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From Physics to Finance

XXXIX Heidelberg Physics Graduate Days

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The banks' role in the economy

The "Banks"





Photo source: © NH1977 / PIXELIO

New players – Fintecs try to disrupt the banking industry

New opportunities arise from disrupting old and inefficient processes in the banking business:

- » Use of Blockchain technologies contemplated in:
 - Money (Bitcoin etc.)
 - > Stock exchange
 - Account management
 - Interbanking transactions
 - > Title register...
- » Brokerage of fix rate invest in foreign countries
- » Assistance in changing the bank-account
- » Support of collection services
- Credit rating with the help of user profiles/social network data (currently not legal in Germany)
- » First Fintechs in Germany are in possession of a bank licence
- » Established banks take notice \rightarrow Cooperation with or acquisition of Fintechs



Quelle: Barkow-Consulting



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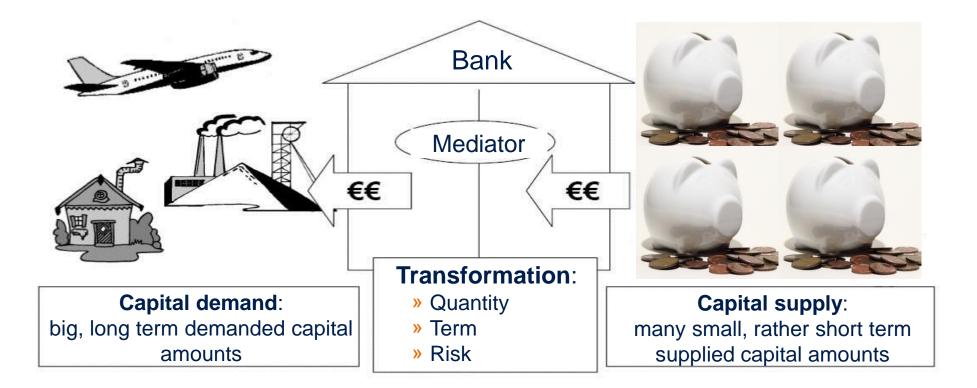
,	Universalbanken (1.635)					
	266	Kreditbanken	4	Großbanken		
			155	Regionalbanken und sonstige Kreditbanken		
three			107	Zweigstellen ausländischer Banken		
pillars	963	Genossenschaftliche Kreditinstitute	963	Kreditgenossenschaften		
	406	Öffentlich-rechtliche Kreditinstitute	397	Sparkassen		
			9	Landesbanken		

Spezi	Spezialbanken (54)						
20	Bausparkassen						
14	Realkreditinstitute						
20	Banken mit Sonderaufgaben						

Stand: Juli 2017

Source: Bankenstatistik, Statistisches Beiheft 1 zum Monatsbericht, Deutsche Bundesbank, September 2017, S. 106

The banks' role – Transforming money

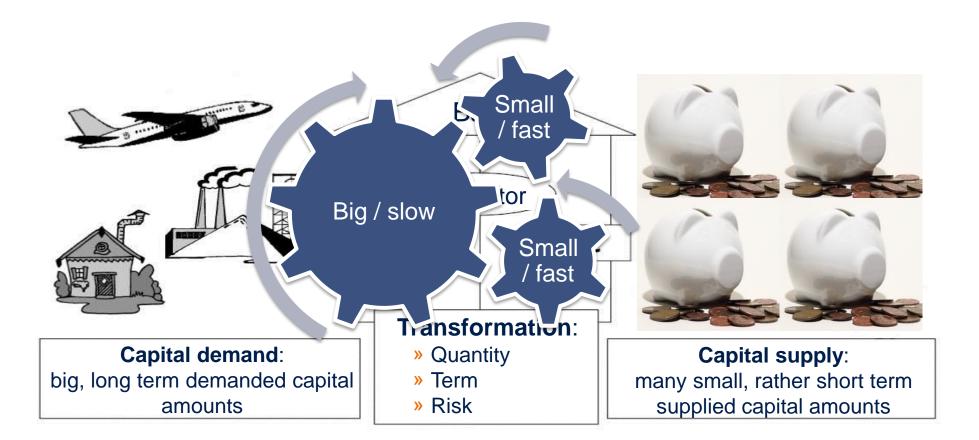


Transformation is at the heart of banking business

Photo source: © segavax, Andreas Hermsdorf / PIXELIO

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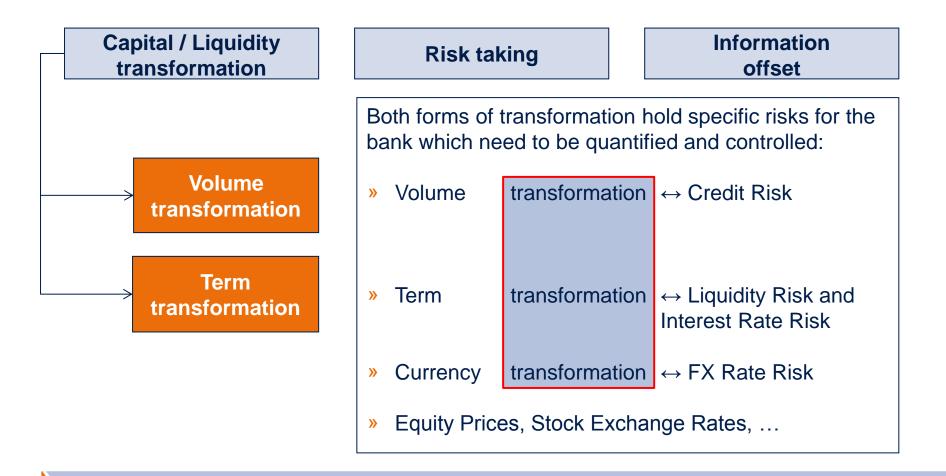
The banks' role – Transforming money



Transformation is at the heart of banking business

Photo source: © segavax, Andreas Hermsdorf / PIXELIO

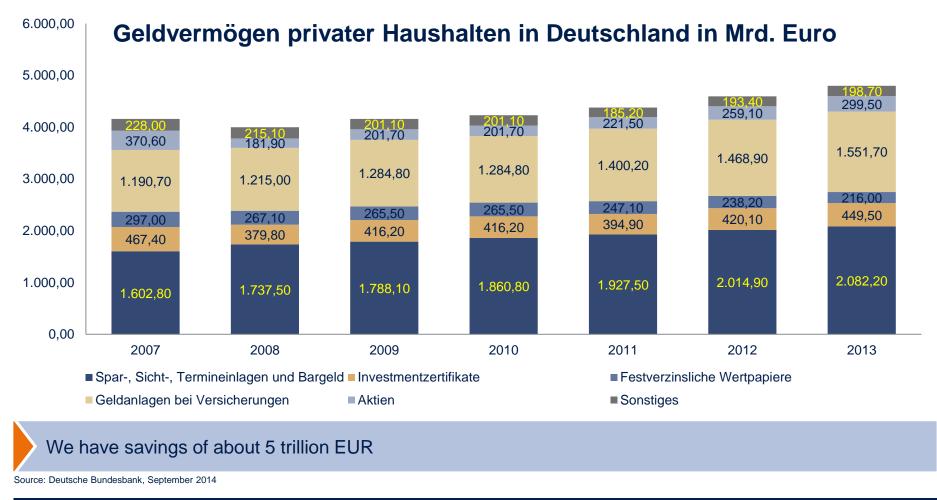
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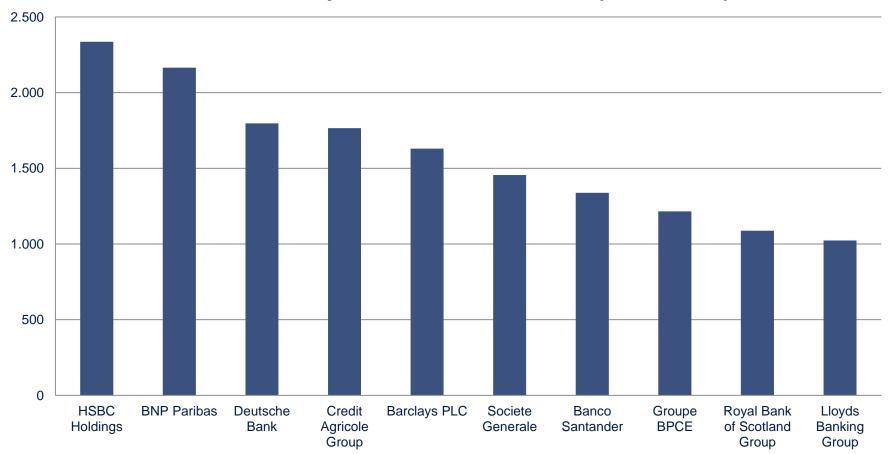


Transformation is at the heart of banking business

German saving behaviour

» Germans still invest the largest part of their capital in savings- / sight- / termdeposits and cash, as well as insurances

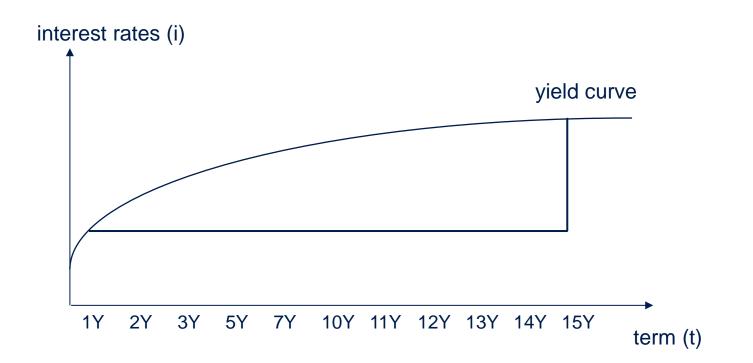




Total Assets of European Banks in trillion € (30.06.2016)

