

# Econophysics II: Detailed Look at Stock Markets and Trading

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# Outline

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- stock markets and trading **in reality**
- two extreme **model scenarios**: Efficient Market Hypothesis, Zero Intelligence Trading
- large scale **data analysis** reveals non–Markovian features
- artificial stock market to **encircle mechanisms**
- trading strategies and **temporal correlations**

# Stock Market Trading in Reality

# Clearing House and Orders

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trading via clearing house, buy and sell offers/orders (bids and asks)

**limit order:** bid or ask for a specific volume at a specific price within a certain time window,

best ask  $a(t)$ , best bid  $b(t)$ , always  $a(t) \geq b(t)$

if equal  $\longrightarrow$  trade, price  $S(t) = a(t) = b(t)$  immediately thereafter

**market order:** buy or sell immediately what is offered,  
 $S(t) = b(t)$  or  $S(t) = a(t)$

# Order Book

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make public to provide all traders with same information



## Bid/Ask-Overview

Bid	Bid Qty		Ask Qty	Ask
44.020	1,845		675	44.040
44.015	577		2,444	44.045
44.010	1,195		2,121	44.050
44.005	1,016		685	44.055
44.000	921		592	44.060
43.995	685		184	44.065
43.990	2,022		184	44.070
43.985	699		103	44.075
43.980	1,146		1,026	44.080
43.975	1,011		380	44.090

limit orders appear in the order book, market orders do not

# Midpoint, Bid–ask Spread, Trade Sign

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in between trades, there is no price !

**bid–ask spread**  $s(t) = b(t) - a(t) < 0$

the higher the trading frequency, the smaller is  $s(t)$

**midpoint**  $m(t) = \frac{a(t) + b(t)}{2}$

immediately after a trade, define **trade sign**

$$\vartheta(t) = \begin{cases} +1 & \text{if } S(t) \text{ is higher than the last } m(t) \\ -1 & \text{if } S(t) \text{ is lower than the last } m(t) \end{cases}$$

positive, if trade was triggered by a market order to buy  
negative, if trade was triggered by a market order to sell

# Traders and Liquidity

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market is **liquid**, if there are always enough shares at a “reasonable” price to ensure that every planned trade can be carried out and if the trading happens continuously

small bid–ask spread  $s(t) = b(t) - a(t)$  is indicator

**limit orders** make a market liquid  $\longleftrightarrow$  **liquidity providers**

**market orders** absorb liquidity  $\longleftrightarrow$  **liquidity takers (“informed”)**

liquidity providers and takers are not static populations, these rôles change constantly

# Model Scenarios



# First Extreme Model Scenario — EMH

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## Efficient Market Hypothesis

Traders always act fully rationally. Market price results from consensus between the traders about the “fair” price. It always exists and reflects quantifiable economic value of asset.

Deviation of market price from “fair” price  $\longrightarrow$  arbitrage  $\longrightarrow$  disappears. Consensus comes about, because group of traders processed all available information. Price can only change if new information arrives. The new information is totally random.

# Second Extreme Model Scenario — ZIT

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## Zero Intelligence Trading

Individual trader is irrational and acts fully randomly. The other traders do not know that and interpret the buy and sell decisions made by others as potentially information driven. Price change is not attributed to new information, it automatically follows from the fact that trading takes place  $\longrightarrow$  demand and supply. There is no fair price, midpoint  $m(t)$  moves as well. Traders immediately accept the new midpoint as the new reference point about which they send out their random buy and sell orders.

# Where is the Truth ?

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both scenarios lead to a Markovian random walk model for price !

both partly compatible with reality, but there are objections:

**EMH:** “fair” price deeply obscure → what is then rational ? — high volatilities incompatible with rational pricing — time scales of trading not consistent with those of information flow

