Fakultät für Physik

UNIVERSITÄT DUISBURG ESSEN

# Econophysics V:

# Credit Risk

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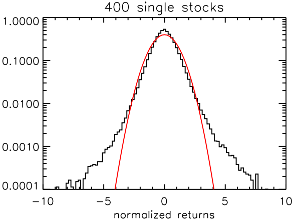
# Outline

- Introduction — What is credit risk ?
- Structural model and loss distribution
- Numerical simulations
- Random matrix approach
- Conclusions — general, present credit crisis

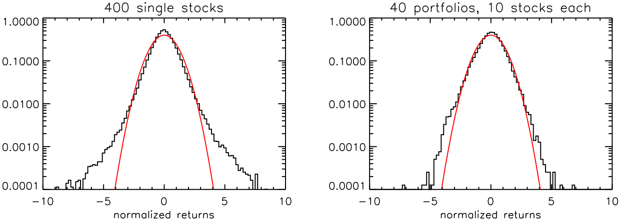
# Introduction

# Diversification in a Stock Portfolio — No Correlations



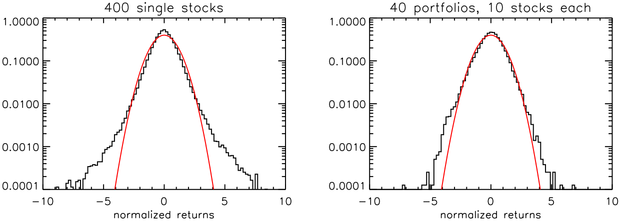

- empirical distribution of normalized returns (400 stocks)

# Diversification in a Stock Portfolio — No Correlations



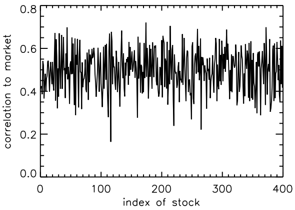

- empirical distribution of normalized returns (400 stocks)
- portfolio: superposition of stocks

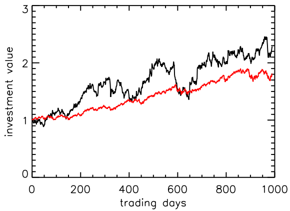
# Diversification in a Stock Portfolio — No Correlations




- empirical distribution of normalized returns (400 stocks)
- portfolio: superposition of stocks
- risk reduction by diversification (no correlations yet!):
  returns are more normally distributed,
  market risk reduced by approx. 50 percent

# Correlations




- stocks highly correlated to overall market
- risk reduction by diversification (with correlations): unsystematic risk can be removed, systematic risk (overall market) remains

# Credits and Stability of the Economy




- credit crisis shakes economy $\longrightarrow$ dramatic instability
- physics: model building based on empirical information
- econophysics: treat economy as complex system

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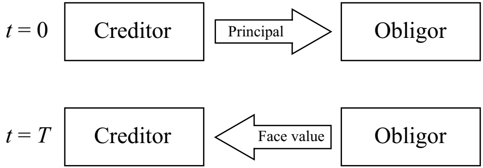
- risk reduction by diversification ?

# Zero–Coupon Bond




- principal: borrowed amount
- face value $F$:
  borrowed amount + interest + risk compensation
- credit contract with simplest cash-flow
- credit portfolio comprises many such contracts

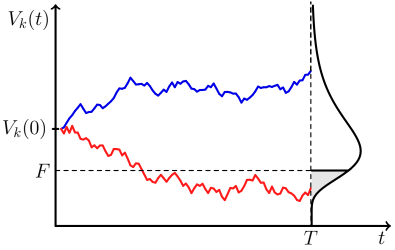
# Defaults and Losses

- default occurs if obligor fails to repay

  $\Rightarrow$ loss between 0 and face value $F$
- possible losses have to be priced into credit contract
- correlations are important to evaluate risk of credit portfolio
- statistical model yields loss distribution