Source: http://www.relbanks.com/, http://www.xe.com

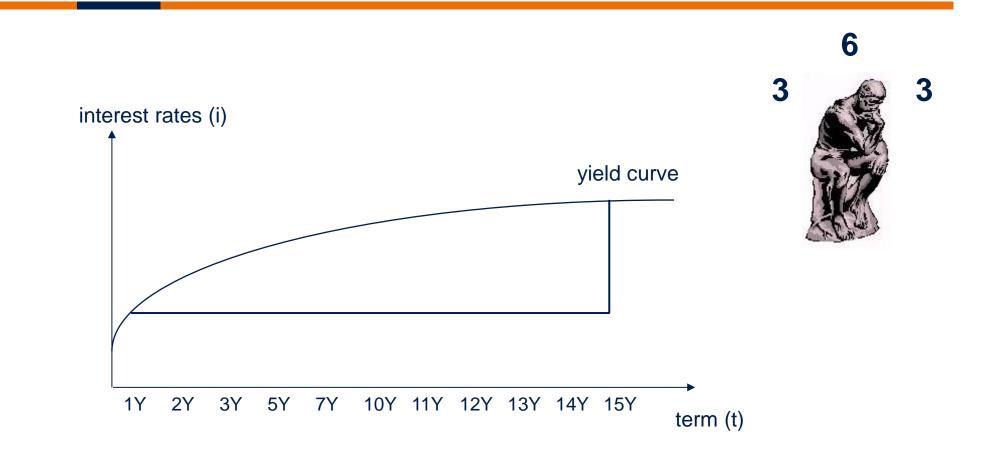
Time series in finance – non-linearity and prediction of the future



Term transformation, i.e., transformation in time, is a major transformation

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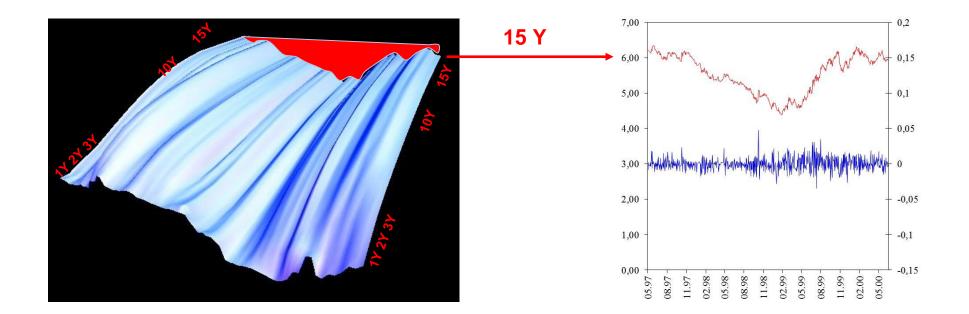
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Term transformation, i.e., transformation in time, is a major transformation

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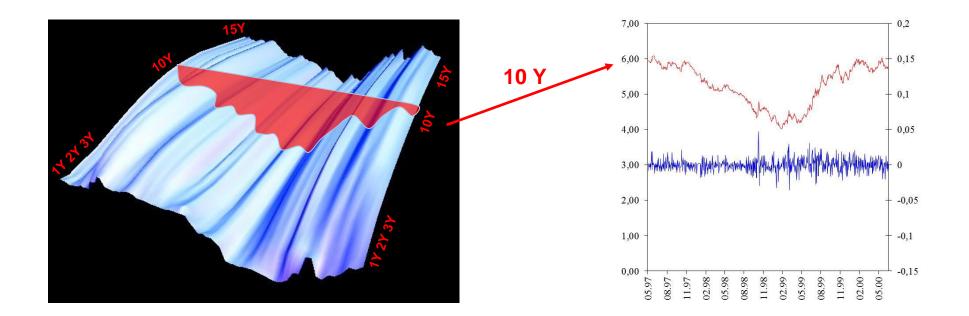
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The change in interest rates follows no simple statistics

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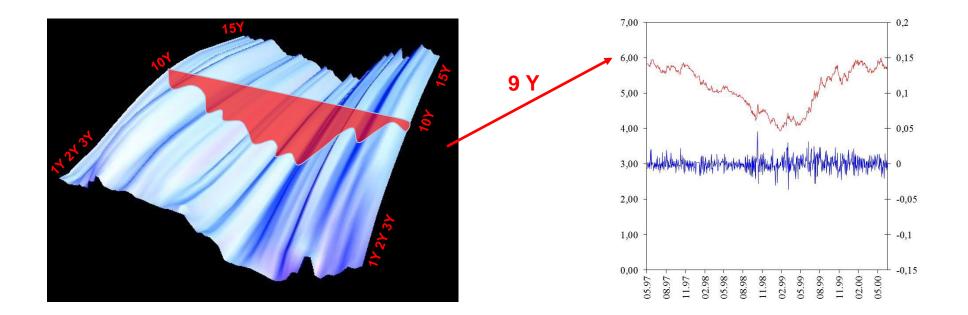
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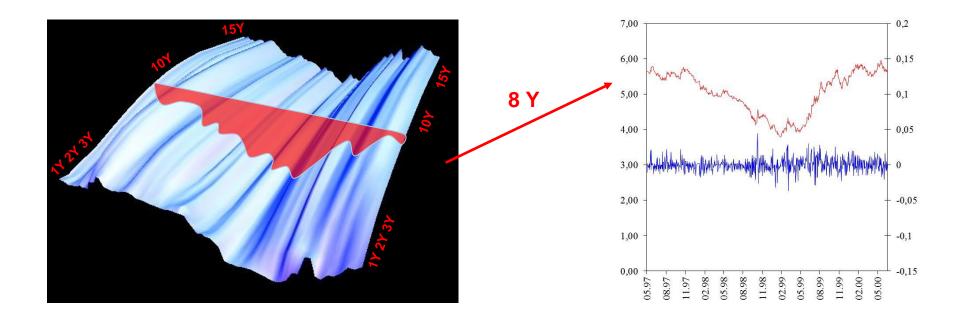
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The change in interest rates follows no simple statistics

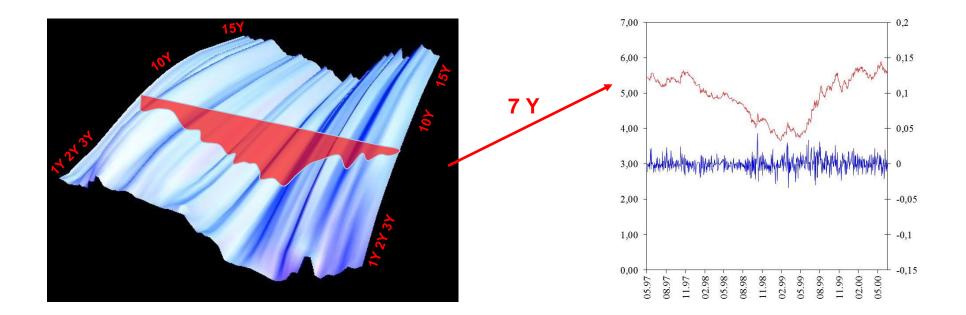
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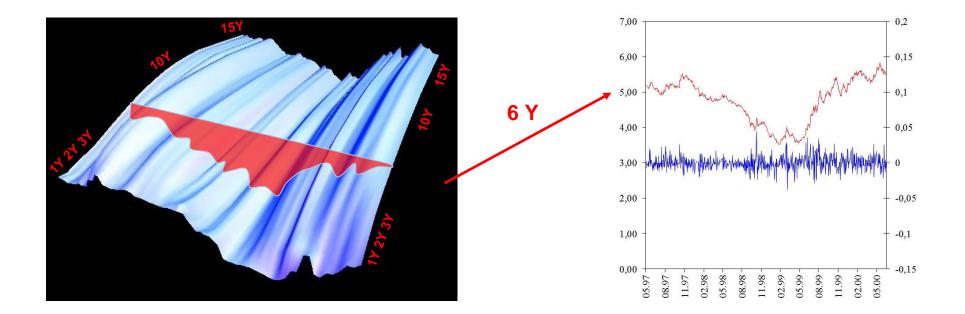
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The change in interest rates follows no simple statistics

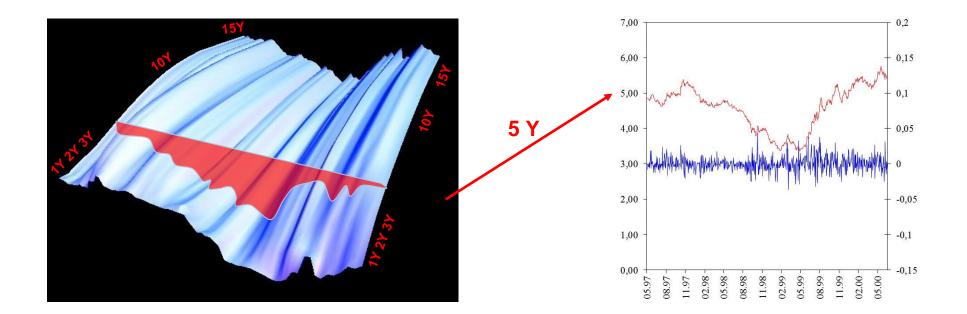
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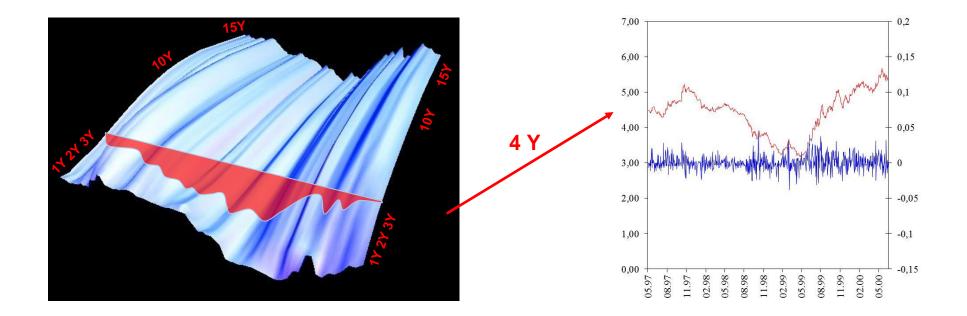
The change in interest rates follows no simple statistics

