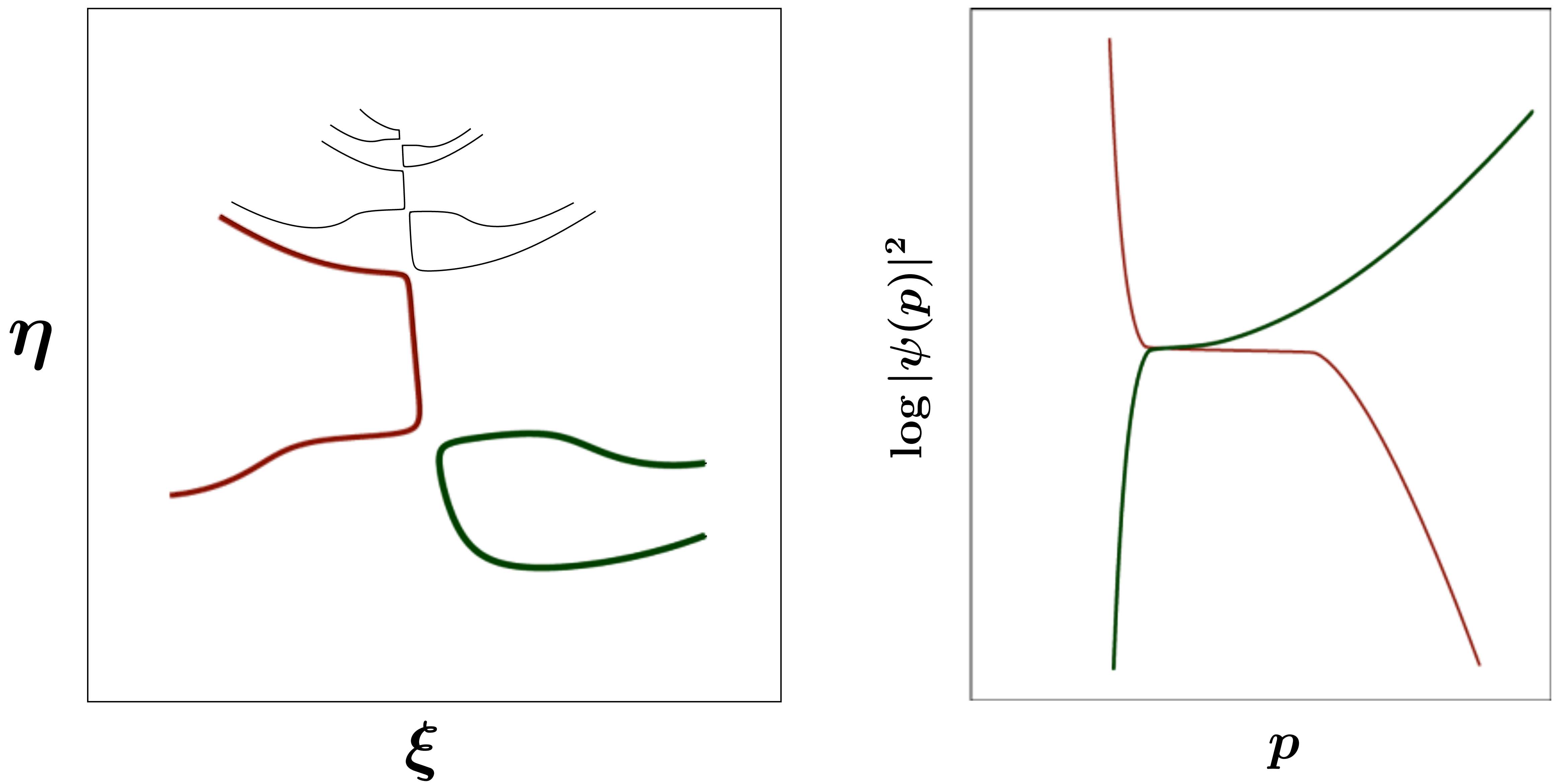


Semiclassical contributions in chained branches

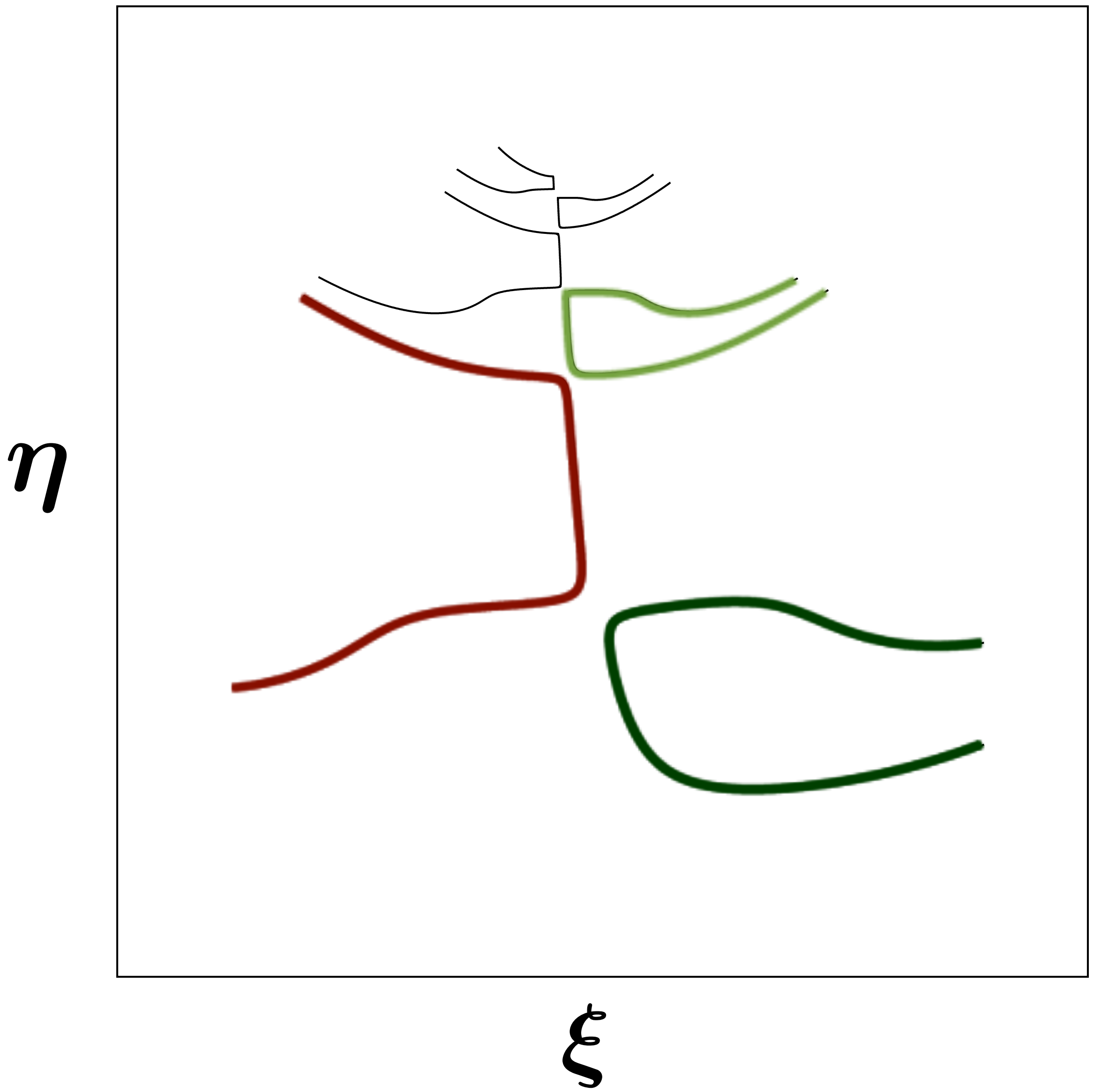