**ZIT:** irrationality not realistic either, traders use information

truth is somewhere in between → need detailed data analysis

# Collecting Empirical Information

# Data Analysis

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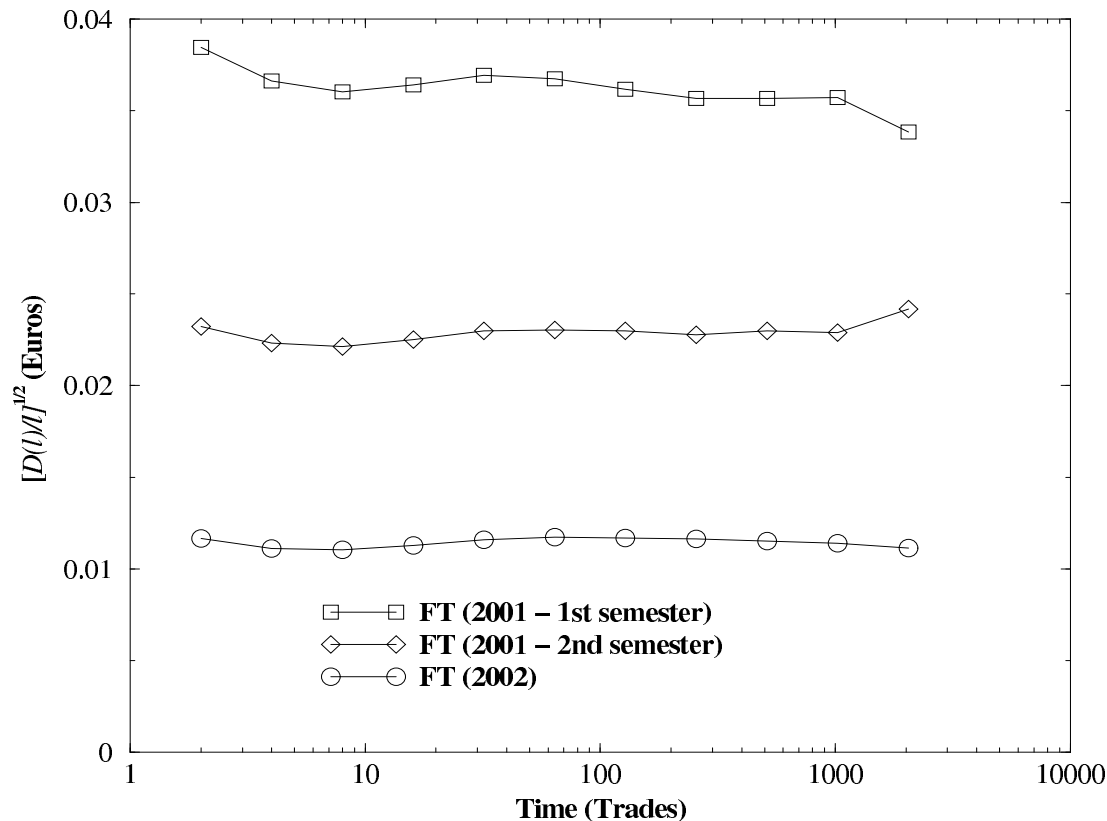
Bouchaud, Gefen, Potters and Wyart, Quantitative Finance 4  
(2004) 176

- fully electronically traded French stocks 2001–2002
- intraday
- high frequency, up to 10000 trades/day
- volumes between a few and 80000 shares
- trade time instead of real time