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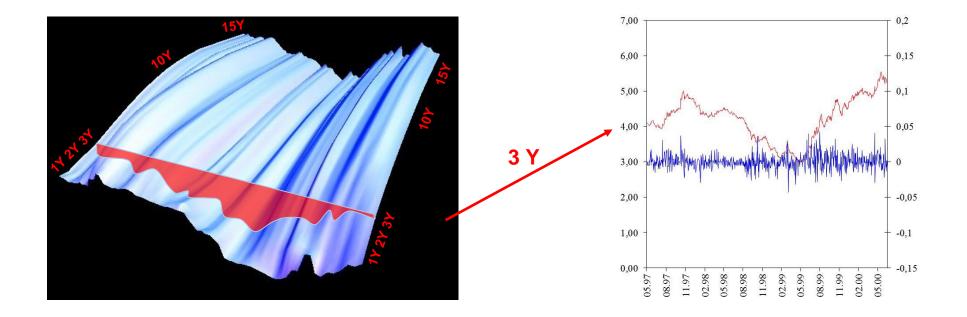
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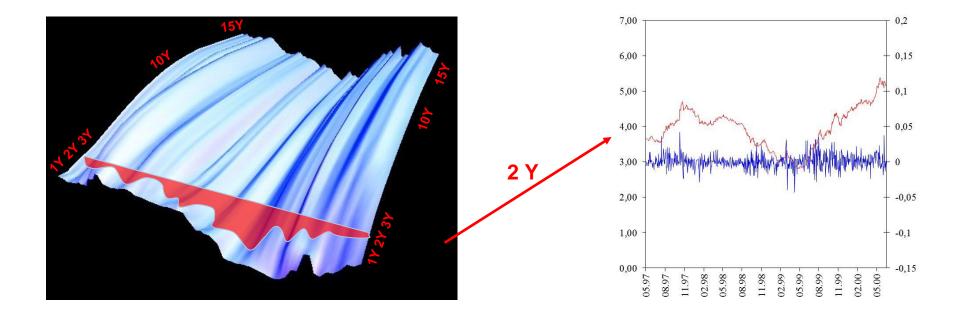
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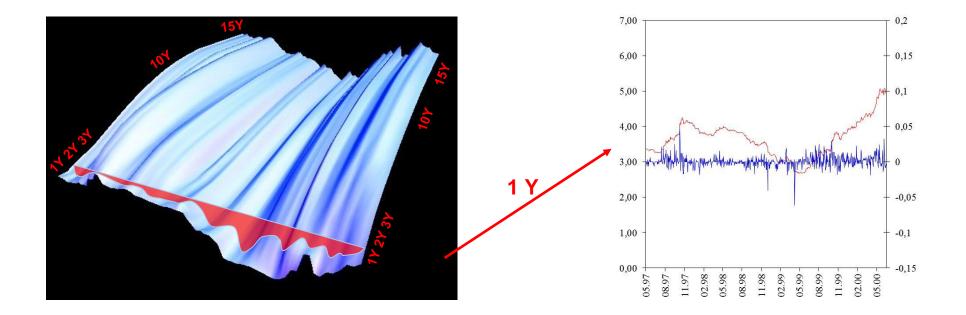
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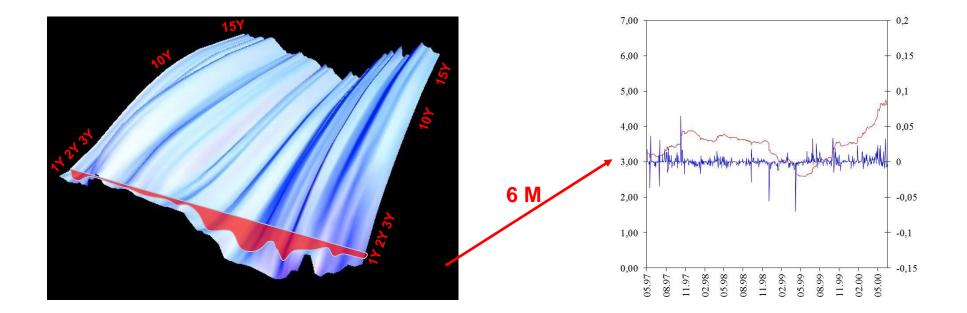
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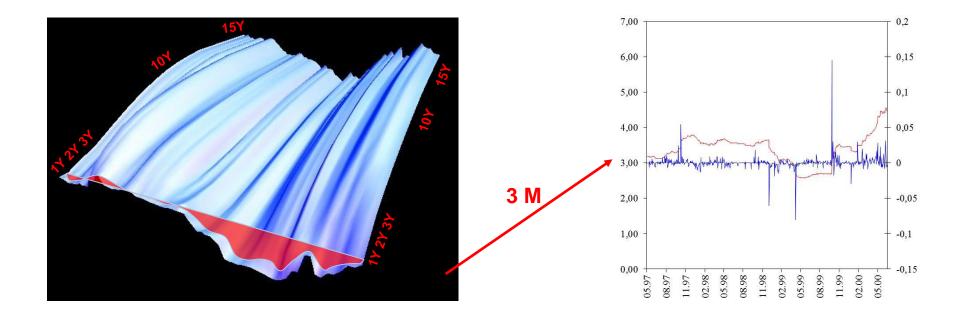
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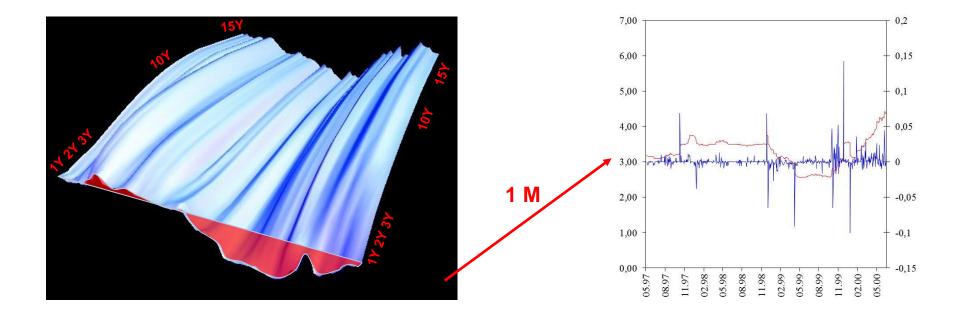
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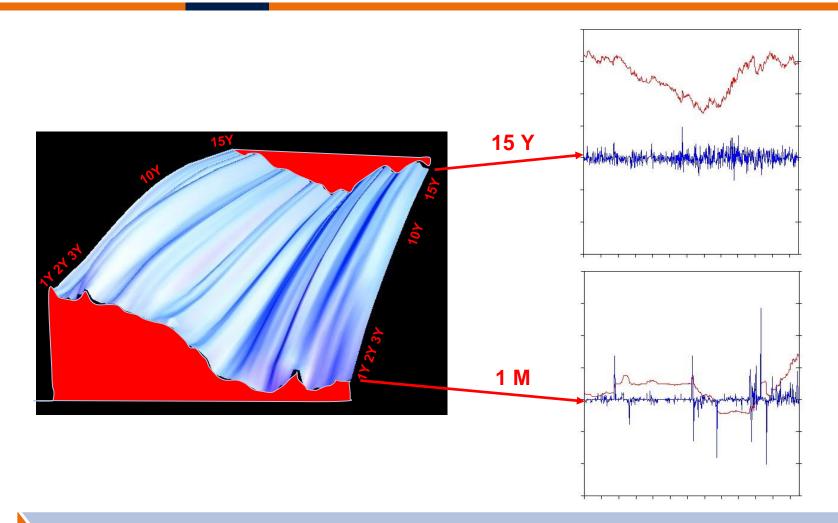
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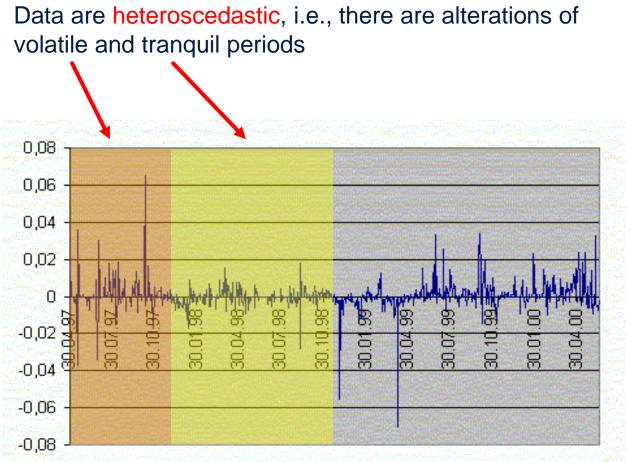
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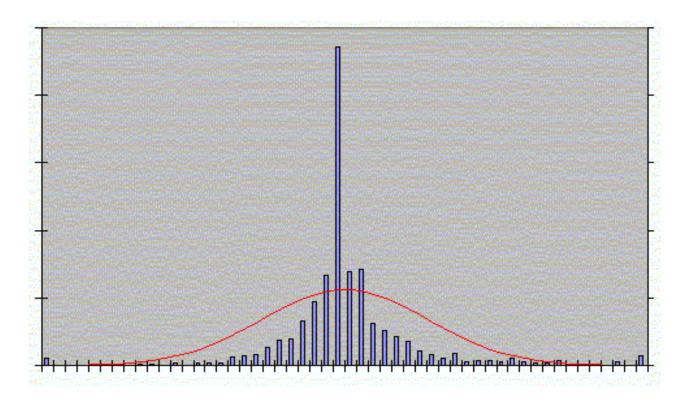
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The change in interest rates follows no simple statistics



