



Lecture Four: Frenkel Excitons in Natural and Artifical Light-Harvesting

Oliver Kühn

For personal use only. References for reprinted figures are available upon request.

Solar Energy Conversion



• photon <u>collection</u> and <u>directed</u> energy transfer



Learning From Nature....

- photosynthetic complexes
- adapted to environment
- high efficiency (95%)



FMO trimer, wikipedia



PSII of higher plants, van Amerongen, 2011



LH2 (Rps. acidophila), T. Pullerits

3

Molecular Aggregates



S. Daehne et al. (2002)

4

spectroscopic signatures (Scheibe & Jelley, 1930s)



Overview

- Frenkel exciton model
- exciton-vibrational coupling
- from electronic structure to absorption
- coherence in photosynthesis
- beyond a reduced system-bath description

Frenkel Exciton Model

- electron-hole pair
 Frenkel exciton
 charge transfer (CT) exciton
 Wannier-Mott exciton
 monomer: electronic excitation
- aggregate: exciton=delocalized superposition state



Aggregate Hamiltonian

 separation of aggregate into m=1...N_{mol} monomeric building blocks



monomer Hamiltonian

Coulomb interaction

adiabatic wave functions & PES

$$H_m^{(\text{el})}(R_m)\varphi_{ma}(r_m; R_m) = V_{ma}(R_m)\varphi_{ma}(r_m; R_m)$$

• aggregate wavefunction (neglecting exchange)

$$\phi_A^{(\mathrm{HP})}(r;R) = \prod_{m=1}^{N_{\mathrm{mol}}} \varphi_{ma_m}(r_m;R_m)$$

7

• adiabatic state representation of aggregate Hamiltonian

$$H_{\text{agg}} = \sum_{m} \sum_{a,b} H_m(ab) |\varphi_{ma}\rangle \langle \varphi_{mb}| + \frac{1}{2} \sum_{m,n} \sum_{a,b,c,d} J_{mn}(ab,cd) |\varphi_{ma}\varphi_{nb}\rangle \langle \varphi_{nc}\varphi_{md}|$$

monomer contribution

$$H_m(ab) = \delta_{a,b}(T_m + V_{ma}) + (1 - \delta_{a,b})\Theta_{m,ab}$$

Coulomb coupling

$$J_{mn}(ab, cd) \equiv \int dr_m dr_n \,\varphi_{ma}^*(r_m) \varphi_{nb}^*(r_n) V_{mn}^{(\text{el-el})}(r_m, r_n) \varphi_{nc}(r_n) \varphi_{md}(r_m)$$

$$+ \qquad \delta_{b,c} \int dr_m \varphi_{ma}^*(r_m) V_{mn}^{(\text{el-nuc})}(r_m, R_n) \varphi_{md}(r_m)$$

$$+ \qquad \delta_{a,d} \int dr_n \varphi_{nb}^*(r_n) V_{mn}^{(\text{nuc-el})}(R_m, r_n) \varphi_{nc}(r_n) + \delta_{a,d} \delta_{b,c} V_{mn}^{(\text{nuc-nuc})}$$

-	-	
r	-	
	-	
	2	
	,	

transition density

$$n_{ab}^{(m)}(\mathbf{x}) = \varrho_{ab}^{(m)}(\mathbf{x}) - \delta_{a,b} \sum_{\mu \in m} e Z_{\mu} \delta(\mathbf{x} - \mathbf{R}_{\mu})$$

• Coulomb matrix elements

$$J_{mn}(ab, cd) = \int d\mathbf{x} d\mathbf{x}' \frac{n_{ad}^{(m)}(\mathbf{x}) n_{bc}^{(n)}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}$$

Beljonne et al., J. Phys. Chem. B <u>113</u>, 6583 2009)

- assume that size of transition densities small w.r.t. intermonomer distance
- dipole moment

$$\mathbf{d}_{mab} = \int d^3 \mathbf{x} \, \mathbf{x} n_{ab}^{(m)}(\mathbf{x}) = \int d^3 \mathbf{x} \, \mathbf{x} \varrho_{ab}^{(m)}(\mathbf{x}) - \delta_{a,b} \sum_{\mu \in m} e Z_\mu \mathbf{R}_\mu$$

Coulomb matrix elements

$$J_{mn}(ab,cd) \approx \frac{\mathbf{d}_{mad}\mathbf{d}_{nbc}}{|\mathbf{X}_{mn}|^3} - 3\frac{(\mathbf{X}_{mn}\mathbf{d}_{mad})(\mathbf{X}_{mn}\mathbf{d}_{nbc})}{|\mathbf{X}_{mn}|^5}$$