# Volatility and Diffusion

subtract drift from  $S(t)$   $\longrightarrow$  detrended price  $Z(t)$

diffusion function  $D(\tau) = \langle (Z(t + \tau) - Z(t))^2 \rangle$



largely constant  
volatility function

$$\sqrt{D(\tau)/\tau}$$

for France–Telecom

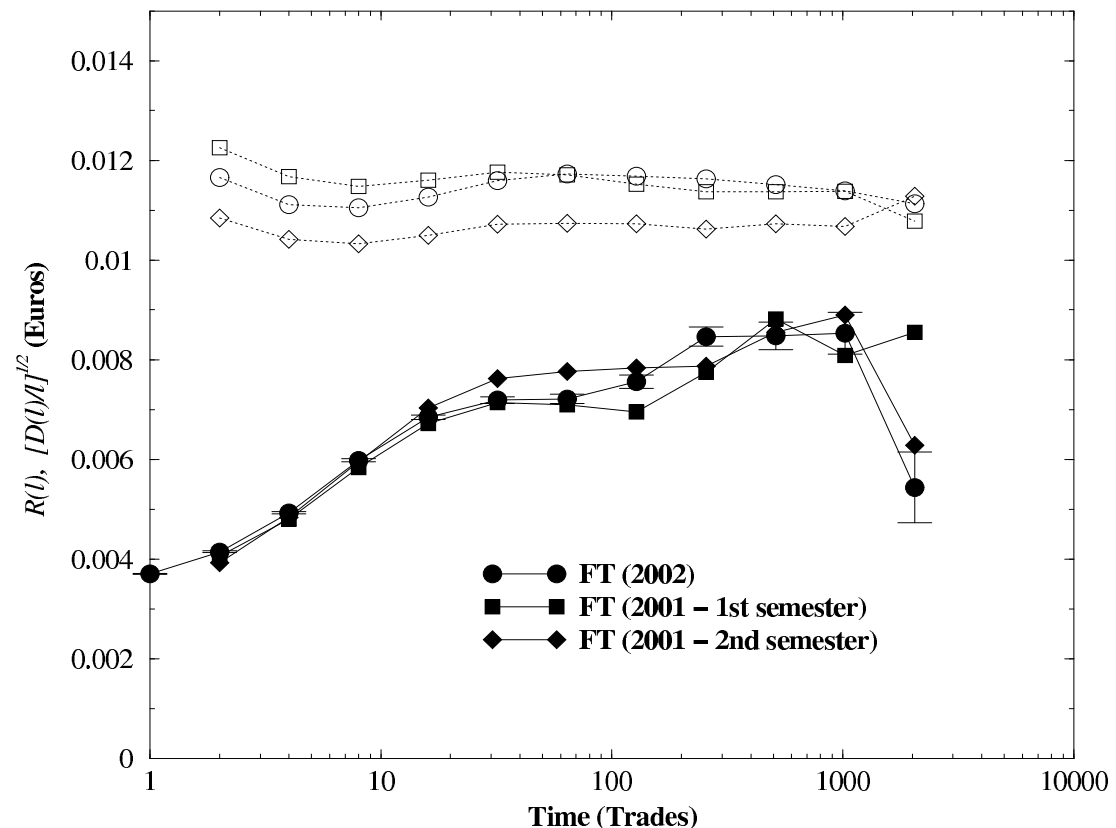
diffusive motion !

# Average Response to Trading

response function  $R(\tau) = \langle (Z(t + \tau) - Z(t)) \vartheta(t) \rangle$

average impact of trading at  $t$  on subsequent price changes

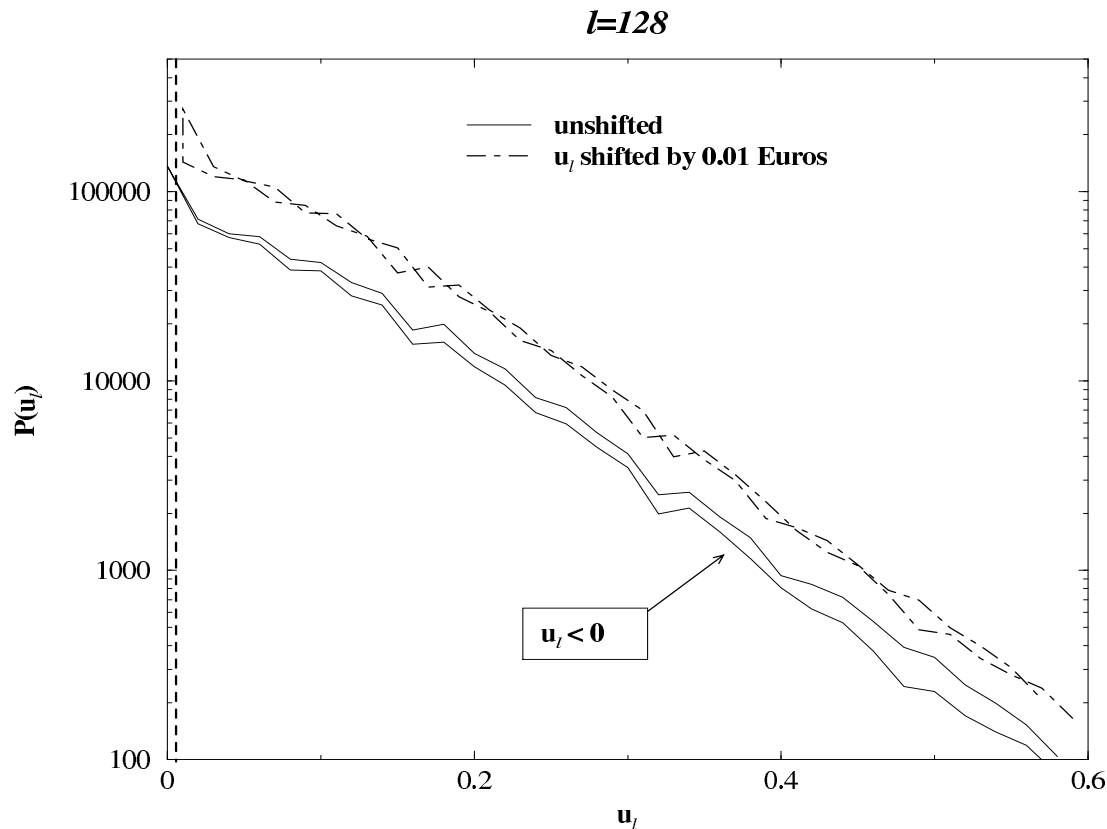
non-zero empirical  
result proves  
non-Markovian  
behavior !