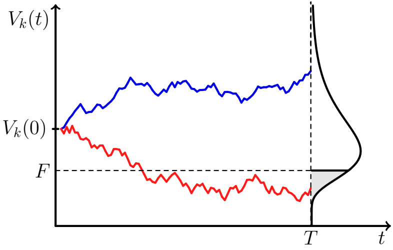
# Modeling Credit Risk

# Structural Models of Merton Type




- microscopic approach
- dynamical description of risk elements $V_k(t),\ k = 1, \ldots, K$
- default occurs if asset value $V_k(T)$ falls below face value $F_k$
- then the (normalized) loss is $L_k = \dfrac{F_k - V_k(T)}{F_k}$

# Structural Models of Merton Type




- microscopic approach
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- default occurs if asset value $V_k(T)$ falls below face value $F_k$
- then the (normalized) loss is $L_k = \dfrac{F_k - V_k(T)}{F_k}$
- e.g. credits with stock portfolio or houses as securities

# Geometric Brownian Motion with Jumps

choose the stock prices as risk elements $V_k(t),\ k = 1, \ldots, K$

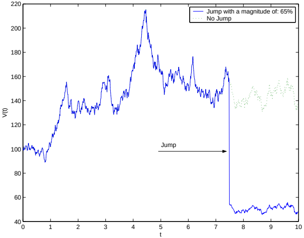
$$\frac{dV_k(t)}{V_k(t)} = \mu_k\, dt + \sigma_k \varepsilon_k(t)\sqrt{dt} + dJ_k(t)$$

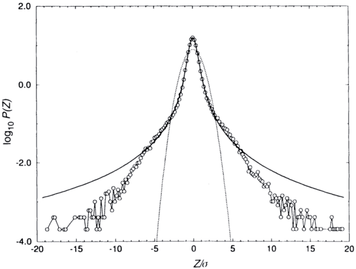
we include jumps !

- deterministic term $\mu_k\, dt$
- diffusion term $\sigma_k \varepsilon_k(t)\sqrt{dt}$
- jump term $dJ_k(t)$, governed by a Poisson process

parameters can be tuned to describe the empirical price and return distributions

# Jump Process and Price or Return Distributions

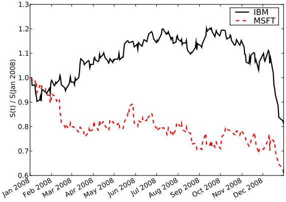



jumps reproduce empirically found heavy tails

# Financial Correlations

stock prices $V_k(t), \; k = 1, \ldots, K$
measured at $t = 1, \ldots, T$




returns $R_k(t) = \dfrac{dV_k(t)}{V_k(t)}$

normalization $M_k(t) = \dfrac{R_k(t) - \langle R_k(t) \rangle}{\sqrt{\langle R_k^2(t) \rangle - \langle R_k(t) \rangle^2}}$

correlation $C_{kl} = \langle M_k(t) M_l(t) \rangle$ , $\qquad \langle u(t) \rangle = \dfrac{1}{T} \sum_{t=1}^{T} u(t)$

$K \times T$ data matrix $M$ such that $C = \dfrac{1}{T} M M^\dagger$

# Inclusion of Correlations in Risk Elements

- $\varepsilon_i(t),\ i = 1, \ldots, I$ set of random variables
- $K \times I$ structure matrix $A$
- correlated diffusion, uncorrelated drift, uncorrelated jumps

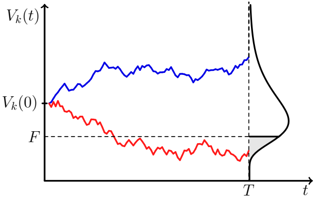
$$\frac{dV_k(t)}{V_k(t)} = \mu_k \, dt + \sigma_k \sum_{i=1}^{I} A_{ki} \varepsilon_i(t) \sqrt{dt} + dJ_k(t)$$

for $T \to \infty$ correlation matrix is $C = AA^\dagger$

covariance matrix is $\Sigma = \sigma C \sigma$ with $\sigma = \mathrm{diag}\,(\sigma_1, \ldots, \sigma_K)$