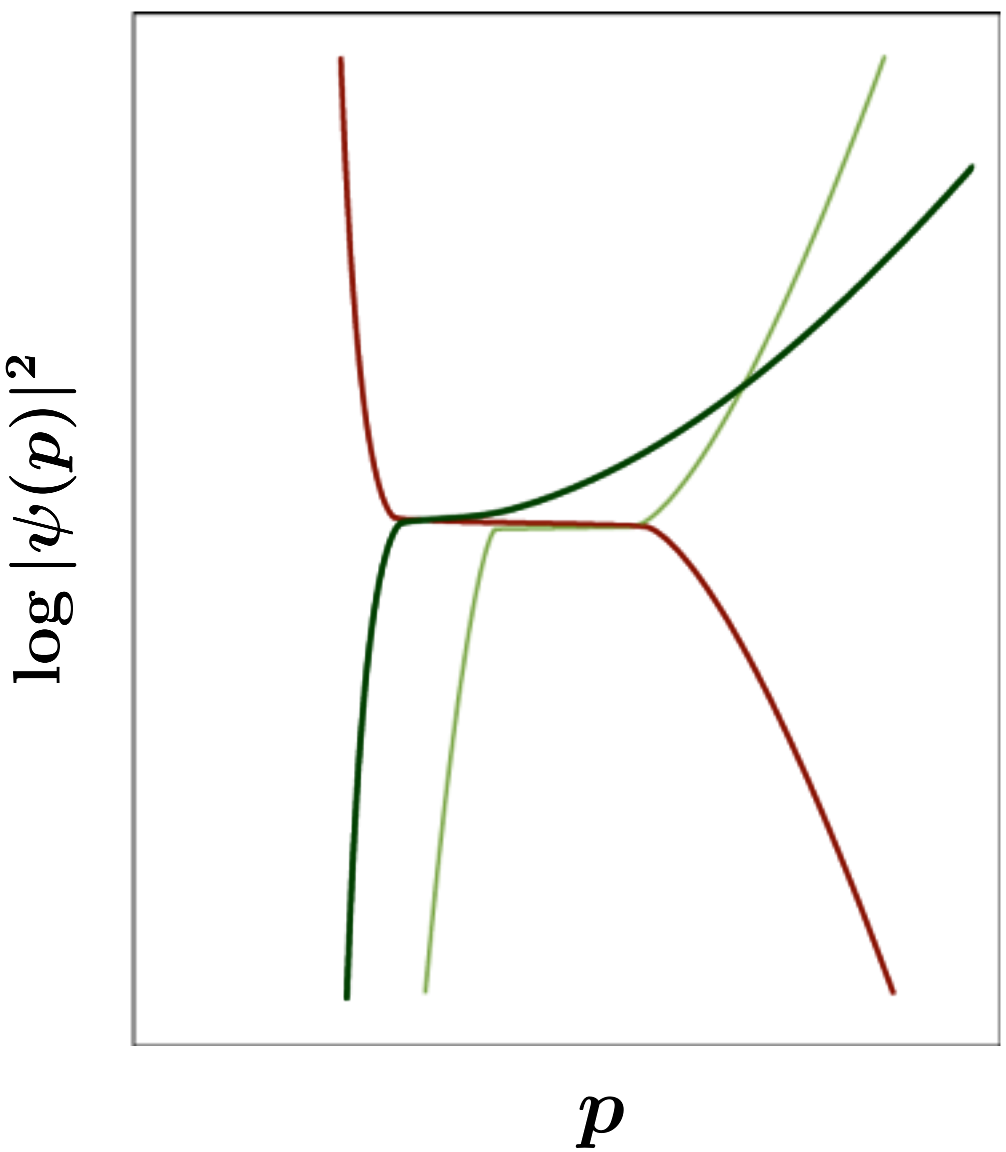


Semiclassical contributions in chained branches