# Distribution of Sign Supplemented Price Changes

sign supplemented price changes  $u(t, \tau) = (Z(t + \tau) - Z(t)) \vartheta(t)$

response  $R(\tau) = \langle u(t, \tau) \rangle$ , diffusion function  $D(\tau) = \langle u^2(t, \tau) \rangle$



distribution  $p(u(t, \tau))$   
for  $\tau = 128$

moment  $R(128) > 0$

small arbitrage

truly informed traders



# Power Law Autocorrelations in Trade Signs

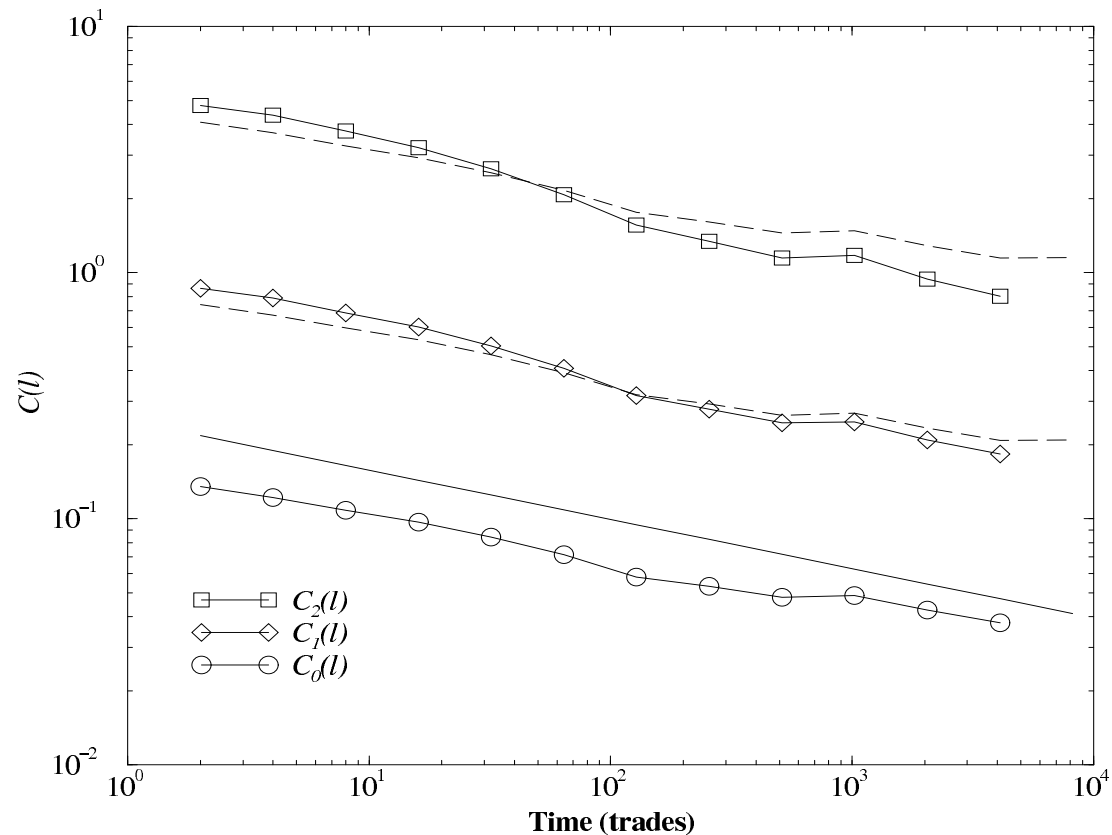
trade sign autocorrelation  $\Theta(\tau) = \langle \vartheta(t + \tau)\vartheta(t) \rangle - \langle \vartheta(t) \rangle^2$