Testing the Dipole Approximation



	Matrix element	Interaction process		
(I)	J _{mn} (gg, gg) J _{mn} (ee, ee)	between charges at molecules <i>m</i> and <i>n</i>		
	J _{mn} (ge, eg) J _{mn} (eg, ge)	$arphi_{n,e}$	• • •	•
(11)	J _{mn} (eg, gg) J _{mn} (gg, ge) J _{mn} (ge, ee) J _{mn} (ee, eg)	between transitions at molecule <i>m</i> with charges at <i>n</i> $\varphi_{n,g}$	$d_{n,eg} \longleftrightarrow$	b
(III)	J _{mn} (eg, eg) J _{mn} (ge, ge)	between $S_0 \rightarrow S_1$ transition at molecule <i>m</i> and $S_1 \rightarrow S_0$ transition at <i>n</i> (and reverse)		
(IV)	J _{mn} (ee, gg) J _{mn} (gg, ee)	simultaneous excitation and deexcitation of molecules <i>m</i> and <i>n</i>		

• Coulomb coupling for TLS with resonant terms only

$$J_{mn}(eg, eg) \equiv J_{mn} = \int d^3 \mathbf{x} d^3 \mathbf{x}' \; \frac{\varrho_{eg}^{(m)}(\mathbf{x})\varrho_{ge}^{(n)}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}$$

dipole approximation

$$J_{mn} \approx \kappa_{mn} \frac{|\mathbf{d}_m| |\mathbf{d}_n^*|}{|\mathbf{X}_{mn}|^3}$$

orientation factor

$$\kappa_{mn} = \mathbf{n}_m \, \mathbf{n}_n - 3(\mathbf{e}_{mn} \, \mathbf{n}_m)(\mathbf{e}_{mn} \, \mathbf{n}_n)$$

absorption of electronic homodimer

$$E_{\alpha_1=\pm} = \frac{\hbar\omega_{eg}}{2} \pm J \qquad |\alpha_1=\pm\rangle = \frac{1}{\sqrt{2}}(|1\rangle \pm e^{-i\arg(J)}|2\rangle)$$

13

J- vs. H-Aggregates



▶ for applications that require strong fluorescence J-aggregates preferable

15

Three-State Approximation

- organic dyes will have more than one excited state
- important whenever more than two photons are absorbed
- for two-photon processes one should take into account states where

$$E_e - E_g \approx E_f - E_e$$

• minimal effective model should include three levels



 classification of aggregate's electronic state according to number of excitations

$$\sum_{A} |\phi_{A}^{(\mathrm{HP})}\rangle \langle \phi_{A}^{(\mathrm{HP})}| = |0\rangle \langle 0| + \sum_{m} |m\rangle \langle m| + \sum_{m,n \ge m} |mn\rangle \langle mn| + \dots$$

zero excitation

$$0\rangle = \prod_{m} |\varphi_{mg}\rangle$$

single excitation

$$|\varphi_{m}\rangle = |\varphi_{me}\rangle \prod_{n \neq m} |\varphi_{ng}\rangle$$

double (local and nonlocal) excitation

$$|mn\rangle = (1 - \delta_{mn})|\varphi_{me}\rangle|\varphi_{ne}\rangle \prod_{k \neq m,n} |\varphi_{kg}\rangle + \delta_{mn}|\varphi_{mf}\rangle \prod_{k \neq m} |\varphi_{kg}\rangle$$

17

Multi (Two)-Exciton Hamiltonian

• restrict Hamiltonian to two excitation space

$$H = H^{(0)} + H^{(1)} + H^{(2)} + H^{(1-2)}$$

$$H^{(0)} = \sum_{m} H_{m,g} |0\rangle \langle 0| \qquad H_{m,a} = T_m + V_{ma}$$
$$H^{(1)} = \sum_{m,n} [\delta_{mn} (H_{m,g} + U_{m,eg}) + J^{(eg)}_{mn}] |m\rangle \langle n|$$

one-exciton motion
$$\ket{m} \leftrightarrow \ket{n}$$

gap coordinate

$$U_{m,ab} = V_{m,a} - V_{m,b}$$



$$J_{mn}(eg, eg) = J_{mn}^{(eg)}$$

$$H^{(2)} = \sum_{l \ge k} \sum_{n \ge m} \{ \delta_{mk} \delta_{nl} [(1 - \delta_{mn}) (H_{m,g} + U_{m,eg} + U_{n,eg}) + \delta_{mn} (H_{m,g} + U_{m,fg})]$$