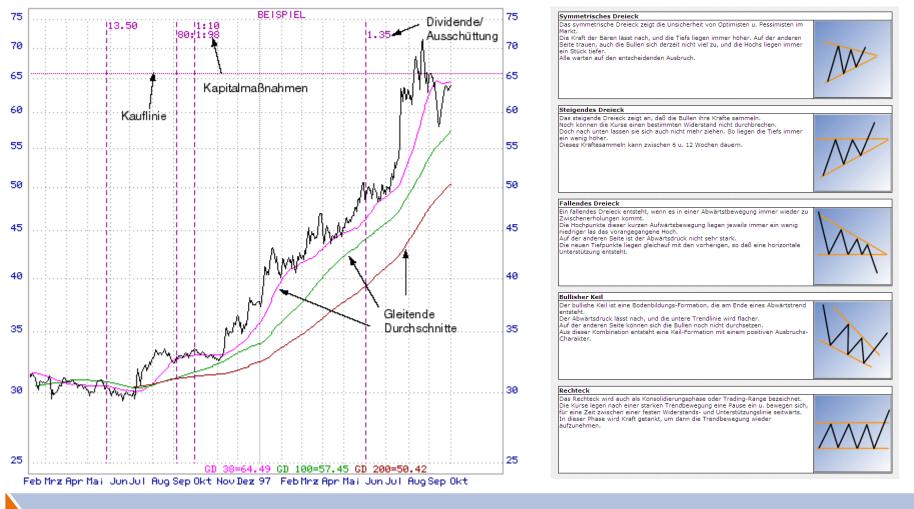
Data are leptocurtic, i.e., the empirical distribution is more pronounced / steeper in the middle of the distribution as the normal distribution and it has more mass in the tails as a normal distribution (fat tails).



How to "explain" the curves – Different approaches



How to "explain" the curves – Different approaches

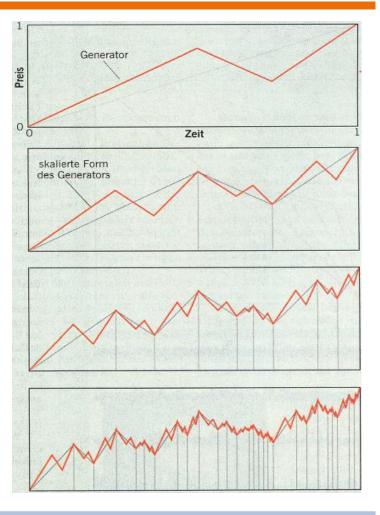


The "Euclidean geometry" approach

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How to "explain" the curves – Different approaches





The fractal geometry approach

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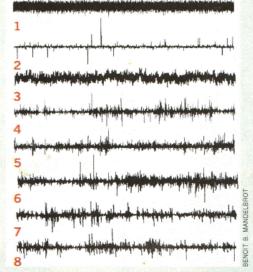
Wie gut können Multifraktale echte Preis-Charts wiedergeben? Vergleichen wir mehrere historische Preisverläufe mit ein paar künstlichen Modellen.

Die erste Kurve ist offensichtlich noch weit von der Realität entfernt. Sie ist au-Berordentlich einförmig und läuft auf einen konstanten Hintergrund kleiner Preisänderungen hinaus, wie das Rauschen beim Radioempfang. Die Volatilität bleibt gleichförmig, ohne plötzliche Sprünge. Wenn das die Aufzeichnung eines historischen Preisverlaufs wäre, würden sich die Veränderungen zwar von Tag zu Tag unterscheiden, aber die Monate würden insgesamt doch sehr gleichartig verlaufen.

Die ziemlich einfache zweite Kurve ist schon besser, denn sie zeigt viele plötzliche Zacken. Aber die stehen isoliert gegen einen unveränderlichen Hintergrund, in

dem die Variabilität der Preise ungefähr gleich bleibt. Das ist bei der dritten Kurve besser getroffen; dafür zeigt sie keine urplötzlichen Sprünge.

Alle drei Diagramme sind mit bloßem Auge als unrealistisch zu erkennen. Woher stammen sie? Kurve 1 folgt einem Modell, das der französische Mathematiker Louis Bachelier (1870 bis 1946) im Jahre 1900 eingeführt hat. Die Preisveränderungen



Welche Kurve ist die gefälschte?

folgen einer Irrfahrt (*random walk*); dazu gehört die Glockenkurve, womit das Modell auf die Portfolio-Theorie hinausläuft. Die Kurven 2 und 3 ergeben sich aus Verbesserungsversuchen von Bacheliers Arbeiten. Die eine entspricht einem Modell, das ich 1963 vorgeschlagen habe (basierend auf Lévy-stabilen Zufallsprozessen) und einem, das ich 1965 publiziert habe (basierend auf *fractional Brownian motion*). Beide sind nur unter sehr speziellen Marktbedingungen sinnvoll.

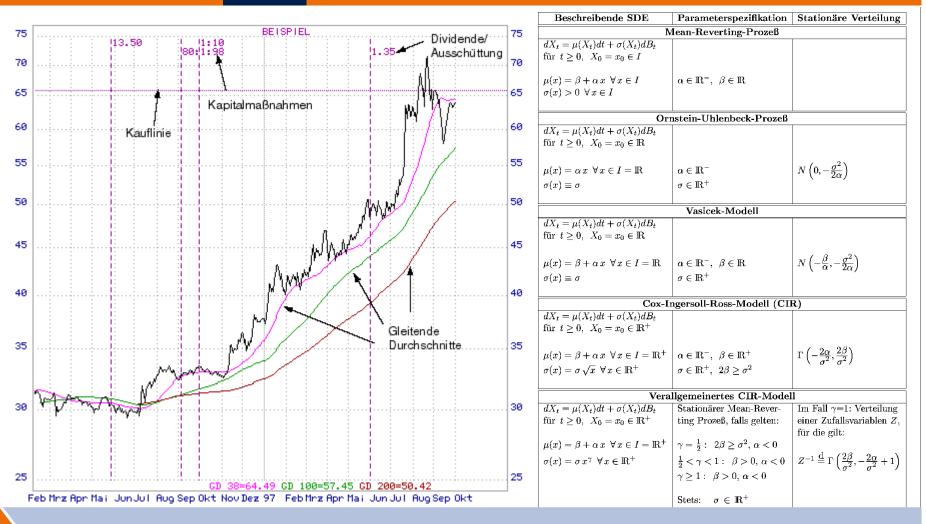
Von den – wichtigeren – fünf unteren Diagrammen beruht wenigstens eines auf echten Marktdaten, und wenigstens ein weiteres ist ein computergeneriertes Beispiel meines letzten multifraktalen Modells. Bevor Sie weiterlesen, versuchen Sie, diese Charts richtig zuzuordnen! Ich hoffe, daß auch Sie auf die Fälschungen hereinfallen.

Tatsächlich sind nur zwei der Charts echte Marktdaten. Chart 5 stellt den Kurs der IBM-Aktie dar und Chart 6 den Wechselkurs DM gegen amerikanische Dollar. Die anderen Kurven (4, 7 und 8) ähneln ihren zwei echten Gegenstücken zwar stark, sind aber vollständig künstlich, erzeugt mit einer weiter verfeinerten Form meines multifraktalen Modells.

The fractal geometry approach

Source: B. B. Mandelbrot, Börsenturbulenzen neu erklärt, Spektrum der Wissenschaft, Mai 1999, 74-77

How to "explain" the curves – Different approaches



The stochastic approach

How to "explain" the curves – Different approaches

Stochastic Calculus

· · · ·	$ dW_t$	dt	Itô Formula
$\mathrm{d}W_t$	dt	0 6	$df(X_t) = f'(X_t) dX_t$
	0		$+rac{1}{2}f''(X_t)(\mathrm{d}X_t)^2$
			$W_s) = \min(t,s)$
Itô-Tan	aka Fori	nula fo	or Local time
$L_t^a(Z)$	$) = Z_{i} -$	$ a - Z_0 $	$ \mathbf{b} - a = \int_0^t \operatorname{sgn}(Z_u - a) \mathrm{d}Z_u (63)$
$f(Z_t) =$	$f(Z_0) + $	$\int_0^t f_t'(Z_t)$	u_u) d $Z_u + \frac{1}{2} \int_{\mathcal{B}} L_t^a(Z), \mu(\mathrm{d}a)$ (64)
			de derivative of <i>f</i> , measure of distributions
			$\exp(\sigma W_T - \frac{1}{2}\sigma^2 T) dIP$
			an motion under $\bar{IP}_{0} = \sup_{\tau} I\!\!E[e^{-r_{d}\tau}g(S_{\tau})]$
			over all stopping times $ au$)
Rogers:	$V_0 =$	$= \inf_{M}$	$\mathbb{E}[\sup_{t} (e^{-r_{d}t}g(S_{t}) - M_{t})]$
	$p_t M_t $		nartingales with $M_0 = 0$
	S. B. L		position Supermartingale
			M a martingale and $A \uparrow$
			(W_r, W_t) on $r \leq s \leq t$
$\sim W_r +$	$\frac{s-r}{t-r}(W$	$V_t = W_r$	$)+\sqrt{rac{(s-r)(r-t)}{t-r}}\mathcal{N}(0,1)$