power law

$$\Theta(\tau) \sim \frac{1}{\tau^\gamma}$$

with  $\gamma < 0$

non-Markovian,  
outrules ZIT idea !



# Modeling the Price Dynamics

# Non-Markovian Model

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$$Z(t) = \sum_{t'=1}^t G_0(t-t') \vartheta(t') \ln V(t') + \sum_{t'=1}^t \varepsilon(t')$$

first term non-Markovian, second Markovian

ansatz for bare impact function  $G_0(\tau) \sim \frac{1}{(1 + \tau/\tau_0)^\beta}$

→  $D(\tau) \sim \tau^{2-2\beta-\gamma}$ , critical exponent  $\beta_c = \frac{1-\gamma}{2}$

$\beta = \beta_c$  diffusive,  $\beta > \beta_c$  sub-diffusive,  $\beta < \beta_c$  super-diffusive

possible to reproduce empirical  $R(\tau)$  for  $\beta \approx \beta_c$  and  $\tau_0 \approx 20$

# Liquidity Takers versus Liquidity Providers

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Reality is **non-Markovian**, interpreted as a **competition**: Consider trader who is “informed” that price of a company will go up. He wants to buy shares, likely by market orders  $\rightarrow$  liquidity taker. Not be wise to place big offer, because this would alert liquidity providers who emit the limit orders to sell (“knows something”). They would place their limit orders at higher price. Liquidity taker is aware  $\rightarrow$  divides his market order into smaller chunks which he places one after the other  $\rightarrow$  introduces temporal autocorrelations  $\Theta(\tau)$ . Liquidity providers want to mean revert price  $\rightarrow R(\tau) \rightarrow 0$  for large  $\tau$ . They do that slowly, because they do not know whether liquidity taker’s information becomes true  $\rightarrow$  maximum of  $R(\tau)$ .  $\rightarrow$  **Persistence**: liquidity providers do not sufficiently mean revert the price  $\rightarrow$  super-diffusive. **Antipersistence**: they mean revert too strongly  $\rightarrow$  sub-diffusive.  $\rightarrow$  Subtle balance between sub- and super-diffusive  $\rightarrow$  effectively diffusive. Compares to balancing a stick on the palm.

# Artificial Stock Market

# Agent Based Modeling

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financial markets are complex systems: many degrees of freedom, non-linear effects, basic processes and time evolution governed by **probabilistic rules**, not by **deterministic equations**

**top-down approach**: schematic models, stochastic processes  
→ successes and limitations

**bottom-up approach**: artificial stock market on computer with virtual traders → agent based modeling

- set up system microscopically and let evolve
- price dynamics and all other macroscopic observables result
- identify crucial mechanisms by encircling them

various examples in biology, social sciences, economics, one of the first is Conway's **Game of Life** (1970)

# Wigner's Caveat

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“It is nice to know that the computer understands the problem. But I would like to understand it too.”

Eugene Paul Wigner, 1902–1995



# Impact of Trading Strategies

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introduce **different types** of traders → top-down element in an otherwise bottom-up approach, not adaptive

- ZeroIntelligenceTrader
- RandomTrader
- EagerTrader
- LiquidityProvider
- RandomInformedTrader
- SerialTrader
- ExpectingTrader