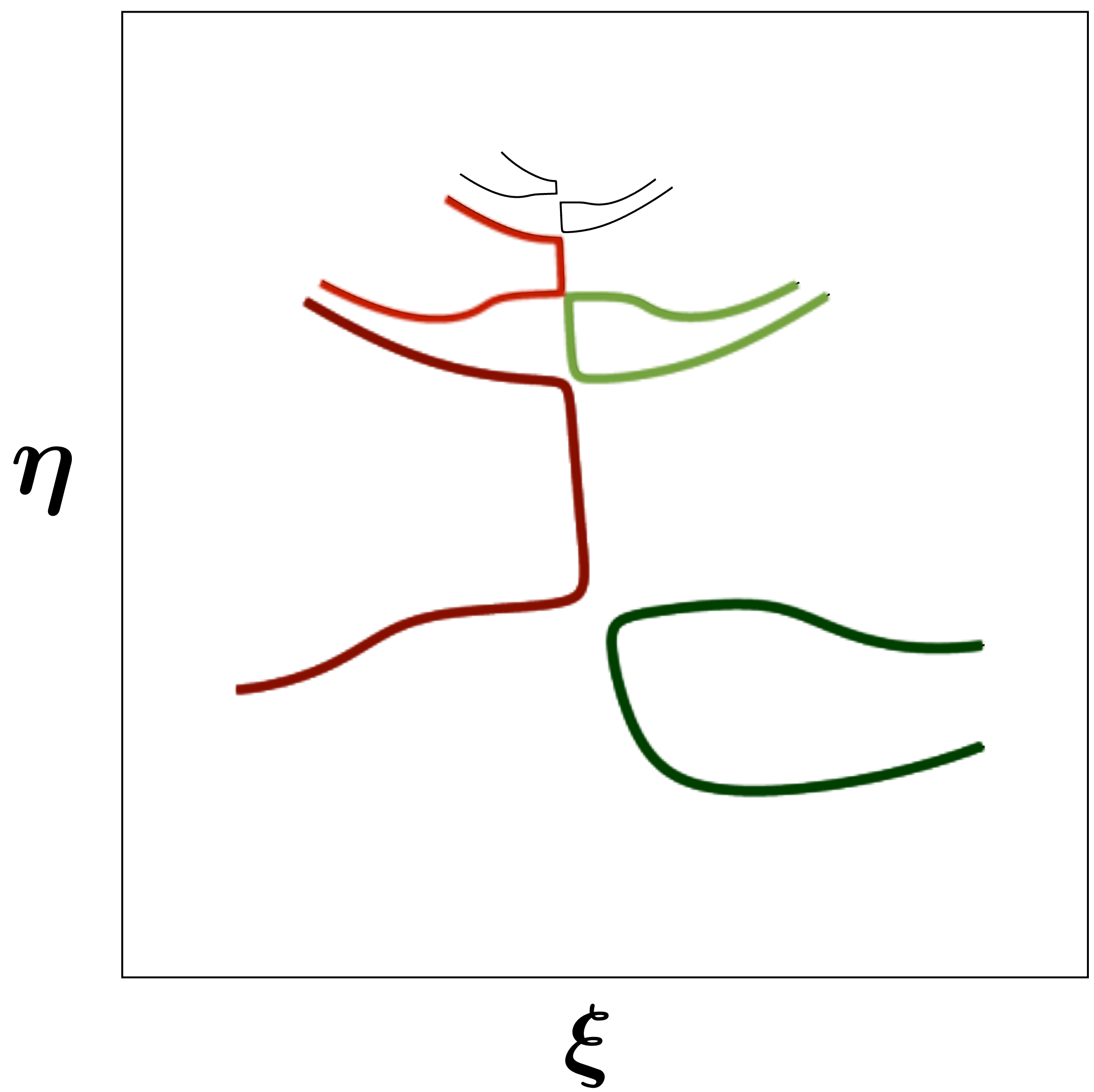


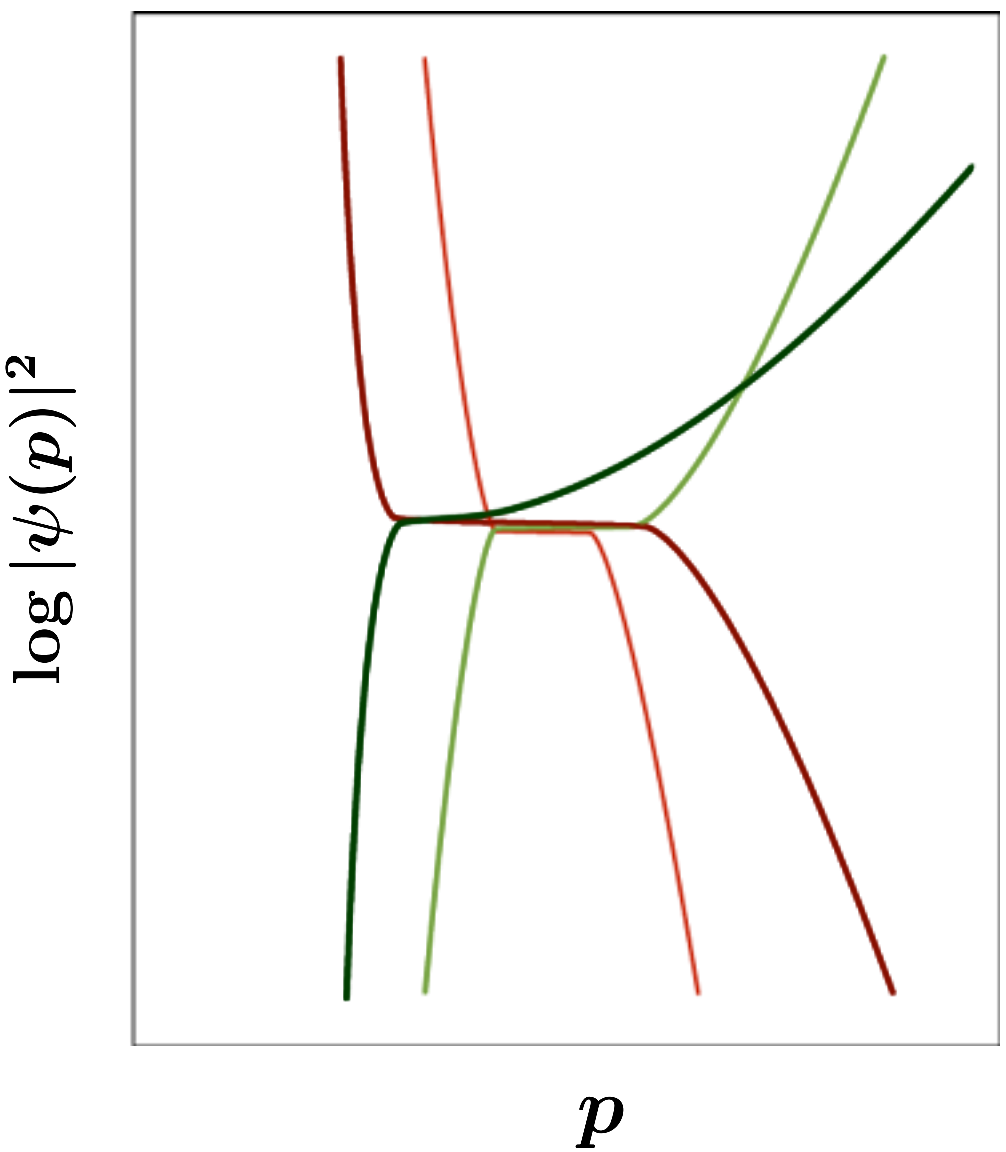
Semiclassical contributions in chained branches