+ $(1 - \delta_{mn}) (1 - \delta_{kl}) [J_{km}^{(eg)} \delta_{nl} + J_{ln}^{(eg)} \delta_{mk} + J_{kn}^{(eg)} \delta_{lm} + J_{lm}^{(eg)} \delta_{kn}]$
+ $(1 - \delta_{mn}) \delta_{kl} [J_{km}^{(fe)} \delta_{kn} + J_{kn}^{(fe)} \delta_{mk}]$
+ $\delta_{mn} (1 - \delta_{kl}) [J_{km}^{(fe)} \delta_{lm} + J_{lm}^{(fe)} \delta_{km}] \} |kl\rangle \langle mn|$
 $J_{mn} (fg, ee) = J_{mn}^{(fe)}$

two-exciton motion
$$|nm\rangle \leftrightarrow |nk\rangle$$
 "fusion/fission" $|0m\rangle \leftrightarrow |nm\rangle$
 \longrightarrow D_m D

Exciton-Exciton Annihilation



$$\Theta_{m,fe} \propto \left[\int dr_{\rm el} \varphi_{mf}(r_{\rm el}; R_{\rm nuc}) \nabla_{\rm nuc} \varphi_{me}(r_{\rm el}; R_{\rm nuc})\right] \nabla_{\rm nuc}$$

Multi-Exciton Eigenstates

• diagonalization of N-exciton Hamiltonian

$$H^{(N)}|\alpha_N\rangle = E_{\alpha_N}|\alpha_N\rangle$$

state expansions

$$|\alpha_1\rangle = \sum_n C_n(\alpha_1)|n\rangle$$
$$|\alpha_2\rangle = \sum_{m>n} C_{mn}(\alpha_2)|mn\rangle + \sum_m C_{mm}(\alpha_2)|mm\rangle$$

exciton bands



Exciton-Vibrational Coupling

• one-exciton Hamiltonian

$$H^{(1)} = \sum_{m,n} [\delta_{mn} (H_{m,g} + U_{m,eg}) + J^{(eg)}_{mn}] |m\rangle \langle n|$$

ground state vibrations

$$H_{m,g} \equiv H_{m,g}(R_m) = T_m + V_{mg}(R_m)$$

▶ gap coordinate for excitation

$$U_{m,eg}(R_m) = V_{m,e}(R_m) - V_{m,g}(R_m)$$



Strategies

- stochastic models (e.g. Haken-Strobel-Reineker)
- harmonic bath models (spectral density):
 - single bath (e.g. intramolecular)
 - multi-mode Brownian oscillator
 - can be used to fit experimental sepectral densities
- atomistic models: molecular dynamics simulations plus electronic structure theory

Huang-Rhys Model

shifted oscillator model for nuclear DOF

$$U_m(Q_{m,\xi}) = E_m + \sum_{\xi} \frac{\hbar\omega_{\xi}}{2} (Q_{m,\xi} - Q_{m,\xi}^{(0)})^2$$

• coupling given by Huang-Rhys factor

$$S_{m,\xi} = \frac{1}{2} Q_{m,\xi}^{(0)\,2}$$

• correlation function

$$C_m(\omega) = 2\pi\omega^2 (1+n(\omega)) [\mathcal{J}_m(\omega) - \mathcal{J}_m(-\omega)]$$

• spectral density

$$\mathcal{J}_m(\omega) = \sum_{\xi} S_{m,\xi} \delta(\omega - \omega_{m,\xi})$$



Huang-Rhys + Coulomb Coupling

• exciton-vibrational states



- single excitation states $|m, M_{e_m}, M_{g_n}\rangle \quad |n, N_{g_m}, N_{e_n}\rangle$
- effective Coulomb coupling

 $J_{mn}\langle M_{e_m}|N_{g_m}\rangle\langle M_{g_n}|N_{e_n}\rangle$



Absorption Spectrum

recall correlation function approach

$$J(t) = \operatorname{tr} \left\{ \mu U_0(t) \mu \rho_{eq} U_0^+(t) \right\}$$
$$\mu = \sum_{\alpha} \mu_{\alpha} |\alpha\rangle \langle 0| + \text{h.c} \qquad U_0(t) |\alpha\rangle = e^{-iE_{\alpha}t/\hbar} |\alpha\rangle \quad \rho_{eq} = |0\rangle \langle 0$$
$$\rightarrow \quad \chi''(\omega) \propto \sum_{\alpha} |\mu_{\alpha}|^2 \delta(\hbar\omega - E_{\alpha})$$

basis set expansion for exciton-vibrational states

$$|\alpha\rangle = \sum_{M_{e_1}, M_{g_2}} C_{\alpha}(2, M_{e_1}, M_{g_2}) |1, M_{e_1}, M_{g_2}\rangle + \sum_{M_{e_1}, M_{g_2}} C_{\alpha}(2, M_{g_1}, M_{e_2}) |2, M_{g_1}, M_{e_2}\rangle$$