**let evolve and see what happens !**

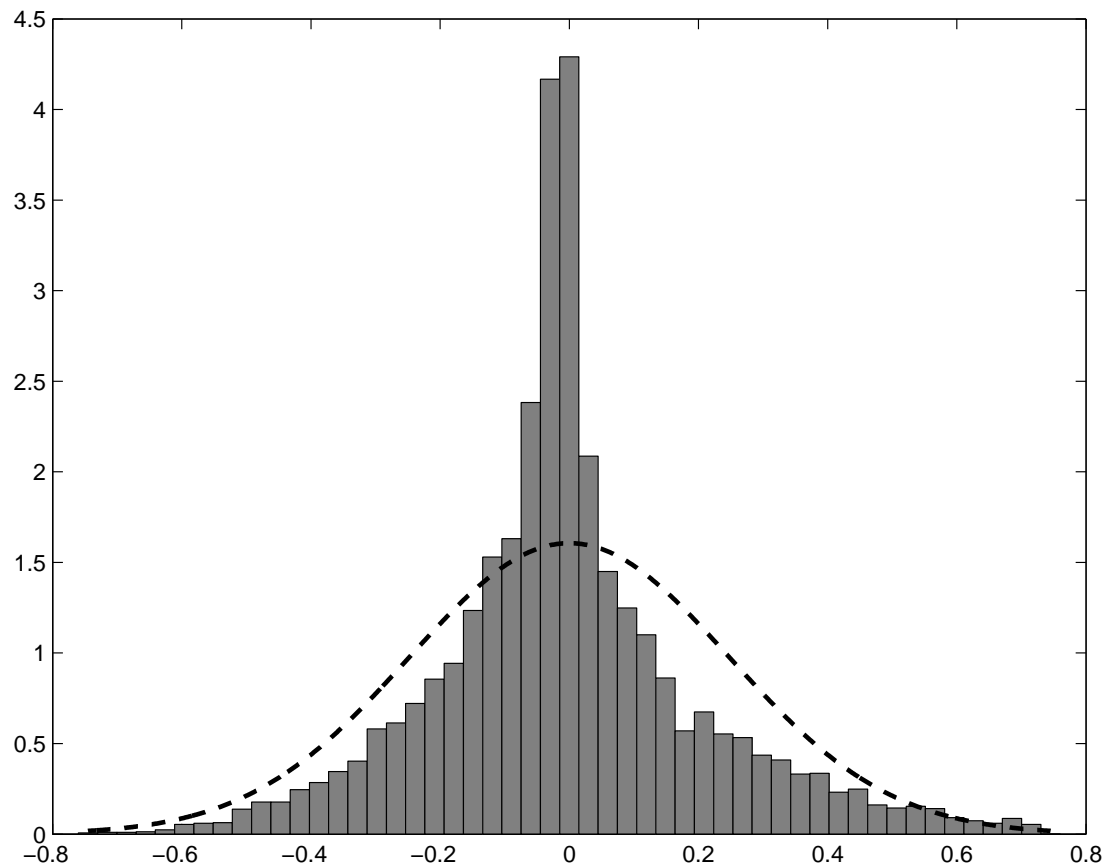
Berseus, Schäfer, Guhr (2007)



# Zero Intelligence Trading

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population of 300 ZeroIntelligenceTraders  
return distribution after 10000 trades



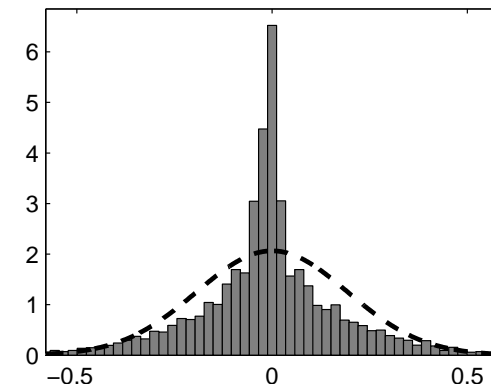
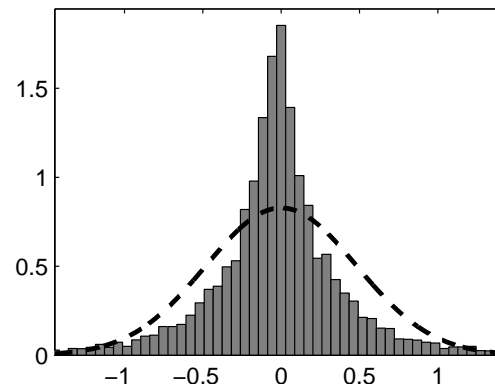
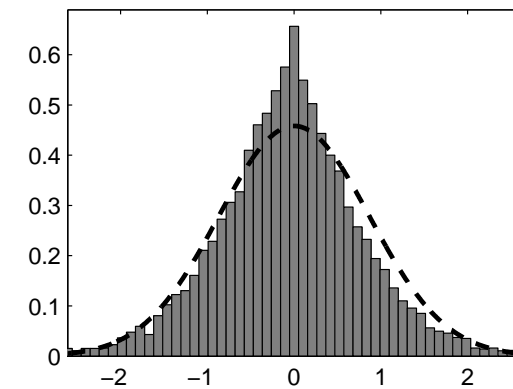
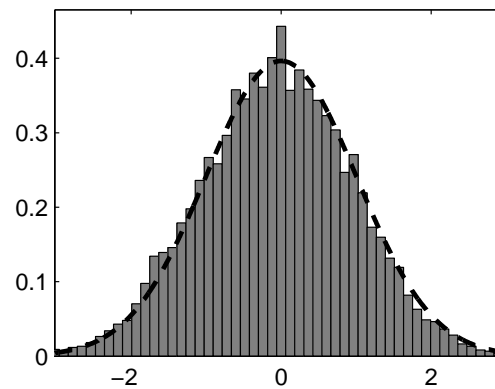
non-Gaussian,  
heavy tails !

