# Lecture Three: <br> Dynamics and Spectroscopy of Hydrogen Bonds 

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## Why Hydrogen Bonds ?

H-bonds \& proton transfer $\rightarrow$ Physics, Chemistry, Biology

- the theoreticians answer:
$>$ H-bond: van der Waals $\rightarrow$ covalent
$>$ kinetic isotope effect (4 isotopes)
$>$ proton is quantum particle
$>$ tunneling even at room temperature
> H-bond dynamics is multidimensional
- M. L. Huggins (1936)
... hydrogen bridge theory will lead to a better understanding of the nature and behavior of complicated organic structures, such as proteins, starch, cellulose, sugar....


## $\mathrm{H}^{+}$in $\mathrm{H}_{2} \mathrm{O}$



## Alpha-Helix


L. Pauling (1951)


## DNA Structure




Freezing OfWater
$>$ fluctuating hydrogen bonded network
spontaneous formation of polyhedral nucleus


## Artificial Water Channels





1D water wire

H-bonded network + fluctuations

concerted water flow (bursts)

## Proton Pump



## Nature Of H-Bonds

- donor-acceptor interaction involving hydrogen
- covalent A-H $\rightarrow \mathrm{H}^{+\delta} \rightarrow$ interaction with B (lone pair or polarizable $\pi$ electrons)
- H-bond interactions: $>$ electrostatic (directionality)
$>$ charge transfer
$>$ dispersion
> exchange repulsion
- H-bond geometry:



## IR Spectra



## Weak H-Bonds

$\mathrm{E}_{\mathrm{B}}=<4 \mathrm{kcal} / \mathrm{mol}$
$\Delta v=<10 \%$
$r_{A B}=3.2-4.2 \AA$
$\mathrm{r}_{\mathrm{BH}}=2.2-3.2 \AA$
directionality
$>$ tunneling


- HCl...benzene (T-form), C-H...B in crystals, forced contacts


## Moderate H-Bonds

$E_{B}=4-15 \mathrm{kcal} / \mathrm{mol}$
$\Delta \nu=10-25 \%$
$r_{A B}=2.5-3.2 \AA$
$r_{B H}=1.5-2.2 \AA$

- neutral D/A:
$>\mathrm{O}-\mathrm{H} . . . \mathrm{O}$
- intramolecular H -bonds

- biological systems:
> packing, solvation, conformation


## Strong H-Bonds

$\mathrm{E}_{\mathrm{B}}=14-40 \mathrm{kcal} / \mathrm{mol}$
$\Delta v=25 \%$
$r_{A B}=2.2-2.5 \AA$
$r_{B H}=1.2-1.5 \AA$

- low/vanishing barrier
> delocalized wave function
sensitive to environment

- [FHF] ${ }^{-}$O-H...O,$~ N-H . . . N^{-}$, enzymes (?), forced contacts


## IR Spectroscopy of Strong H-Bonds in Gas Phase

## Protonated Ammonia Clusters $\mathrm{NH}_{4}{ }^{+}\left(\mathrm{NH}_{3}\right)_{n}$

$n=1$
$n=3$



- N...H...N HBs
- $\mathrm{NH}_{3}$ channels
- proton wires


## $\mathrm{NH}_{3}$-Networks

AmtB NH3 channel (E. coli)

$\mathrm{NH}_{3}$-wires


7-hydroxyquinoline• $\left(\mathrm{NH}_{3}\right)_{3}$

[^0]IR Spectrum of $\mathrm{N}_{2} \mathrm{H}_{7}{ }^{+}$


# $\mathrm{N}_{2} \mathrm{H}_{7}{ }^{+}$: 6D Model 

- kinetic energy for non-Cartesian coordinates
model coordinates
- assumptions:
- use constraints to restrict motions to model coordinates
- no kinetic coupling between proton and $\mathrm{NH}_{3}$ fragments
- torsion decoupled from vibrations