Reflection Principle $M_T \stackrel{\Delta}{=} \max_{0 \le t \le T} W_t$ $I\!\!P[M_T > m, W_T < b] = I\!\!P[W_T > 2m - b]$ (65) $\mathbb{P}[M_T > m] = 2\mathbb{P}[W_T > m]$ (66)Martingale Representation Theorem (Δ -Hedge) Each martingale X can be represented by $X_t = X_0 + \int_0^t \delta(u) \, \mathrm{d}W_u$ for an adapted δ Lévy's Characterization A continuous martingale M with $M_t^2 - t$ being a martingale is a Brownian motion *Itô-Process*: $dX_t = b_t$, $dt + \sigma_t dW_t$ for adapted b and σ Semimartingale: $X_t = M_t + A_t$, where M is a local martingale and A is a càdlàg adapted process of locally bounded variation Dambis / Dubins-Schwarz A continuous lo- (\mathcal{F}_t) -martingale is a time-changed cal Brownian motion: $M_t = W_{\langle M \rangle_t}$, where $\tau_u = \inf\{t: \langle M \rangle_t > u\}$ and W_{τ_u} is an (\mathcal{F}_{τ_u}) -Brownian motion *Bessel Process* $R_t = |\mathbf{W}_t|$, the Euclidean norm of an n-dimensional Brownian motion $\mathrm{d}R_t = \mathrm{d}W_t + \frac{n-1}{2R_t}\,\mathrm{d}t$ (67)

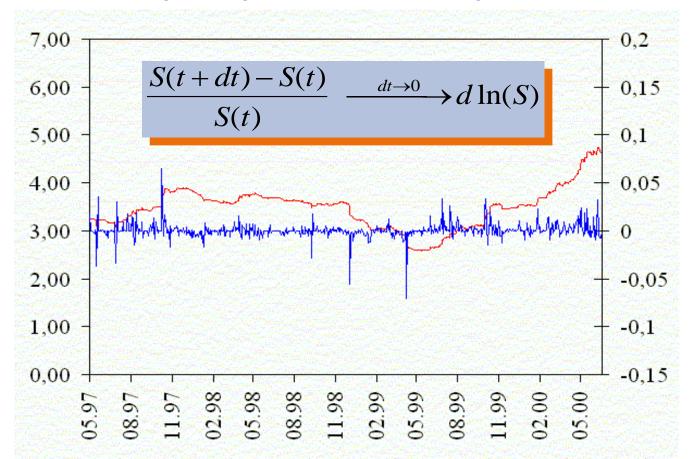
$$\mathrm{d}R_t^2 = 2\sqrt{R_t^2}\,\mathrm{d}W_t + n\,\mathrm{d}t \tag{68}$$

How to "explain" the curves – Different approaches

Stochastic Calculus \cdot dW_t dt $It\partial$ Formula dW_t dt 0 $df(X_t) = f'(X_t) dX_t$ dt 0 0 $+\frac{1}{2}f''(X_t)(dX_t)^2$ $varW_t = t;$ Cov $(W_t, W_s) = min(t, s)$ $It\partial$ -Tanaka Formula for Local time	Reflection Principle $M_T \triangleq \max_{0 \le t \le T} W_t$ $\mathbb{IP}[M_T > m, W_T < b] = \mathbb{IP}[W_T > 2m - b]$ (65) $\mathbb{IP}[M_T > m] = 2\mathbb{IP}[W_T > m]$ (66) Martingale Representation Theorem (Δ -Hedge) Each martingale X can be represented by $X_t = X_0 + \int_0^t \delta(u) dW_u$ for an adapted δ L for $t < 0$
$\int_{0}^{t} dx = \int_{0}^{t} dx = \int_{0$	Lévy's Characterization A continuous martingale M with M^2 – their a martingale is a Brown
$L_t^a(Z) = Z_t - a - Z_0 - a - \int_0^t \operatorname{sgn}(Z_u - a) \mathrm{d}Z_u (63)$	
$f(Z_t) = f(Z_0) + \int_0^{\infty} f'_t(Z_u) \mathrm{d}Z_u + \frac{1}{2} \int_{\mathbb{R}} L^u_t(Z), \mu(t) \mathrm{Stor}_{Variat}$ $f'_t \text{ is the left-hand side derivative of } f, \mathrm{m}$ $\mu = f'' \text{ in the sense of distributions}$ $Girsanov \text{ With } d\tilde{I} \stackrel{\Delta}{=} \exp(\sigma W_T - \frac{1}{2}\sigma^2 T) \mathrm{d}$ $\tilde{W}_t - \sigma t \text{ is a Brownian motion under } \bar{I} \stackrel{P}{=}$ $American Option V_0 = \sup_{\tau} IE[e^{-\tau_d \tau}]$	Illiavin, A. Thalmaier,tionchastic Calculus oftionchastic Calculus of $x_t = b_t, dt + \sigma_t dW_t$ for adapted bions in Mathematical $tringale: X_t = M_t + A_t$, where M isFinance,martingale and A is a càdlàg adaptedSpringers of locally bounded variation/ Dubins-SchwarzA continuous lo- \mathcal{F}_t)-martingaleis a time-changedprovinianmotion: $M_t = W_{\langle M \rangle_t}$, where
payoff g (maximize over all stopping times τ) Rogers: $V_0 = \inf_M \mathbb{E}[\sup_t (e^{-r_d t}g(S_t) - M_t)]$ minimize over all martingales with $M_0 = 0$ and $\sup_t M_t \in L^1$ Doob-Meyer Decomposition Supermartingale $Y_t = Y_0 + M_t - A_t$, M a martingale and $A \uparrow$ Brownian Bridge $B_s (W_r, W_t)$ on $r \leq s \leq t$ $\sim W_r + \frac{s-r}{t-r}(W_t - W_r) + \sqrt{\frac{(s-r)(r-t)}{t-r}}\mathcal{N}(0, 1)$	The function $M_t = W(M)_t$, where $\tau_u = \inf\{t : \langle M \rangle_t > u\}$ and W_{τ_u} is an (\mathcal{F}_{τ_u}) - Brownian motion Bessel Process $R_t = \mathbf{W}_t $, the Euclidean norm of an <i>n</i> -dimensional Brownian motion $dR_t = dW_t + \frac{n-1}{2R_t} dt$ (67) $dR_t^2 = 2\sqrt{R_t^2} dW_t + n dt$ (68)

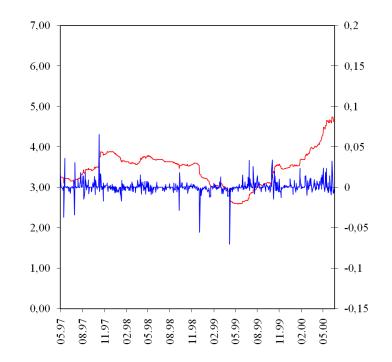
P. Malliavin, A. Thalmaier, Stochastic Calculus of Variations in Mathematical Finance, Springer

Modelling the logarithmical price change



» Basic model: $X_t = \sigma_t Z_t$ with

{Z_t} is IID with mean 0, variance 1, e.g. N(0,1) very simple: fixed σ , more advanced: { σ _t} is a volatility process



» GARCH model

 $X_t = \sigma_t Z_t$ GARCH(p,q) process (General AutoRegressive Conditional Heteroscedastic)

$$\sigma_{t}^{2} = c_{0} + c_{1}X_{t-1}^{2} + \dots + c_{p}X_{t-p}^{2} + \beta_{1}\sigma_{t-1}^{2} + \dots + \beta_{q}\sigma_{t-q}^{2} .$$

Special case ARCH(1)

$$X_{t}^{2} = (c_{0} + c_{1}X_{t-1}^{2})Z_{t}^{2}$$
$$= c_{1}Z_{t}^{2}X_{t-1}^{2} + c_{0}Z_{t}^{2}$$
$$= A_{t}X_{t-1}^{2} + B_{t}$$

Interest rate models

» Stochastic volatility models

 $X_t = \sigma_t Z_t$

 σ_t is a second process, independent of Z_t Model for the volatility (Taylor 1986)

$$\log \sigma_t^2 = \alpha_0 + \alpha_1 \log \sigma_{t-1}^2 + \alpha_2 \varepsilon_t, \ \{\varepsilon_t\} \sim \text{IID N}(0,1)$$

Stochastic recurrence model

$$X_{t} = X_{t-1}\varepsilon_{t} + \eta_{t} \text{ mit } \{\varepsilon_{t}, \eta_{t}\} \sim \text{IID}$$

» Extensions to the basic GARCH model

General formula: Bilinear (Granger / Andersen 1978): ARCH(1, 1) (Engle 1982): GARCH(1, 1) (Bollerslev 1986): EGARCH (Nelson 1990):

$$r_{t} = \sigma_{t} \mathcal{E}_{t}$$

$$\sigma_{t}^{2} = r_{t-1}^{2}$$

$$\sigma_{t}^{2} = c_{0} + c_{1} r_{t-1}^{2}$$

$$\sigma_{t}^{2} = c_{0} + c_{1} r_{t-1}^{2} + c_{2} \sigma_{t-1}^{2}$$

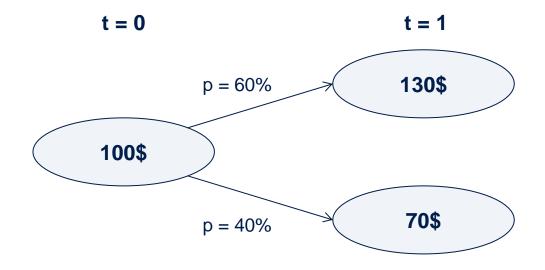
$$\log(\sigma_{t}) = c_{0} + c_{1}\log(\sigma_{t-1}) + \frac{c_{2}\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}}} + c_{3}\left(\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}}} - \sqrt{\frac{2}{\pi}}\right)$$