# Heavy Tails and Order Book

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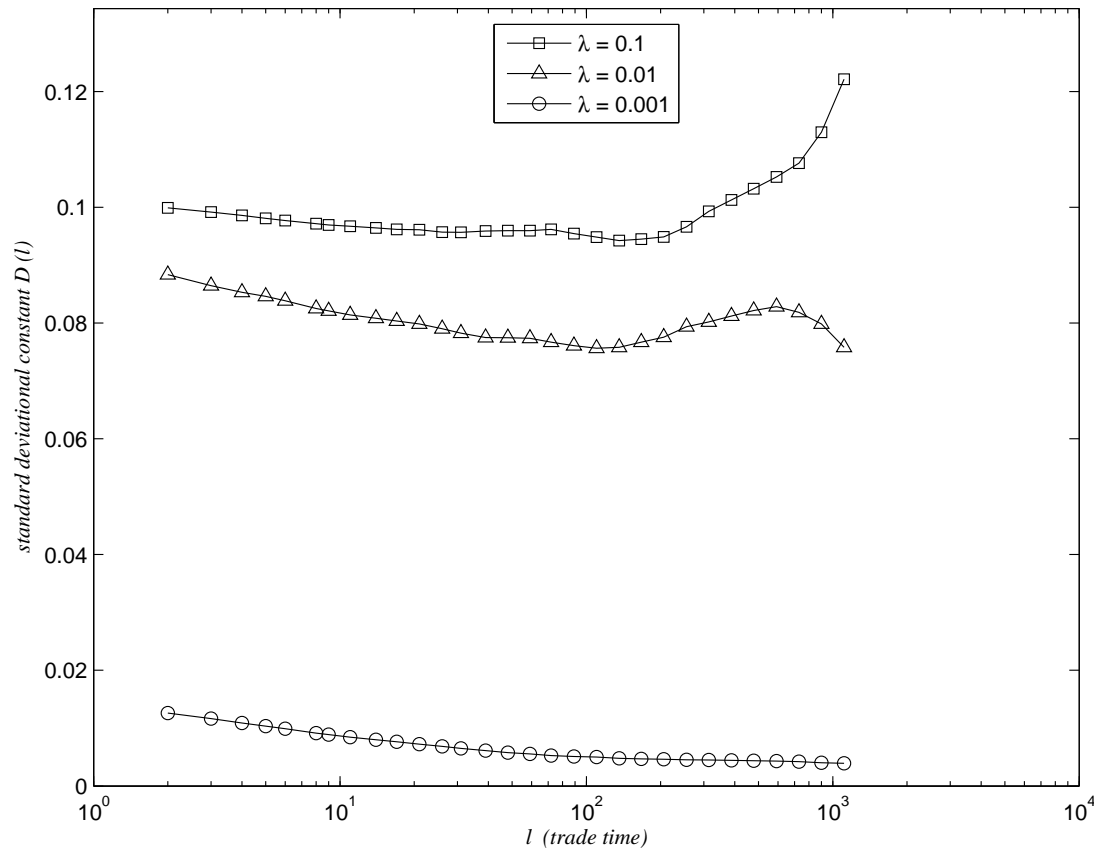
population of 300 traders, distributions of price differences  
EagerTraders and three versions of RandomTraders