# Loss Distribution

# Individual Losses




normalized loss at maturity
$t = T$

$$L_k = \frac{F_k - V_k(T)}{F_k}\Theta(F_k - V_k(T))$$

if default occurs

# Portfolio Loss Distribution

- homogeneous portfolio
- portfolio loss $L = \frac{1}{K} \sum_{k=1}^{K} L_k$
- stock prices at maturity $V = (V_1(T), \ldots, V_K(T))$
- distribution $p^{(\mathrm{mv})}(V, \Sigma)$ with $\Sigma = \sigma C \sigma$

want to calculate

$$p(L) = \int d[V] p^{(\mathrm{mv})}(V, \Sigma)\, \delta \left( L - \frac{1}{K} \sum_{k=1}^{K} L_k \right)$$

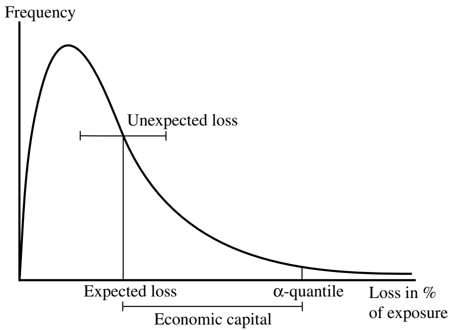
# Large Portfolios

Real portfolios comprise several hundred or more individual contracts $\longrightarrow$ $K$ is large.

Central Limit Theorem: For very large $K$, portfolio loss distribution $p(L)$ must become Gaussian.

Question: how large is "very large" ?

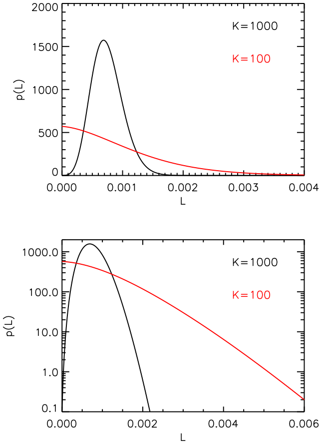
# Typical Portfolio Loss Distributions




- highly asymetric, heavy tails, rare but drastic events
- mean of loss distribution is called expected loss (EL)
- standard deviation is called unexpected loss (UL)
- kurtosis excess (KE) to measure heavy tails: $\gamma_2 = \mu_4/\mu_2^2 - 3$

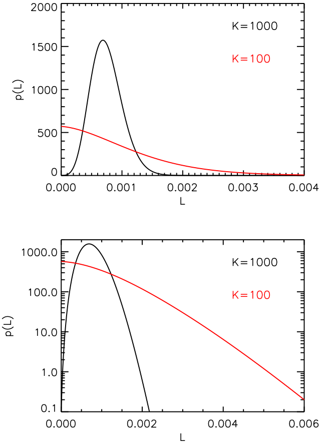
# Simplified Model — No Jumps, No Correlations

- analytical, good approximations
- slow convergence to Gaussian for large portfolio
- kurtosis excess of uncorrelated portfolios scales as $1/K$



# Simplified Model — No Jumps, No Correlations

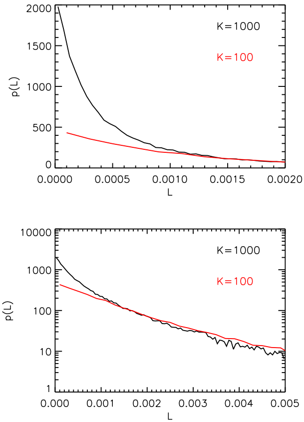
- analytical, good approximations
- slow convergence to Gaussian for large portfolio
- kurtosis excess of uncorrelated portfolios scales as $1/K$
- diversification works slowly, but it works!