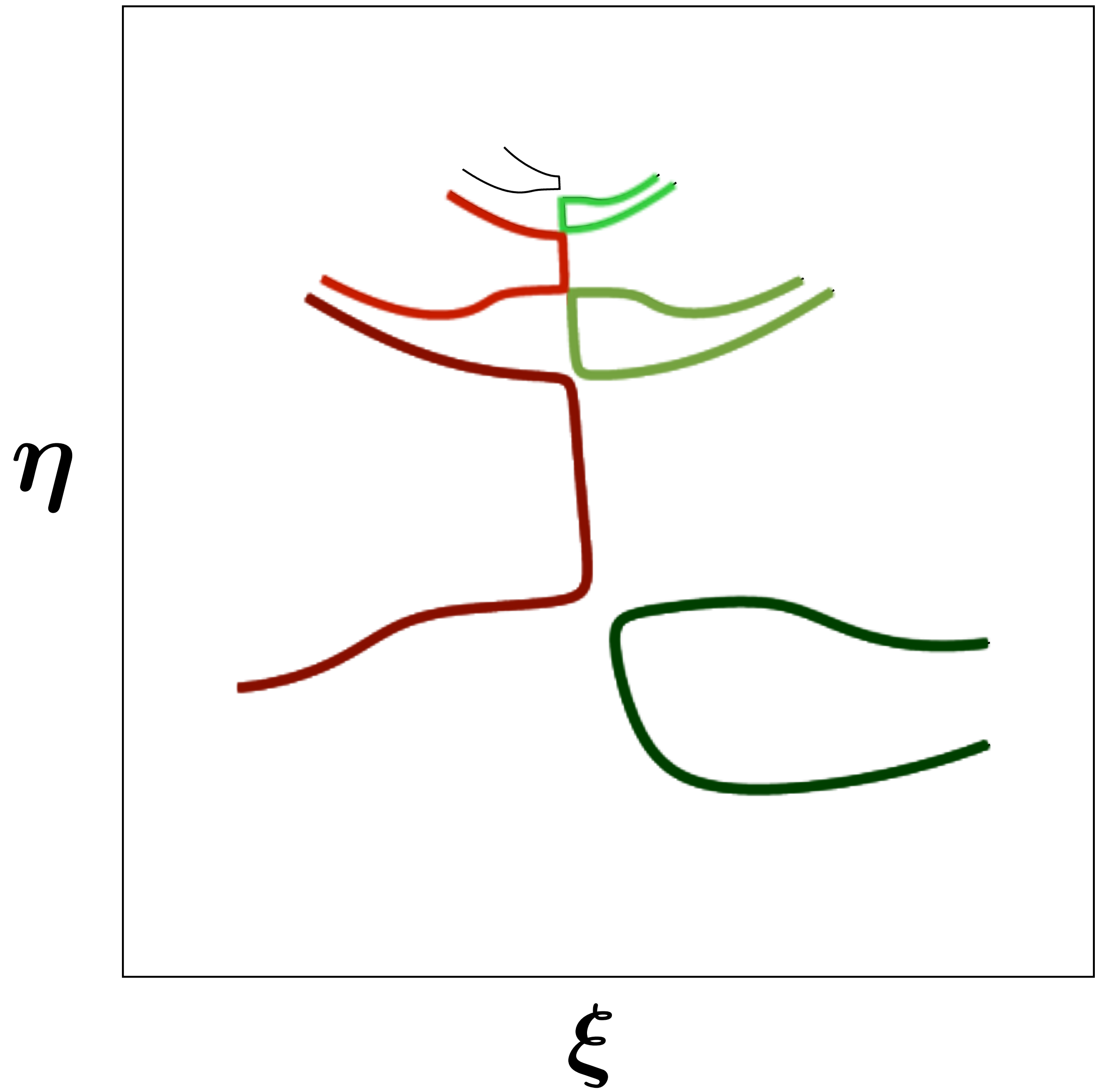


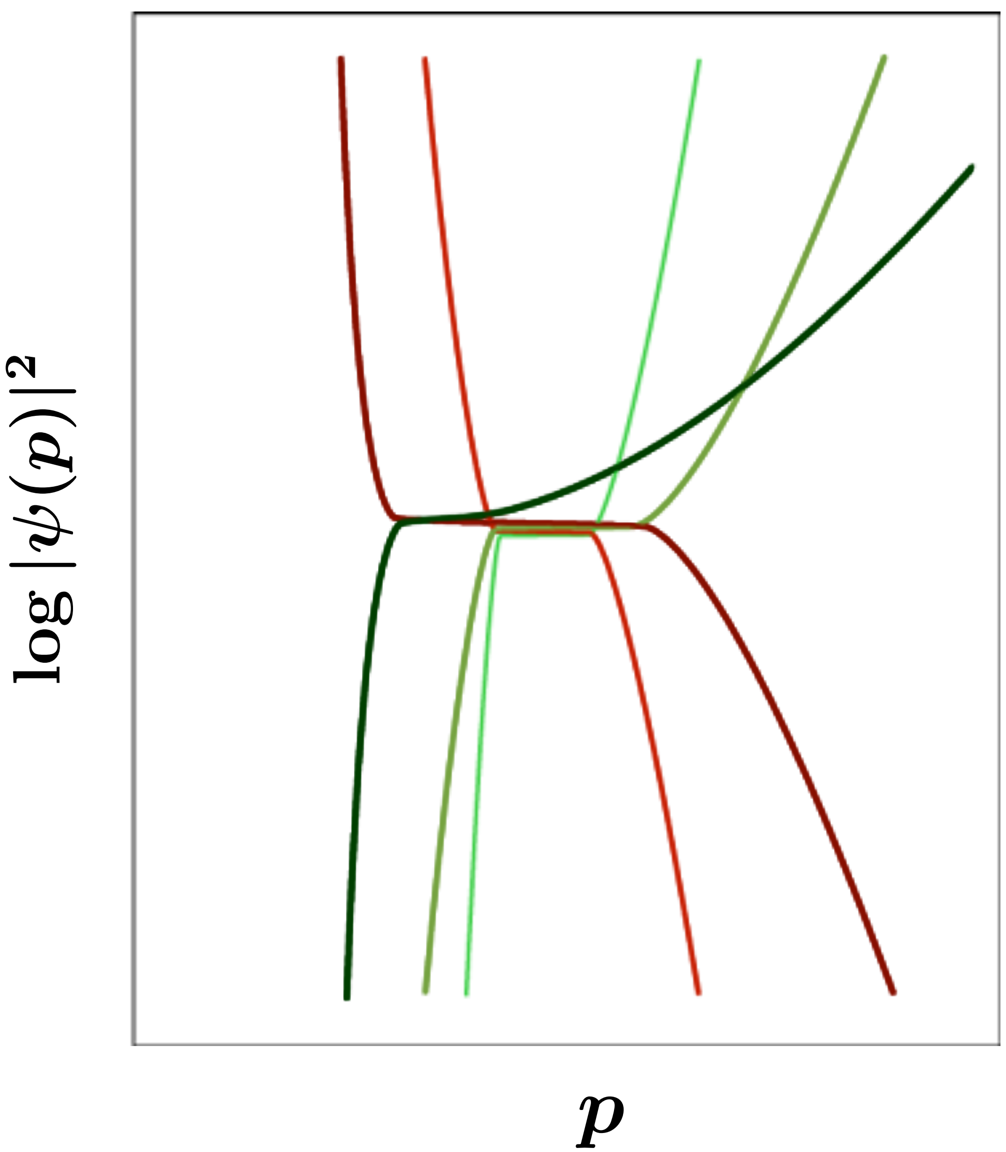
Semiclassical contributions in chained branches