Further: ARCH-M, AARCH, NARCH, PARCH, PNP_ARCH, STARCH, SWARCH, Component-ARCH, IARCH, multiplicative ARCH

For weather derivatives e.g. the ARFIMA-FIGARCH approach is used

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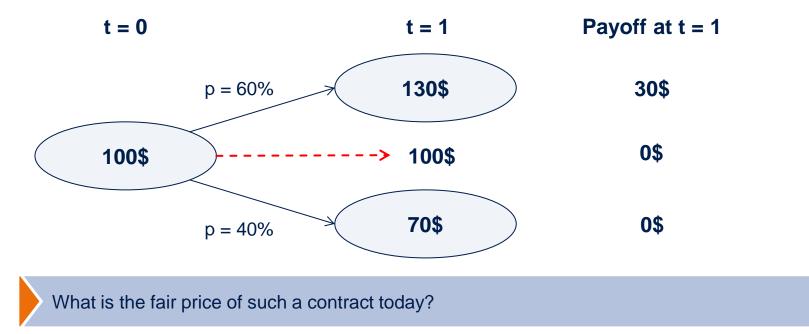
Suppose we have a stock that is worth 100\$ today. Tomorrow we have two scenarios: the stock can go up to 130\$ with empirical probability of 60% or it can go down to 70\$ with empirical probability of 40%.



What is the fair price of such a contract today?

Suppose we have a stock that is worth 100\$ today. Tomorrow we have two scenarios: the stock can go up to 130\$ with empirical probability of 60% or it can go down to 70\$ with empirical probability of 40%.

Now define the following contract: The holder of the contract has the right to buy the stock tomorrow for 100\$. If the price tomorrow is 130\$, the holder can buy the stock for 100\$ and immediately sell it for 130\$, thus making a profit of 30\$. If the price tomorrow is 70\$ the holder will not use his right to buy the stock for 100\$ since he can buy it in the market for 70\$.

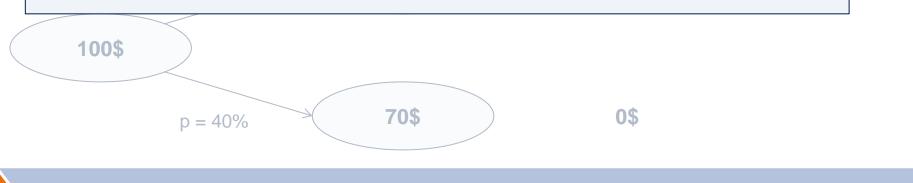


to

Suppose we have a stock that is worth 100\$ today. Tomorrow we have two scenarios: the stock can go up to 130\$ with empirical probability of 60% or it can go down to 70\$ with empirical probability of 40%.

Now define the following contract: The holder of the contract has the right to buy the stock

Suppose we find somebody who pays us the expected profit of (60%*30\$) 18\$ for such a contract.



What is the fair price of such a contract today?

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blder

We could then accept the 18\$ and do the following: we buy $\frac{1}{2}$ of the stock today for 50\$ and wait until tomorrow. If the stock goes up we have to deliver one stock for the price of 100\$, so we have to buy another $\frac{1}{2}$ for 65\$. Having spent a total of 115\$ and received a total of 118\$ we make a profit of 3\$. If the stock goes down we don't have to deliver the stock and can sell our $\frac{1}{2}$ stock for 35\$, adding the 18\$ we got for the contract and subtracting the 50\$ we paid for the $\frac{1}{2}$ stock at t = 0 again gives us a profit of 3\$.

	Money spent	Money received	Profit
130\$	 Buy ½ stock at t = 0: -50\$ Buy ½ stock at t = 1: -65\$ Total -115\$ 	 » Initial contract: 18\$ » Delivery of 1 stock: 100\$ » Total 118\$ 	3\$
100\$	// Iotal=1159	" IUtai 1105	υψ
♣	» Buy ½ stock at t = 0: -50\$	 » Initial contract: 18\$ » Sell $\frac{1}{2}$ stock at t = 1: 35\$ 	
70\$	» Total -50\$	» Total 53\$	3\$

We make a profit of 3\$, no matter what happens tomorrow!

We could then accept the 18\$ and do the following: we buy $\frac{1}{2}$ of the stock today for 50\$ and wait until tomorrow. If the stock goes up we have to deliver one stock for the price of 100\$, so we have to buy another $\frac{1}{2}$ for 65\$. Having spent a total of 115\$ and received a total of 118\$ we make a profit of 3\$. If the stock goes down we don't have to deliver the stock and can sell our $\frac{1}{2}$ stock for 35\$, adding the 18\$ we got for the contract and subtracting the 50\$ we paid for the $\frac{1}{2}$ stock at t = 0 again gives us a profit of 3\$.

	Money spent	Money received	Profit
130\$,	 0\$ » Initial contract: 18\$ 5\$ » Delivery of 1 stock: 100\$ 	
	» Total -115\$	» Total 118\$	3\$
100\$			
	Buy ½ stock at t = 0: -5	0\$ » Initial contract: 18\$	
		Sell ½ stock at t = 1: 35\$	
70\$	» Total -5	0\$ » Total 53\$	3\$

We make a profit of 3\$, no matter what happens tomorrow!

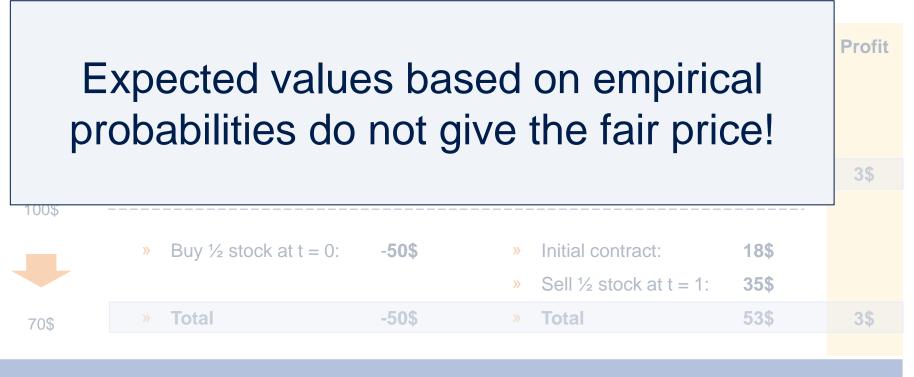
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		Sell ½ stock at t = 1: 35\$	
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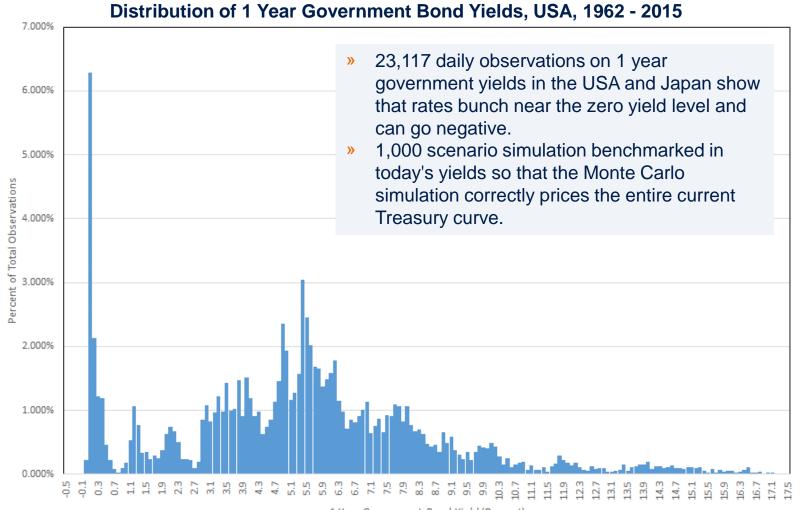
We make a profit of 3\$, no matter what happens tomorrow!

Options in finance

We could then accept the 18\$ and do the following: we buy $\frac{1}{2}$ of the stock today for 50\$ and wait until tomorrow. If the stock goes up we have to deliver one stock for the price of 100\$, so we have to buy another $\frac{1}{2}$ for 65\$. Having spent a total of 115\$ and received a total of 118\$ we make a profit of 3\$. If the stock goes down we don't have to deliver the stock and can sell our $\frac{1}{2}$ stock for 35\$, adding the 18\$ we got for the contract and subtracting the 50\$ we paid for the $\frac{1}{2}$ stock at t = 0 again gives us a profit of 3\$.



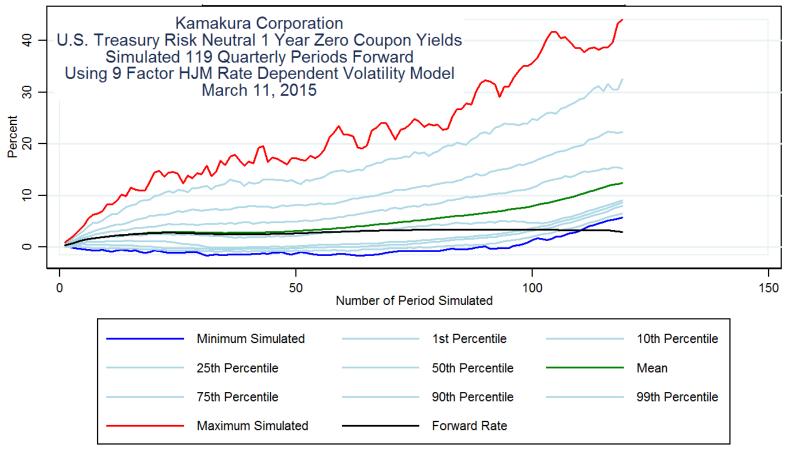
We make a profit of 3\$, no matter what happens tomorrow!