# Numerical Simulations

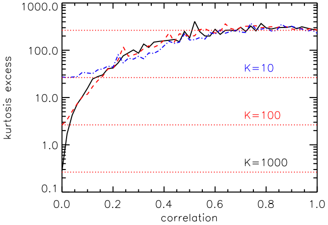
# Numerical Simulations: Influence of Correlations, No Jumps

fixed correlation $C_{kl} = c, \; k \neq l$ , and $C_{kk} = 1$

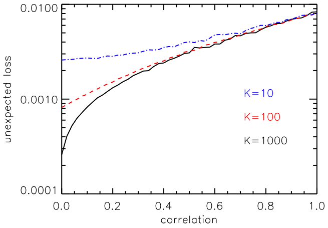



$c = 0.2$ $c = 0.5$

# Tail as Function of Fixed Correlation



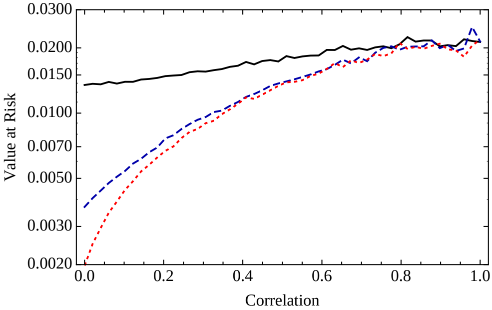

kurtosis excess




standard deviation (UL)

- limiting tail behavior quickly reached
- diversification does not work, it does not reduce risk !
- standard deviation decreases, bad measure for credit risk

# Value at Risk versus Fixed Correlation




$$\int_0^{\mathrm{VaR}} p(L)dL = \alpha$$

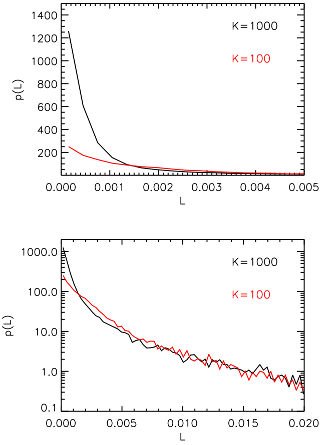
here $\alpha = 0.99$

$K = 10, 100, 1000$

99% quantile, portfolio losses are with probability 0.99 smaller than VaR, and with probability 0.01 larger than VaR

# Numerical Simulations: Correlations and Jumps

- correlated jump–diffusion
- fixed correlation $c = 0.5$
- jumps change picture only slightly
- tail behavior stays similar with increasing $K$



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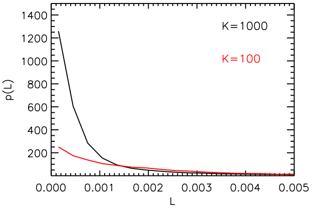
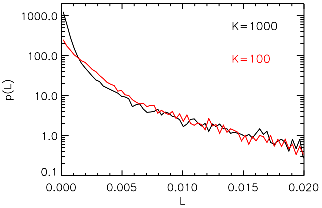


# Random Matrix Approach

# Search for Generic Features

- large portfolio $\rightarrow$ large $K$
- correlation matrix $C$ is $K \times K$
- "second ergodicity": spectral average $=$ ensemble average
- set $C = WW^\dagger$ and choose $W$ as random matrix

# Search for Generic Features

- large portfolio $\rightarrow$ large $K$
- correlation matrix $C$ is $K \times K$
- "second ergodicity": spectral average = ensemble average
- set $C = WW^\dagger$ and choose $W$ as random matrix
- additional motivation: correlations vary over time

# Price Distribution at Maturity

Brownian motion, $V = (V_1(T), \ldots, V_K(T))$, price distribution

$$p^{(\mathrm{mv})}(V,\Sigma) = \frac{1}{\sqrt{2\pi T}^K}\frac{1}{\sqrt{\det \Sigma}} \exp\left(-\frac{1}{2T}(V-\mu T)^\dagger \Sigma^{-1}(V-\mu T)\right)$$