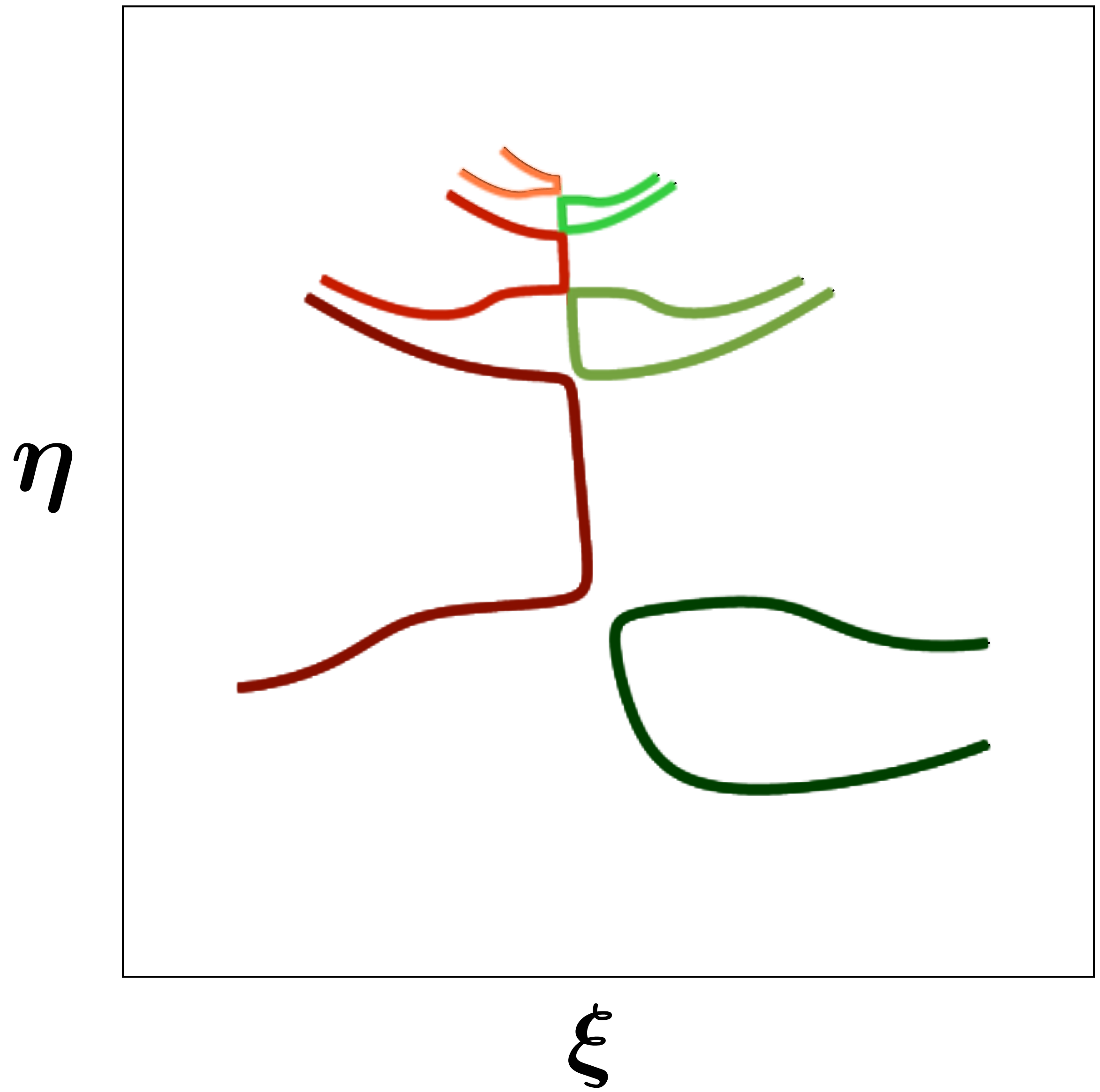


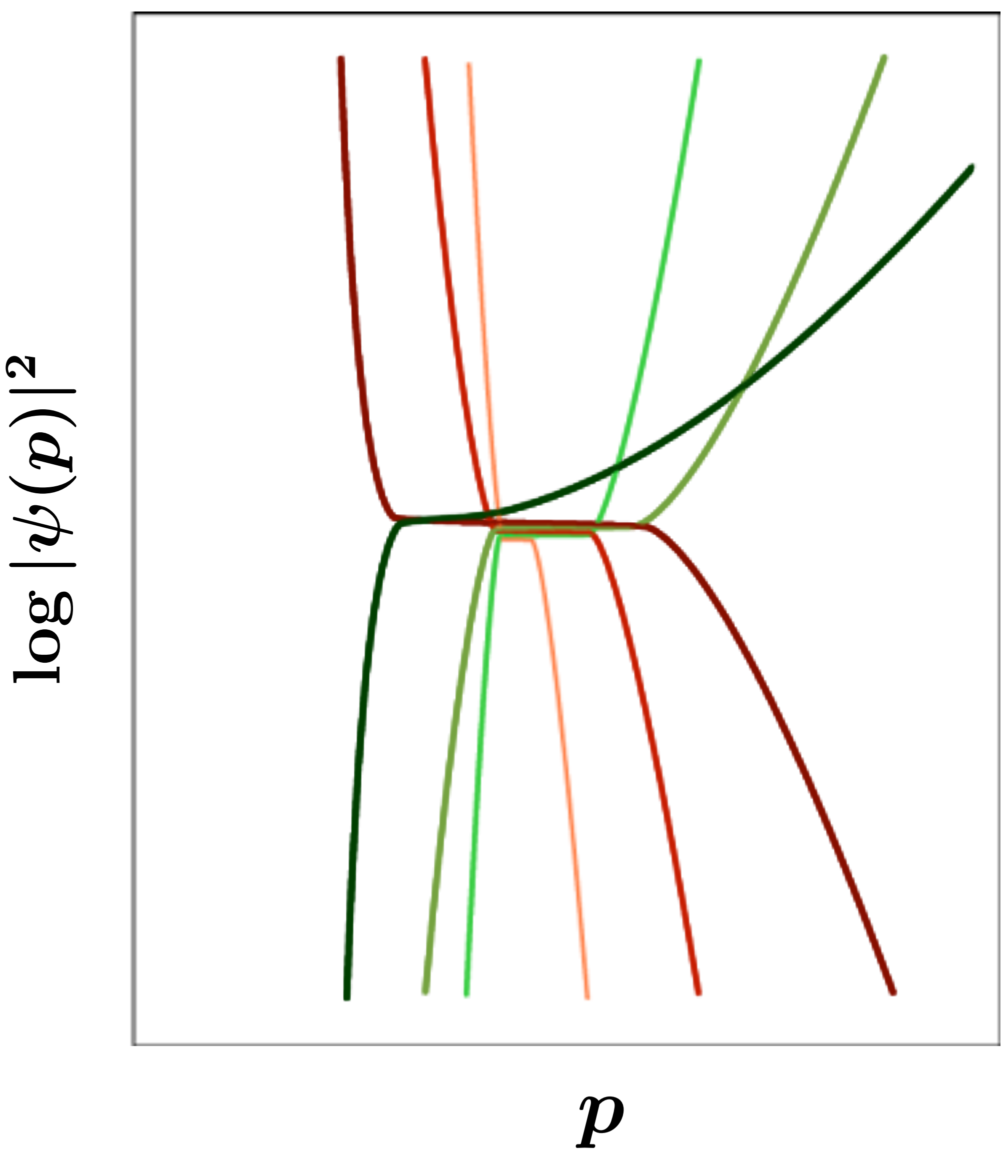
Semiclassical contributions in chained branches