• electronic character of certain state

$$C_{\rm el} = \langle \alpha | \sum_{m=1,2} |m, 0, 0 \rangle \langle 0, 0, m | \ | \alpha \rangle$$



• exciton-vibrational states (by matrix diagonalization)

M. Schröter et al., Phys. Rep. 567, 1 (2015)

Perylene Bisimides

- high performance color pigments
- fluorescence quantum yield up to unity in solution, drops in solid state
- optical properties shaped by bay substituents
- type of arrangement in aggregate depends on imide substituents





H. Marciniak et al., J. Phys. Chem. A 115, 648 (2011)

Vertical So-Sn Excitation Spectrum

- TDDFT/B3LYP (6-311G*)
 - response function formalism
 - based on ground state KS density

E/eV	f	excitation	one-electron excitation	λ/nm
2.13	0.59	$\pi_{pbi} o \pi^*_{pbi}$	${}^{1}S_{1}(274-275)$	582.2
2.74	0.22	$\pi_{per+bay} \to \pi^*_{pbi}$	$^{1}S_{2}(273-275)$	453.2
4.22	0.28	$\pi_{pbi} ightarrow \pi^*_{pbi}$	$^{1}S_{34}(274-281)$	293.8
4.25	0.05	$\pi_{pbi=o} o \pi^*_{pbi}$	¹ CT (250-275)	292.0
4.25	0.16	$\pi_{per+bay} o \pi^*_{pbi}$	${}^{1}S_{36}(272-277)$	291.6
4.48	0.07	$\pi_{\sf pbi} o \pi^*_{\sf bay}$.	¹ CT (274-283)	278.2
4.67	0.05	$\pi_{bay} o \pi^*_{pbi}$	¹ CT (262-276)	265.7
4.70	0.05	$\pi_{pbi=o} \to \pi^*_{pbi}$	¹ CT (256-276)	263.8



Exploring the Coulomb Coupling



DFTB Mulliken transition charges of PBI

- $S_0 \rightarrow S_1$ transition
- remove side groups
- lateral displacement



0.1 8 0.08 4 0.06 $\Delta y/ {\rm \AA}$ o 0.04 0.02 -4 0 J/eV -0.02 -8 -5 0 5 -10 10 $\Delta x/\text{\AA}$

Coupling of the dimer shifted along x- and y-direction

dimer structures (DFTB simulated annealing)



H-bonded

• weaker π -stacking



J _{mn} /eV (TBFE)	J _{mn} /eV (DFT)	
0.091	0.103	
0.068	0.076	
-0.030	-0.034	
- 0.019	-0.021	
	J_{mn} /eV (TBFE) 0.091 0.068 - 0.030 - 0.019	

Franck-Condon active modes (TDDFT, B3LYP, 6-311G*)



Linear Absorption

• 10 mode model (200-1600 cm⁻¹, J_{mn}=-500 cm⁻¹)



dilution of FC progression upon increasing aggregate size

LH2: Photosynthetic Antenna



Rb. capsulatus [1]



 $^{[1]}$ S.J. Schmitt et al.; $^{[2]}$ N. Hunter; $^{[3]}$ T. Pullerits



LH2 complex (Rps. acidophila) [3]





35

Quantum Master Equation

• one-exciton reduced density matrix

$$\frac{\partial \rho_{\alpha\beta}}{\partial t} = i(\omega_{\alpha\beta} + \gamma_{\alpha\beta})\rho_{\alpha\beta} - \delta_{\alpha\beta}\sum_{\alpha'} (k_{\alpha'\alpha}\rho_{\alpha'\alpha'} - k_{\alpha\alpha'}\rho_{\alpha\alpha})$$