$$
\begin{array}{rlrl}
T= & -\frac{\hbar^{2}}{2 \mu_{p}}\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}+\frac{\partial^{2}}{\partial z^{2}}\right)-\frac{\hbar^{2}}{2 \mu_{R}} \frac{\partial^{2}}{\partial R^{2}} & & g(u)=\left(1-u^{2}\right)\left(3 m_{\mathrm{H}}+m_{\mathrm{N}}\right) /\left(3 m_{\mathrm{H}} u^{2}+m_{\mathrm{N}}\right) \\
& -\frac{\hbar^{2}}{2 I_{0}} \sum_{i=1,2} \frac{\partial}{\partial u_{i}} g\left(u_{i}\right) \frac{\partial}{\partial u_{i}}, & u_{p}=\frac{\cos \theta_{i}}{7 m_{\mathrm{H}}\left(3 m_{\mathrm{H}}+m_{\mathrm{N}}\right)} & I_{0}=3 m_{\mathrm{H}} R_{\mathrm{NH}}^{2} \\
\mu_{R}=\frac{1}{2}\left(3 m_{\mathrm{H}}+m_{\mathrm{N}}\right)
\end{array}
$$

- potential energy and dipole moment surfaces
- n -mode correlation expansion

$$
\begin{aligned}
& V(\mathbf{Q})=\sum_{i} V^{(1)}\left(Q_{i}\right)+\sum_{i<j} V^{(2)}\left(Q_{i}, Q_{j}\right)+\sum_{i<j<k} V^{(3)}\left(Q_{i}, Q_{j}, Q_{k}\right)+\ldots \\
& V^{(2)}\left(Q_{i}, Q_{j}\right)=V(\mathbf{Q})-\sum_{i} V^{(1)}\left(Q_{i}\right) \\
& \mu(\mathbf{Q})=\sum_{i} \mu^{(1)}\left(Q_{i}\right)+\sum_{i<j} \mu^{(2)}\left(Q_{i}, Q_{j}\right)
\end{aligned}
$$

- recall MCTDH

$$
\begin{gathered}
i \hbar \dot{A}_{J}=\sum_{K} H_{J K} A_{K} \\
H_{J K}=\int d \mathbf{Q} \phi_{j_{1}}^{(1)}\left(Q_{1}\right) \ldots \phi_{j_{f}}^{(f)}\left(Q_{f}\right) H \phi_{k_{1}}^{(1)}\left(Q_{1}\right) \ldots \phi_{k_{f}}^{(f)}\left(Q_{f}\right)
\end{gathered}
$$

- preferential: sum of products form

$$
H=\sum_{r=1}^{s} c_{r} \prod_{\kappa=1}^{f} h_{r}^{(\kappa)} \rightarrow \quad H_{J K}=\sum_{r=1}^{s} c_{r} \prod_{\kappa=1}^{f}\left\langle\phi_{j_{\kappa}}^{(\kappa)}\right| h_{r}^{(\kappa)}\left|\phi_{k_{\kappa}}^{(\kappa)}\right\rangle
$$

## $\mathrm{N}_{2} \mathrm{H}_{7}{ }^{+}$: ZPE Effect



- reaction barrier: $\sim 350 \mathrm{~cm}^{-1}$ (>kT at RT)
- structure determined by ZPE effect


K. Asmis, M. Johnson, O.K. et al., Angew. Chem. 46, 8691 ('07)


## NH-Stretching Mode



- extreme red-shift due to strong H-bond


## Dissipative H-Bond Wave Packets




- oscillations in IR pump-probe spectra


Experiment
$\mathrm{T}_{1}\left(\mathrm{v}_{\mathrm{OD}}\right) \sim 200 \mathrm{fs}$
$\mathrm{T}_{\text {cool }}$ ~ 20 ps
$\mathrm{V}_{\mathrm{osz}} \sim 100 \mathrm{~cm}^{-1}$


## A Simple Model



- adiabatic separation of highand low-frequency modes
- Franck-Condon like progression

absorption
- excitation of wave packets possible
- theoretically reproduced with reaction surface model



## Vibrational Energy Relaxation

- two-color pump-probe spectroscopy



| Probe |  | Pump | $\bar{\uparrow}$ |
| :---: | :---: | :---: | :---: |
| $\overline{\delta_{O H}}$ | $\overline{\delta_{O H}}$ | $\bar{\uparrow}$ | $v_{C O}$ |
| $\bar{Z}$ |  |  |  |
|  |  |  |  |

$$
\begin{aligned}
& \mathrm{T}_{1}\left(\mathrm{v}_{\mathrm{OH}}\right)=200 \mathrm{fs}, \mathrm{~T}_{1}\left(\delta_{\mathrm{OH}}\right)=800 \mathrm{fs} \\
& \text { relaxation via } \delta_{\mathrm{OH}}=1(>30 \%) \\
& \mathrm{T}_{\text {cool }} \sim 20 \mathrm{ps}
\end{aligned}
$$