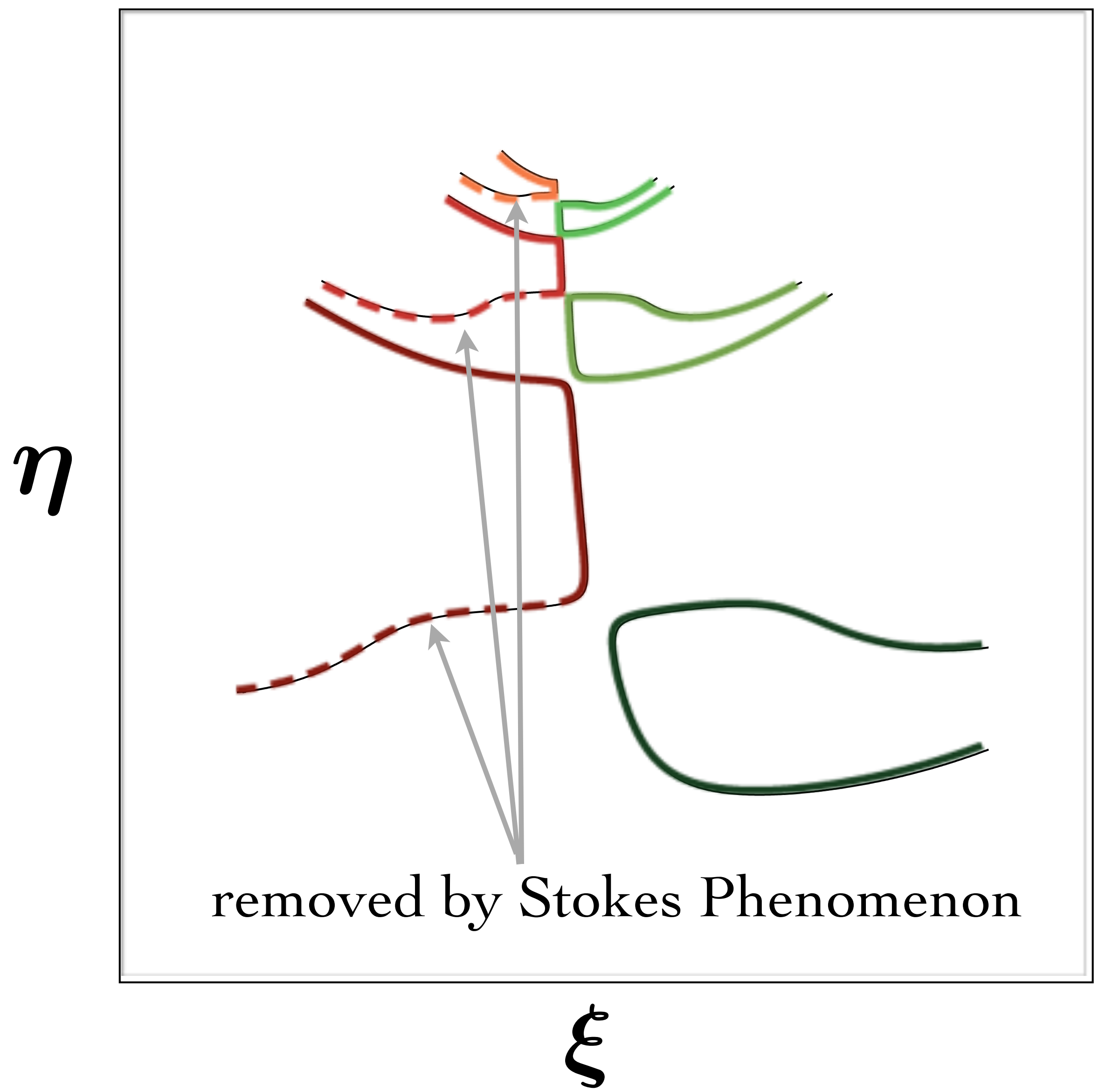


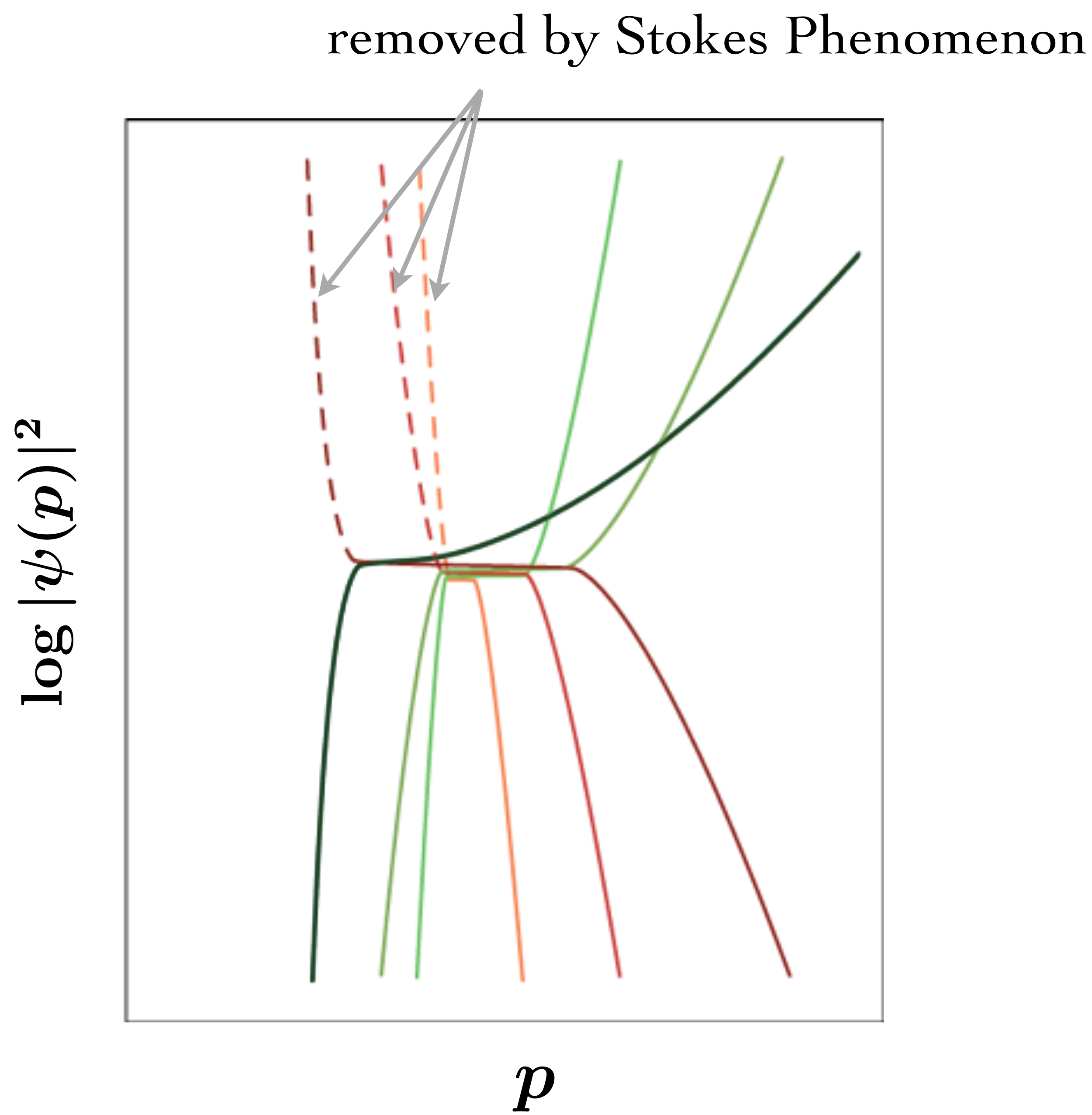
Semiclassical contributions in chained branches