Caldeira-Leggett type model (one bath)

$$H_{\rm R} = \sum_{j} \frac{\hbar\omega_j}{2} \left(-\frac{\partial^2}{\partial Q_j^2} + Q_j^2 \right) \qquad \qquad H_{\rm S-R} = \sum_{m} |m\rangle \langle m| \sum_{j} \hbar\omega_j g_{m,j} Q_j$$

relaxation rates

$$k_{ab} = 2\Gamma_{ab,ba}(\omega_{ab}) = \sum_{u,v} K_{ab}^{(u)} K_{ba}^{(v)} C_{uv}(\omega_{ab})$$

$$\rightarrow \quad k_{\alpha\beta} = 2\pi \sum_{m} |C_m(\alpha)|^2 |C_m(\beta)|^2 \omega_{\alpha\beta}^2 (1 + n(\omega_{\alpha\beta})) [J_m(\omega_{\alpha\beta}) - J_m(-\omega_{\alpha\beta})]$$

static disordered via sampling of transition energies

37

Effective Oscillator Model

$$J(\omega) = \sum_{j} S_{j} \delta(\omega - \omega_{j})$$

experimental spectral density



B850 Linear Absorption

• absorption of LH2 with depleted B800 ring (exp. J. Herek)



39

B850: Pump-Probe Spectroscopy



Quasi-Stationary Spectra

• pump-probe absorption band shape sensitive to disorder strength



Kühn et al., J. Chem. Phys. 107, 4154 (1997)

Dynamics & Coherence



42

41

Green Sulfur Bacteria

- inhabit oxygen poor, sulfur rich environments
- adapt to low light conditions



http://universe-review.ca

Fenna-Matthews-Olsen (FMO) complex





Fenna-Matthews-Olsen (FMO) Complex



en.wikipedia.org; N. Lambert et al. Nat. Phys. 9, 10 (2012)

Exciton Hamiltonian

	/ 310	-97.9	5.5	-5.8	6.7	-12.1	-10.3	37.5
$H_{\rm S} =$	-97.9	230	30 .1	7.3	2.0	11.5	4.8	7.9
	5.5	30.1	0	-58.8	-1.5	-9.6	4.7	1.5
	-5.8	7.3	-58.8	180	-64.9	-17.4	-64.4	-1.7
	6.7	2.0	-1.5	-64.9	405	89.0	-6.4	4.5
	-12.1	11.5	-9.6	-17.4	89.0	320	31.7	-9.7
	-10.3	4.8	4.7	-64.4	-6.4	31.7	270	-11.4
	37.5	7.9	1.5	-1.7	4.5	-9.7	-11.4	505.0



J. Moix et al., J. Phys. Chem. Lett. 2, 3045 (2011).

BChl a Spectral Density

• parametrization of experimental spectral density



¹ M. Wendling et al., J. Phys. Chem. B <u>104</u>, 5825 (2000)

- Quantum Master Equation
 - structured spectral density
 - no info about vibrations
- Schrödinger equation
 - model: 7 electronic states + 74 vibrational modes per site

Population and Coherence Dynamics



- oscillation frequencies (220, 160 cm⁻¹) differ from bare electronic value (208 cm⁻¹)
- dephasing due to high-dimensional vibrational space

J. Schulze, O.K., JPC B 119, 6211 (2015), J. Schulze, O.K. et al., arXiv:1602.03973 [physics.chem-ph]

Vibrational/Vibronic Dynamics

 Q_2



vibronic excitation at site m

 $H_m^{(\text{vibro})} = \sum_{\xi \in m} \frac{\omega_{m,\xi}}{2} \left(-\frac{\partial^2}{\partial Q_{m,\xi}^2} + Q_{m,\xi}^2 + 2\sqrt{2S_{m,\xi}}Q_{m,\xi} \right) |m\rangle\langle m|$



vibrational excitation at sites 1 and 2 (dynamic picture)

vibronic excitation at site 3 (resonance picture)

Synopsis

Hamiltonian

Born-Oppenheimer potential energy surfaces normal modes QM/MM Caldeira-Leggett **Dynamics**

Schrödinger equation MCTDH Quantum Master Equation spectral density

Spectroscopy

nonlinear polarization phase matching response functions lineshape function

Thanks To

- Rostock
 - Jan Schulze
 - Per-Arno Plötz
 - Marco Schröter
 - Sergey Ivanov
 - Y. Yan

- Berlin
 - V. May
 - H. Naundorf
 - K. Heyne
 - M. Petkovic
 - Y. Yang
 - G. Paramonov
 - ▶ G. Krishnan
- Hans-Dieter Meyer (Heidelberg)
- Stefan Lochbrunner and group (Rostock)
- Thomas Niehaus (Lyon)
- T. Pullerits, V. Sundström (Lund)



DFG

Quatar National Research Fund