## 5D Dissipative Model

4-mode correlation potential - B3LYP/6-31+G(d,p)

$v_{\mathrm{HB}}=63 \mathrm{~cm}^{-1}$

$\gamma_{1}=792 \mathrm{~cm}^{-1}$


$$
\gamma_{2}=690 \mathrm{~cm}^{-1}
$$




$\qquad$

$$
\text { labeling of states } \longrightarrow\left(\mathrm{v}_{v}, \mathrm{v}_{\delta}, \mathrm{v}_{\gamma_{1}}, \mathrm{v}_{\gamma_{2}}\right)
$$

- system-bath model
- low-frequency H -bond mode

$$
H_{\mathrm{S}-\mathrm{R}}^{(\mathrm{HB})}=Q_{\mathrm{HB}} \sum_{\xi} c_{\xi} x_{\xi}
$$

- out-of-plane deformation


$$
H_{\mathrm{S}-\mathrm{R}}^{(\gamma)}=Q_{\gamma} \sum_{i, \xi} g_{\gamma, i, \xi} q_{i} x_{\xi}
$$

$\rightarrow$ 3rd order model | $v_{\gamma_{1}} / v_{\gamma_{2}}=1$ |
| :---: | :---: |
| system |
| $v_{\gamma_{1}} / v_{\gamma_{2}}=0$ |

- potential energy curves and IR spectrum

- cascaded energy relaxation

K. Heyne et al. JPCA 108, 6083 (2004)


## Hydrogen Bonds in DNA





Guanine
Cytosine

## Base Pairs in Gas Phase:The Quest for the Structure

- IR-UV Double Resonance Spectra


Kleinermanns et al., ChemPhysChem, 4, 838 (2003)

## A-T Isomers

HF/6-31G(d,p)


$1038 \mathrm{~cm}^{-1}$

WC


B

$1433 \mathrm{~cm}^{-1}$

## Optimized Structures

| HF <br> $6-31 \mathrm{G}(\mathrm{d}, \mathrm{p})$ | $\mathrm{DFT} / \mathrm{B} 3 \mathrm{LYP}$ <br> $6-31+$ <br> $+\mathrm{G}(\mathrm{d}, \mathrm{p})$ | $\mathrm{MP2}$ <br> TZP |
| :---: | :---: | :---: | :---: |
| $0 \mathrm{~cm}^{-1}$ | $0 \mathrm{~cm}^{-1}$ | $0 \mathrm{~cm}^{-1}$ |
| $395 \mathrm{~cm}^{-1}$ | $364 \mathrm{~cm}^{-1}$ | $406 \mathrm{~cm}^{-1}$ |
| $266 \mathrm{~cm}^{-1}$ | $466 \mathrm{~cm}^{-1}$ | $420 \mathrm{~cm}^{-1}$ |

Target Modes


$$
\begin{aligned}
& \mathrm{v}_{\mathrm{as}}=3530 \mathrm{~cm}^{-1} \\
& \mathrm{v}_{\mathrm{sy}}=3326 \mathrm{~cm}^{-1} \\
& \mathrm{v}_{\mathrm{NH}}=3295 \mathrm{~cm}^{-1}
\end{aligned}
$$


$v_{\mathrm{as}}=3698 \mathrm{~cm}^{-1}$

$v_{s y}=3411 \mathrm{~cm}^{-1}$


$$
v_{N H}=2981 \mathrm{~cm}^{-1}
$$

A

B

$\mathrm{v}_{\mathrm{as}}=3683 \mathrm{~cm}^{-1}$
$v_{\mathrm{as}}=3689 \mathrm{~cm}^{-1}$