non-Gaussian  
when order book  
becomes  
more important



# Mixed Populations — Volatility Function

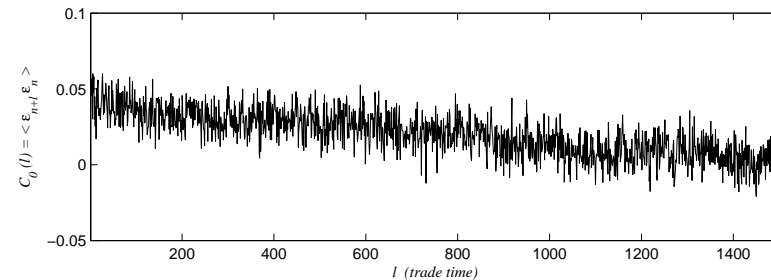
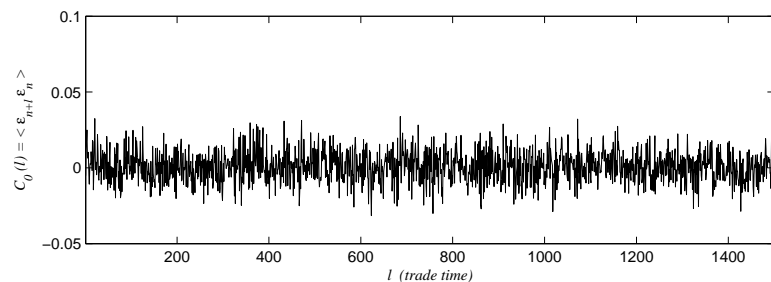
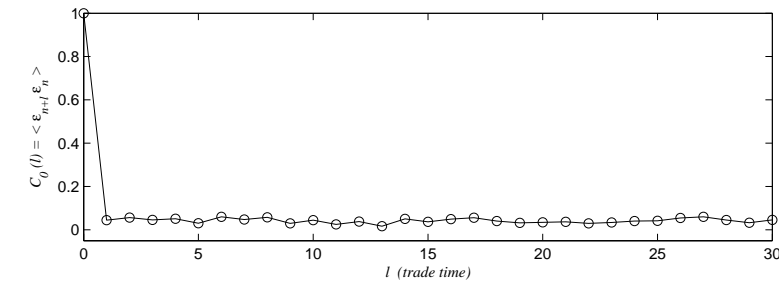
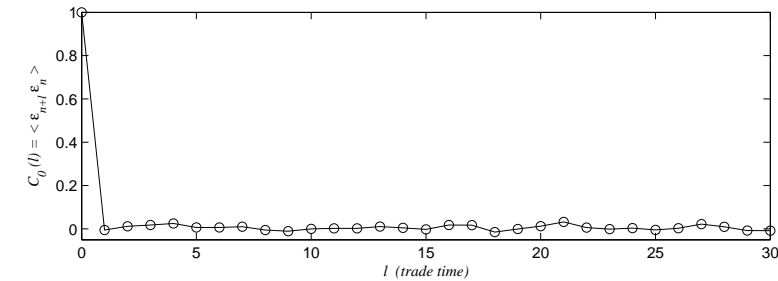
LiquidityProvider and three versions of RandomInformedTraders



largely diffusive

depends sensitively on likeliness to emit market orders

# Mixed Populations — Trade Sign Autocorrelations



Liquidity Providers,  
Serial Traders

Liquidity Providers,  
Expecting Traders

Expecting Trader waits for gap between  $m(t)$  and “fair” price,  
Serial Trader does not  $\longrightarrow$  trade sign autocorrelations

# Summary and Conclusions

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- stock markets and trading **in reality**
- two extreme **model scenarios**: Efficient Market Hypothesis, Zero Intelligence Trading
- large scale **data analysis** reveals non-Markovian features
- schematic **top-down** stochastic model
- artificial stock market as **bottom-down** approach
- heavy tails are **order book effect**
- trading strategies sensitively determine **temporal correlations**