1 Year Government Bond Yield (Percent)

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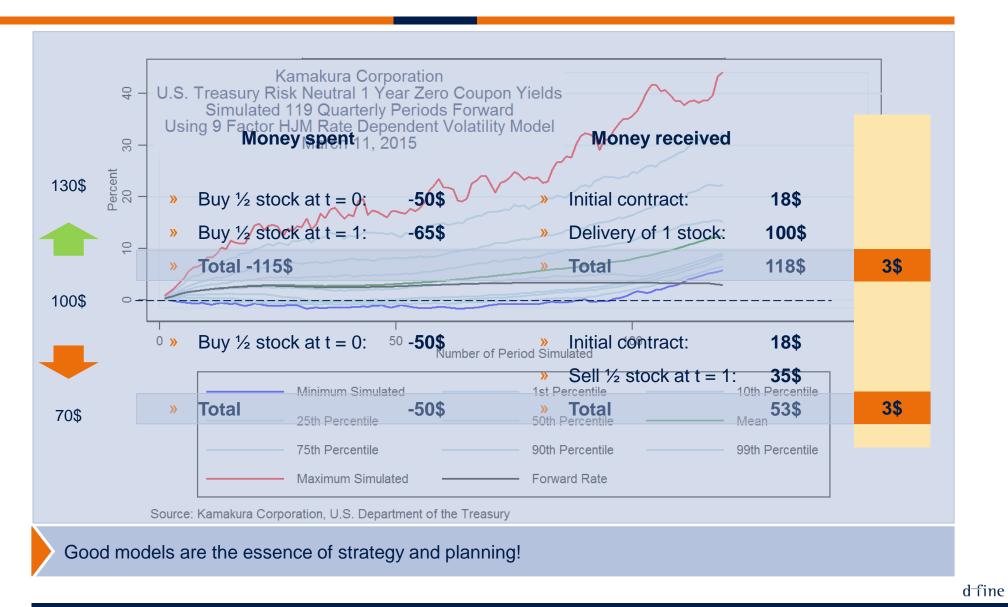
One thousand scenarios for the U.S. treasury curve



Source: Kamakura Corporation, U.S. Department of the Treasury

Good models are the essence of strategy and planning!

One thousand scenarios for the U.S. treasury curve (2/2)



Physical models applied to financial markets

- The application of stochastic methods to questions from the world of finance is nowadays an established standard.
- » Many well understood paradigms from physics can be applied to problems arising in a financial context. Time will tell which of them will also have practical relevance.
- > Ising models, chaos theory, fractals, etc.



The statistical physics approach

2017-10-09 | From Physics to Finance | Time series in finance – non-linearity and prediction of the future (42/55)

Stock markets and quantum dynamics: a second quantized description

F. Bagarello



https://www.flickr.com/photos/bankenverband/9930916773, Jochen Zick

Stock markets and quantum dynamics: a second quantized description

F. Bagarello

- » Toy model of a stock market based on the following assumptions:
 - > Our market consists of L traders exchanging a single kind of share;
 - > The total number of shares, N, is fixed in time;
 - > A trader can only interact with a single other trader: i.e. the traders feel only a two-body interaction;
 - > The traders can only buy or sell one share in any single transaction;
 - > The price of the share changes with discrete steps, multiples of a given monetary unit;
 - When the tendency of the market to sell a share, i.e. the market supply, increases then the price of the share decreases;
 - > For our convenience the supply is expressed in term of natural numbers;
 - > To simplify the notation, we take the monetary unit equal to 1.

Article: F. Bagarello, J. Phys. A, 6823-6840 (2006)

» The formal Hamiltonian of the model is the following operator: $\widetilde{H} = H_0 + \widetilde{H}_l$, where

$$H_0 = \sum_{l=1}^{L} a_l a_l^{\dagger} a_l + \sum_{l=1}^{L} \beta_l c_l^{\dagger} c_l + o^{\dagger} o + p^{\dagger} p$$

$$\widetilde{H}_{l} = \sum_{i,j=1}^{L} p_{ij} \left(a_{i}^{\dagger} a_{j} \left(c_{i} c_{j}^{\dagger} \right)^{P} + a_{i} a_{j}^{\dagger} \left(c_{j} c_{i}^{\dagger} \right)^{\widehat{P}} \right) + o^{\dagger} p + p^{\dagger} o$$

» where $\hat{P} = p^{\dagger}p$ and the following commutation rules are used:

»
$$[a_l, a_n^{\dagger}] = [c_l, c_n^{\dagger}] = \delta_{ln}I$$
 $[p, p^{\dagger}] = [o, o^{\dagger}] = I$

- » All other commutators are zero.
- » We further assume that $p_{ii} = 0$
- > Number, price, cash and supply operators: $a_l^{\ddagger}, p^{\ddagger}, c_l^{\ddagger}, o^{\ddagger}$
- » The states of the market are: $\omega_{\{n\};\{k\};\boldsymbol{0};\boldsymbol{M}}(.) = \langle \varphi_{\{n\};\{k\};\boldsymbol{0};\boldsymbol{M}}, \varphi_{\{n\};\{k\};\boldsymbol{0};\boldsymbol{M}} \rangle$

» where
$$\{n\} = n_1, n_2, ..., n_L, \{k\} = k_1, k_2, ..., k_L$$
 and

$$\varphi_{\{n\};\{k\};O;M} = \frac{(a_1^{\dagger})^{n_1} \dots (a_L^{\dagger})^{n_L} (c_1^{\dagger})^{k_1} \dots (c_L^{\dagger})^{k_L} (o^{\dagger})^{O} \dots (p^{\dagger})^{M}}{\sqrt{n_1! \dots n_L! k_1! \dots k_L! O! M!}} \varphi_0$$

» φ_0 is the vacuum of the model: $a_j\varphi_0 = c_j\varphi_0 = p\varphi_0 = o\varphi_0 = 0, for j = 1, 2, ..., L$

» The time evolution for the observables, e.g., the price

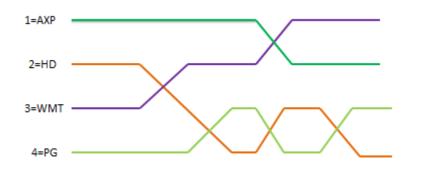
```
\frac{dX(t)}{dt} = ie^{iHt}[H,X]e^{-iHt} = i[H,X(t)]
```

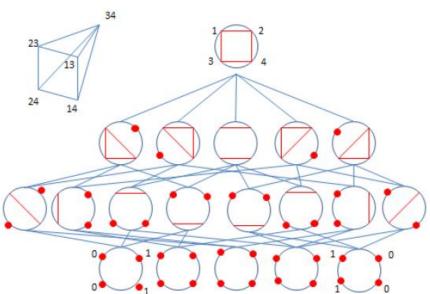


Article: F. Bagarello, J. Phys. A, 6823-6840 (2006), Foto: https://www.flickr.com/photos/bankenverband/9930916773, Jochen Zick

Crossing Stocks and the Positive Grassmannian I: The Geometry behind Stock Market

Removals of crossings in the permutation associated to stock market reside in the decomposition of the positive Grassmannian $G^+(2,4)$ labeled by the stock market polytope in positroid cells as is depicted in the figure 11.





The combinatorial approach

Pictures from O. Racorean, Geometry and Topology of the Stock Market, 2013

2017-10-09 | From Physics to Finance | Time series in finance – non-linearity and prediction of the future (47/55)

From the currency rate quotations onto strings and brane world scenarios D. Horváth, R. Pincak

We are currently in the process of transfer of modern physical ideas into the neighboring field called econophysics. The physical statistical view point has proved fruitful, namely, in the description of systems where many-body effects dominate. However, standard, accepted by physicists, bottom-up approaches are cumbersome or outright impossible to follow the behavior of the complex economic systems, where autonomous models encounter the intrinsic variability.

The "cosmological" approach

Article Physica A 391 (2012) 5172-5188

Physical models applied to financial markets – Selected books

R. Mantegna, H. Stanley Correlations and Complexity in Finance Cambridge University Press	L. Wille New Directions in Statistical Physics Econophysics, Bioinformatics, and Pattern Recognition Springer	M. Small Applied Nonlinear Time Series Applications in Physics, Physiology and Finance World Scientific Series on Nonlinear Science, Series A Vol. 52	F. Abergel, B. Chakrabarti, A. Chakraborti, A.Ghosh (Ed) Econophysics of Systemic Risk and Network Dynamice Systemic Risk and Network Dynamics Springer
B. Mandelbrot Fractals and Scaling in Finance Discontinuity, Concentration, Risk Springer	O. Racorean Geometry and Topology of the Stock Market Quantum Computer generation of quants CreateSpace	H. Kleinert Path Integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets World Scientific	B. Baaquie Quantum Finance Path Integrals and Hamiltonians for Options and Interest rates Cambridge