$$
v_{\mathrm{sy}}=3386 \mathrm{~cm}^{-1}
$$


$v_{N H}=3098 \mathrm{~cm}^{-1}$

$v_{\mathrm{sy}}=3442 \mathrm{~cm}^{-1}$

$v_{N H}=3284 \mathrm{~cm}^{-1}$

## Potential Energy Surfaces

- expand PES in normal mode coordinates

$$
\mathbf{Q}=\left\{Q_{a s}, Q_{s y}, Q_{N H}\right\}
$$

- use (exact) 3-mode expansion

$$
V(\mathbf{Q})=\sum_{i} V^{(1)}\left(Q_{i}\right)+\sum_{i<j} V^{(2)}\left(Q_{i}, Q_{j}\right)+\sum_{i<j<k} V^{(3)}\left(Q_{i}, Q_{j}, Q_{k}\right)
$$

$$
\begin{array}{ll}
V^{(1)}: & \text { MP2 energies on numerical grid } \\
V^{(2)}+V^{(3)}: & \text { DFT up to 4th order derivative }
\end{array}
$$

- 1-mode dipole moment

$$
\mu(\mathbf{Q}) \approx \sum_{i} \mu^{(1)}\left(Q_{i}\right)
$$

## Structure vs. IR Absorption



## Dynamics of DNA Base Pairs in Solution


correlated dynamics

anticorrelated dynamics

- H-Bond dynamics / correlations
- vibrational energy flow
- environmental effects

$\longrightarrow$ IR spectrosopy


## Solvated Base Pairs



## QM/MM Trajectory

- 9-ethyl-8-phenyladenine : 1-cyclohexyluracil in $100 \mathrm{CDCl}_{3}$ at 298 K



## N-H...N HB Geometry



## Lineshape Model



- IR absorption spectrum

$$
\sigma(\omega)=\frac{1}{\pi} \operatorname{Re} \int_{0}^{\infty} d t \exp \left\{i\left(\omega-\left\langle\omega_{10}\right\rangle\right) t-t / 2 T_{1}\right\} J(t)
$$

- phenomenological decay (non-adiabatic
 transitions)
- cumulant approximation

$$
\begin{aligned}
J(t) & \simeq \exp \left\{-g_{10}(t)\right\} \\
g_{10}(t) & \equiv \int_{0}^{t} d \tau \int_{0}^{\tau} d \tau^{\prime}\left\langle\delta \omega_{10}\left(\tau^{\prime}\right) \delta \omega_{10}(0)\right\rangle_{\mathrm{eq}}
\end{aligned}
$$



## On-The-Fly Potentials



## On-the-Fly Correlations





N-H...N

$$
\begin{aligned}
& \Gamma_{\text {FWHM }}=56(36) \mathrm{cm}^{-1} \\
& \Gamma_{\mathrm{FWHM}}^{\mathrm{exp}}=53 \mathrm{~cm}^{-1}
\end{aligned}
$$



## 2nd Order Cumulant Approximation

- quantum-classical approximation

- non-Gaussian statistics of fluctuations for N-H...O
hydrogen bond


## Gap-Autocorrelation N-H...N

$$
C(t)=\sum_{j} S_{j} \omega_{j}^{2}\left[\operatorname{coth}\left(\hbar \omega_{j} / 2 k T\right) \cos \omega_{j} t+i \sin \omega_{j} t\right]
$$



- reconstruction of spectral density


## Nonlinear Spectroscopy of Base Pairs

- open questions
role of population relaxation $\frac{1}{T_{2}}=\frac{1}{2 T_{1}}+\frac{1}{T_{2}^{*}}$
pump-probe spectroscopy:
$\mathbf{k}_{1}=\mathbf{k}_{2}=\mathrm{k}_{\text {pump }} \quad \mathbf{k}_{s}=\mathbf{k}_{3}=\mathbf{k}_{\text {probe }}$
- correlated motion

2D spectroscopy:

$$
\mathbf{k}_{s}=-\mathbf{k}_{1}+\mathbf{k}_{2}+\mathbf{k}_{3}
$$


$\longrightarrow$ model for excited state absorption needed

## Overtone Excitations





S. Woutersen et al., JCP 121, 5381 (2004) Y. Yan, O.K., JPCB 111, 5254 (2011)

## 2D IR Spectroscopy


(i) ground state bleaching/ stimulated emission
(ii) excited state absorption
(iii) cross peak absorption

- correlated dynamics



[^0]:    S. Khademi et al. Science 305, 1587 (2004)
    C. Tanner et al. Science 302, 1736 (2003)