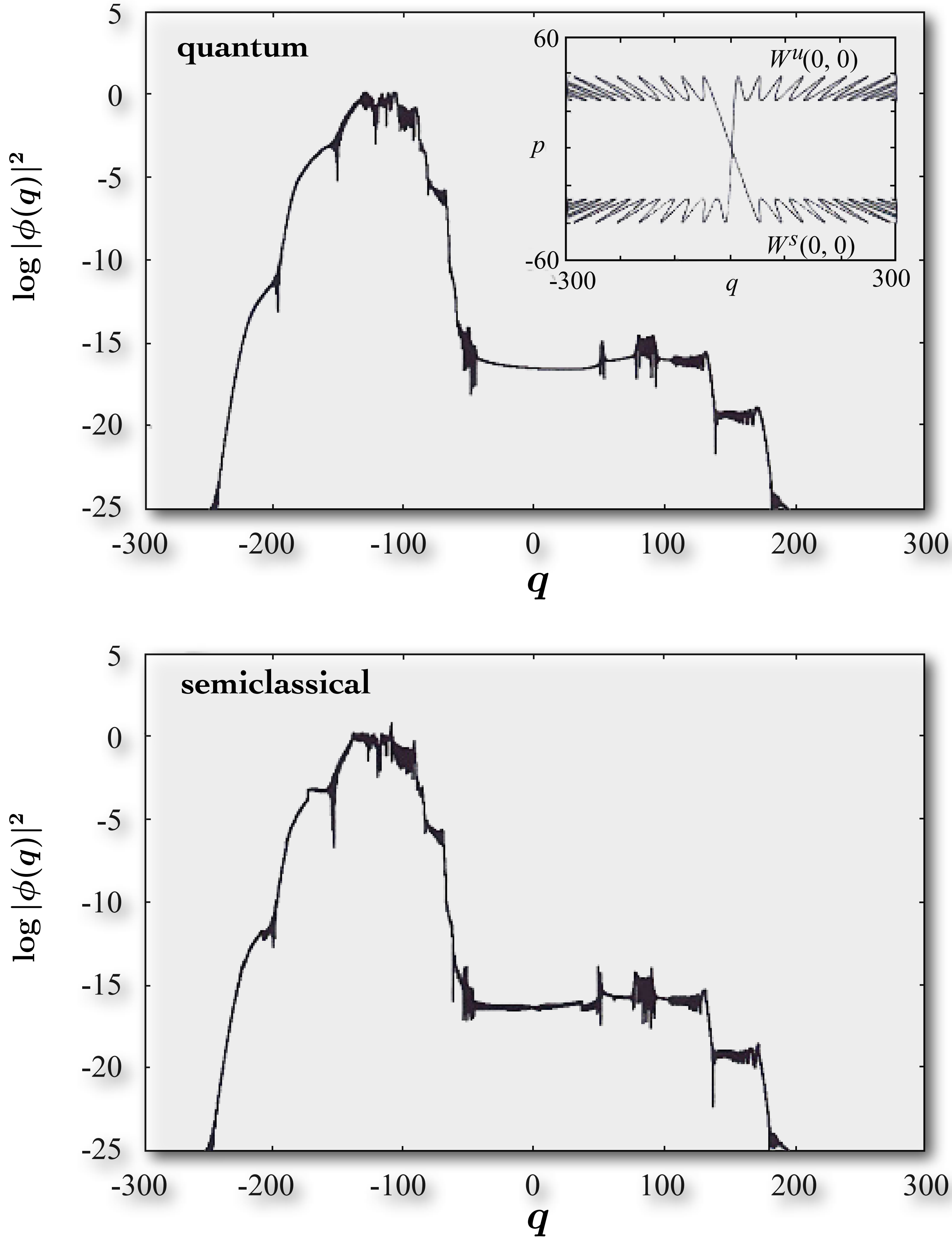




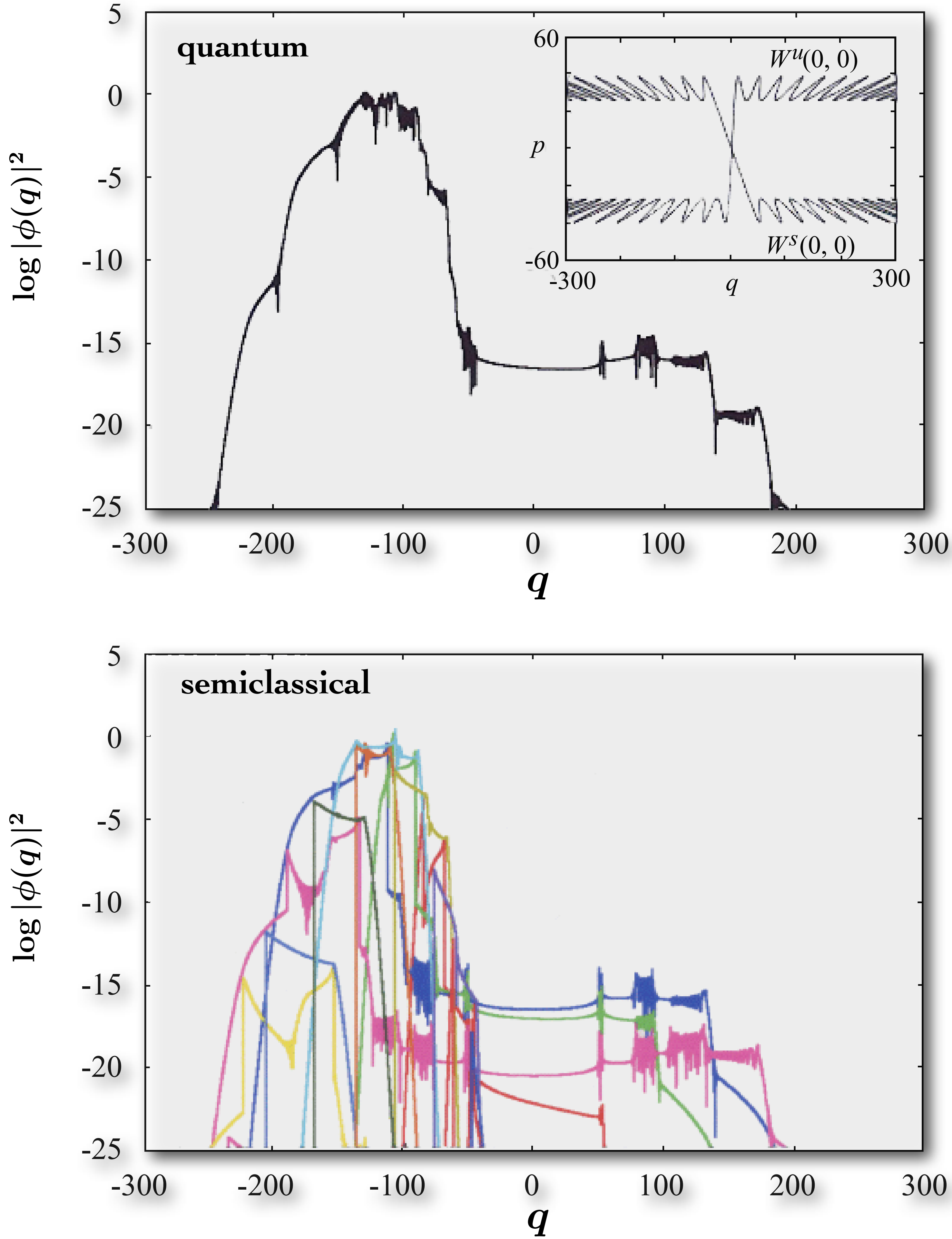
Semiclassical contributions in chained branches