$C = WW^\dagger$ with $W$ rectangular real $K \times N$,
$N$ free parameter, such that $\Sigma = \sigma WW^\dagger \sigma$

assume Gaussian distribution for $W$ with variance $1/N$

$$p^{(\mathrm{corr})}(W) = \sqrt{\frac{N}{2\pi}}^{KN} \exp\left(-\frac{N}{2}\mathrm{tr}\, W^\dagger W\right)$$

average correlation is zero, that is $\langle WW^\dagger \rangle = 1_K$

# Average Price Distribution

$$\langle p^{(\mathrm{mv})}(\rho)\rangle = \int d[W] p^{(\mathrm{corr})}(W) p^{(\mathrm{mv})}(V, \sigma W W^\dagger \sigma)$$

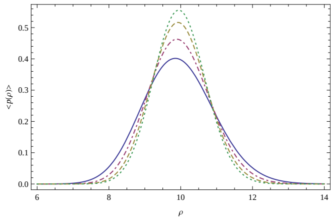
$$= \sqrt{\frac{N}{2\pi T}}^{K} \frac{2^{1-\frac{N}{2}}}{\Gamma(N/2)} \rho^{\frac{N+K-1}{2}} \sqrt{\frac{N}{T}}^{\frac{N-K}{2}} \mathcal{K}_{\frac{N-K}{2}} \left(\rho \sqrt{\frac{N}{T}}\right)$$

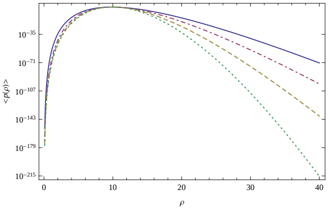
with hyperradius $\rho = \sqrt{\sum_{k=1}^{K} \frac{V_k^2(T)}{\sigma_k^2}}$

similar to statistics of extreme events

easily transferred to geometric Brownian motion

# Heavy Tailed Average Price Distribution



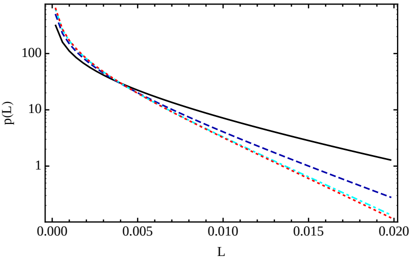




$K = 50$ and $N = K, 2K, 5K, 30K$

$N$ smaller $\longrightarrow$ stronger correlated $\longrightarrow$ heavier tails

# Average Loss Distribution

$$\langle p(L)\rangle = \int d[V]\langle p^{(\mathrm{mv})}(\rho)\rangle\,\delta\left(L - \frac{1}{K}\sum_{k=1}^{K} L_k\right)$$




$\langle C_{kl}\rangle = 0\ , \quad k \neq l$

$N = 5 \ \rightarrow \ \mathrm{std}\,(C_{kl}) = 0.45$

$K = 10, 100, 1000, 10000$

best case scenario, heavy tails remain, little diversification benefit

# General Conclusions

- uncorrelated portfolios: diversification works (slowly)
- correlations lead to extremely fat–tailed distribution
- fixed correlations: diversification does not work
- ensemble average reveals generic features of loss distributions
- average correlation zero, but still: heavy tails remain, little diversification benefit

# Conclusions in View of the Present Credit Crisis

- credit contracts with high default probability, e.g. houses as securities
- credit institutes resold the risk of credit portfolios, grouped by credit rating
- lower ratings $\rightarrow$ higher risk and higher potential return
- problems:
  - rating agencies rated way too high
  - effect of correlations underestimated
  - benefit of diversification overestimated

R. Schäfer, M. Sjölin, A. Sundin, M. Wolanski and T. Guhr,
*Credit Risk - A Structural Model with Jumps and Correlations,*
Physica **A383** (2007) 533

M.C. Münnix, R. Schäfer and T. Guhr,
*A Random Matrix Approach to Credit Risk,*
`arXiv:1102.3900`

both ranked for several months among the top–ten new credit risk papers on `www.defaultrisk.com`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