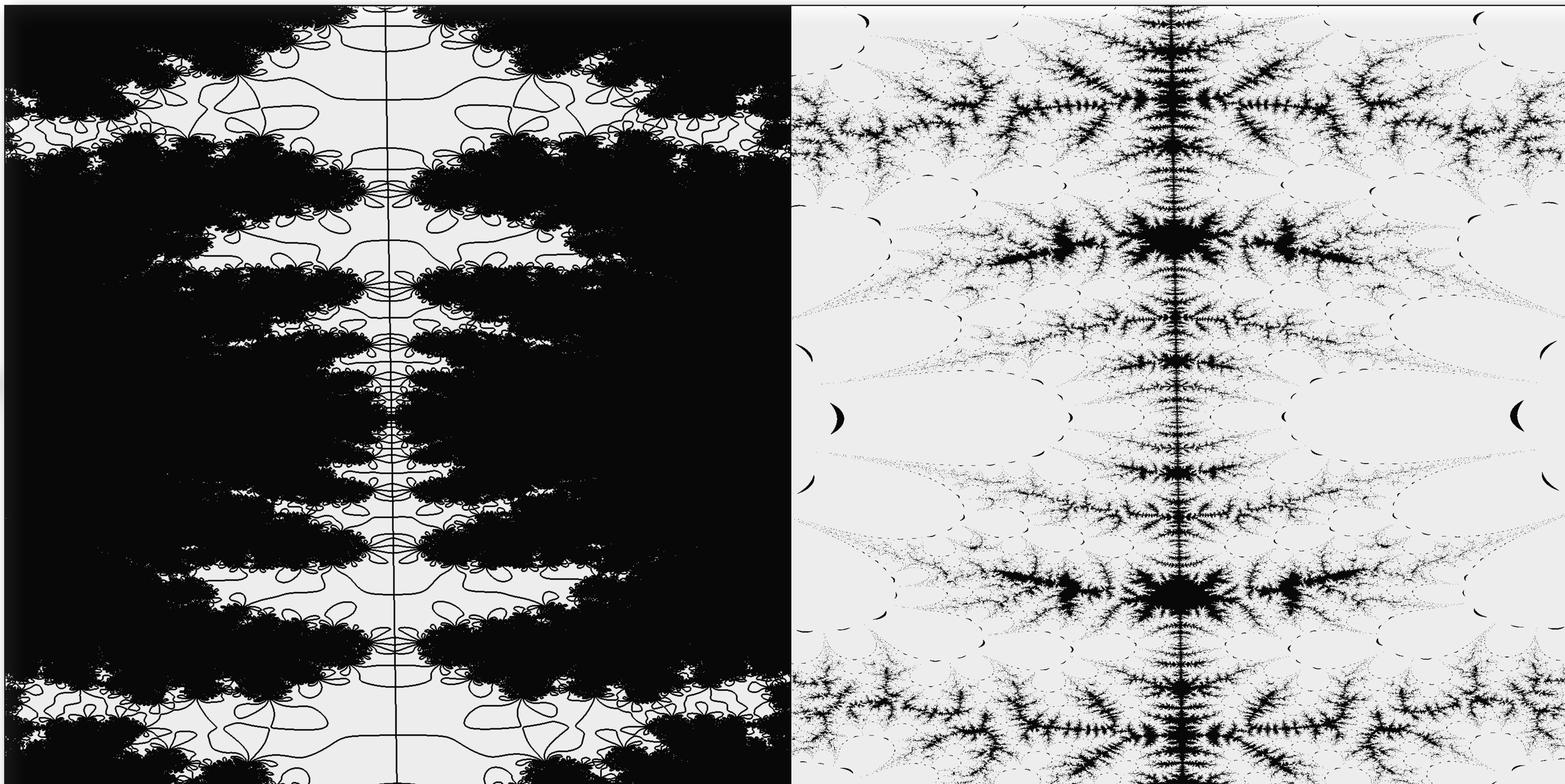
Onishi et al 2003



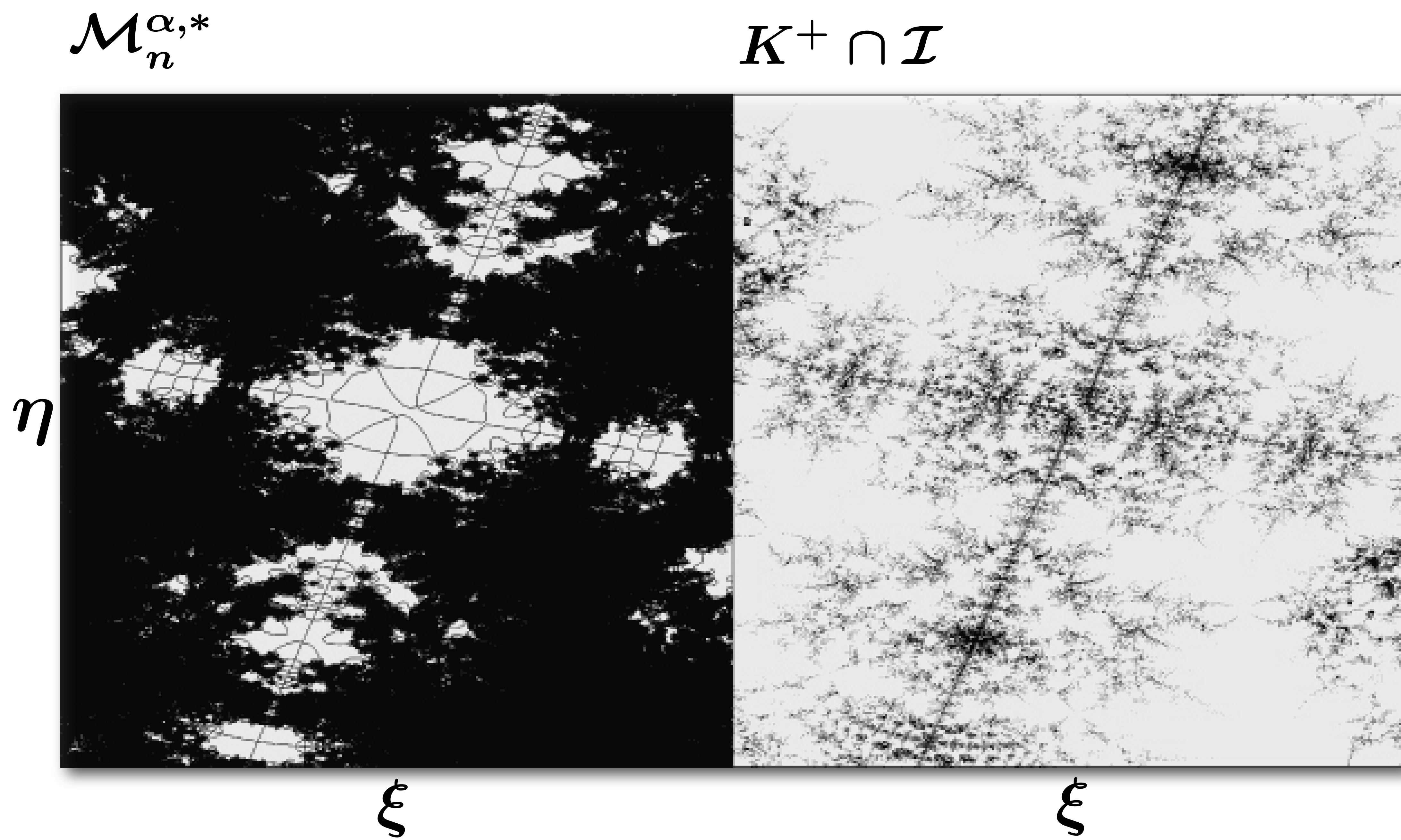
Onishi et al 2003

$\mathcal{M}_n^{\alpha,*}$ $\quad$ $K^+ \cap \mathcal{I}$

η



ξ $\quad$ ξ


$\mathcal{M}_n^{\alpha,*}$
$K^+ \cap \mathcal{I}$
η
ξ
ξ