Chapman, Hall

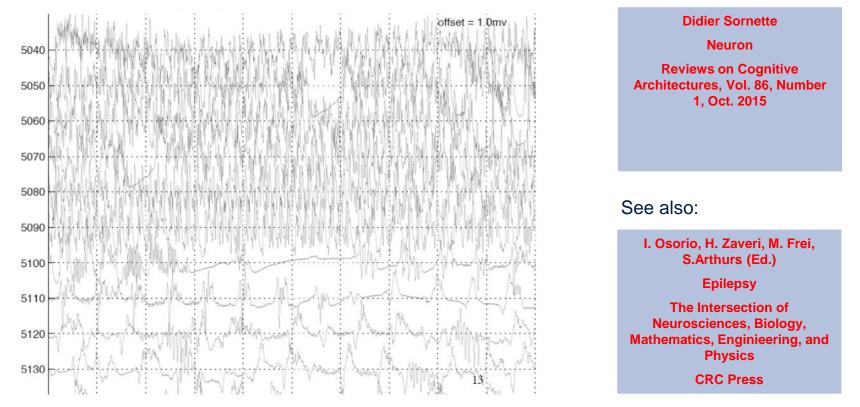
Computational Neuroscience

A Comprehensive Approach

CRC Mathematical Biology and Mediscience Series

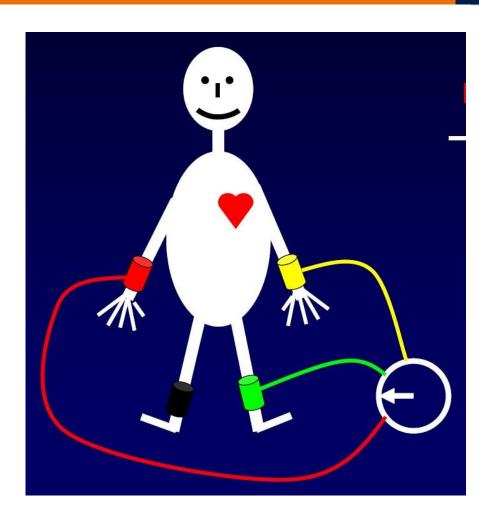
Mathematical/physical models in finance – The "patient" financial markets

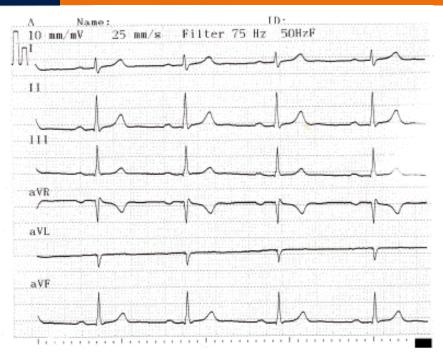
Parallels between Earthquakes, Financial crashes and epileptic seizures Didier Sornette



Our models "fit" in different areas of research – mathematical structures can by analysed by analogies

The "patient" financial markets

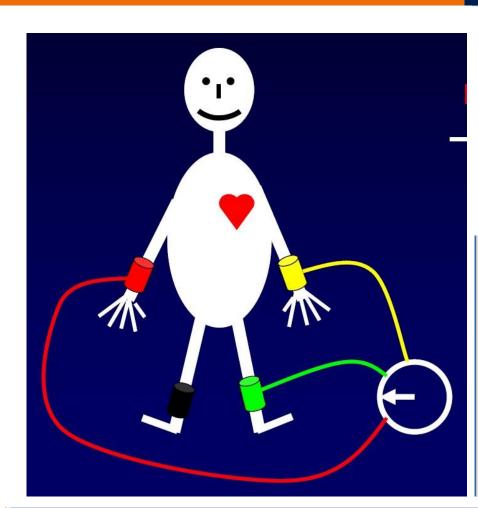


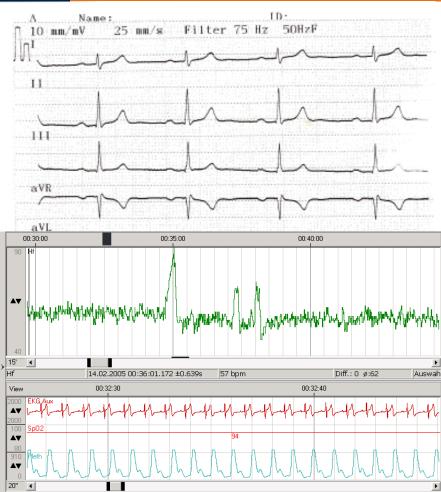


Our models "fit" in various fields of science

2017-10-09 | From Physics to Finance | Time series in finance - non-linearity and prediction of the future (52/55)

The "patient" financial markets





Our models "fit" in various fields of science

The "patient" financial markets



Our models "fit" in various fields of science – exploring mathematical structures via analogy

Photo source: © NH1977 / PIXELIO

2017-10-09 | From Physics to Finance | Time series in finance – non-linearity and prediction of the future (54/55)

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Physical models applied to financial markets – Implementation

Eldgenössische Technische Hochschule Zürich Swiss Federal Institute of Technology Zurich	About us People Research Teaching Publications Seminars ETH Risk Center Real Estate Observatory Financial Crisis Observatory Books Interviews Essays Presentations Inspiring Articles	Contact Sitemap Help Search Q
ETH Zurich - D-MTEC - Welcome t	o the Chair of Entrepreneurial Risks - Financial Crisis Observatory	
Financial Crisis Observatory Description Highlights Is there an oil bubble?	Financial Crisis Observatory The Financial Crisis Observatory (FCO) is a scientific platform aimed at testing and quantifying rigorously, in a systematic way and on a large scale the hypothesis that financial markets exhibit a degree of inefficiency and a potential for predictability, especially during regimes when bubbles develop.	Entrepreneurial Risks
Pertinent articles Websites and Blogs Predictions Market Anxiety Measures RSS Feed The Financial Crisis: How Much Longer and Deeper?	FCO Cockpit Syntheses	FCO Blog The <u>FCO blog</u> discusses how the system approach allows one to develop diagnostic methods and predictions of crises.
	2015 - 1st February 2015: <u>Synthesis report</u> - 1st January 2015: <u>Synthesis report</u>	FCO RSS Feed
	2014 - 1st December 2014: <u>Synthesis report</u> - 1st November 2014: <u>Synthesis report</u> - 1st October 2014: <u>Synthesis report</u> and <u>detailed calculations</u> - <u>1st April 2014</u> - <u>1st February 2014</u>	
	December 2013 : Financial crisis risk monitoring and positive and negative bubble risk maps become available from the Financial Crisis Observatory.	

The mechanics of the balance sheet – an engineers approach

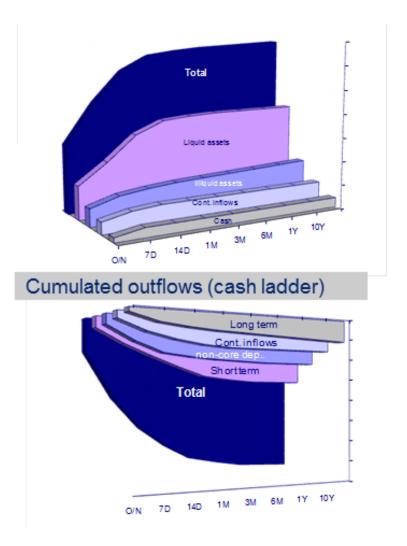
Inflows and outflows

Mechanics of the balance sheet



Averaged balance sheet total of the big German banks: 490 bn Euros

Source: Bundesbankstatistik, July 2011

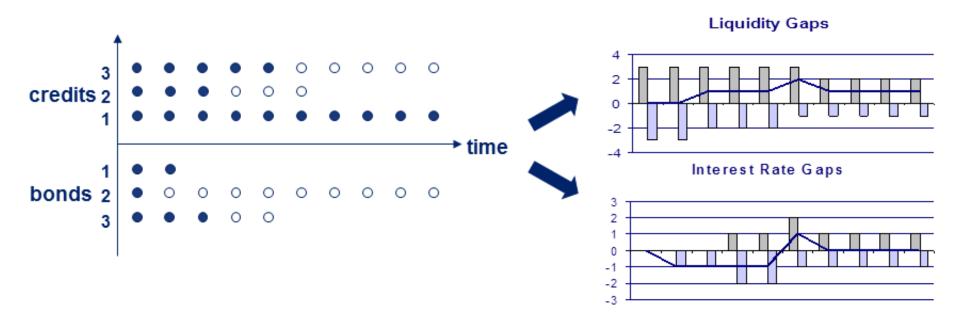


2017-10-09 | From Physics to Finance | The mechanics of the balance sheet – an engineers approach (1/7)

Counting and labelling monetary units in time

Consolidation: The ball model

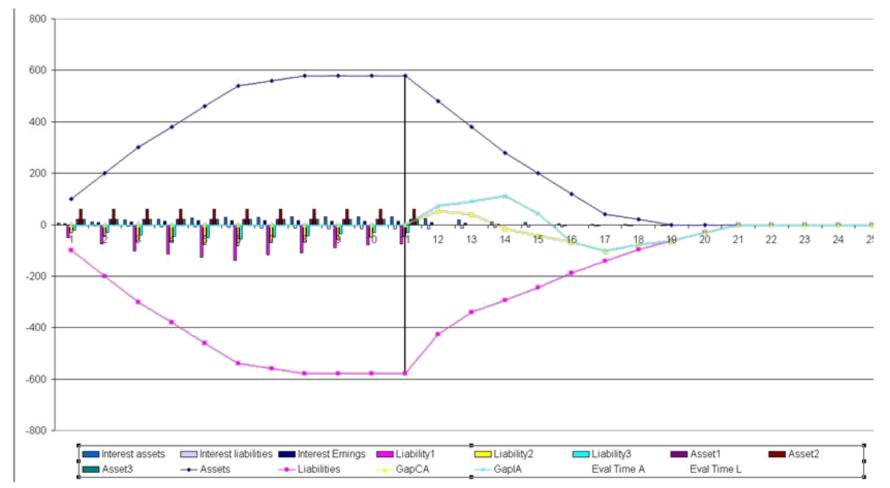
Purpose: Simultaneous consideration of interest rate risk and liquidity risk



- ... capital commitment, no interest rate commitment
- Capital and interest rate commitment

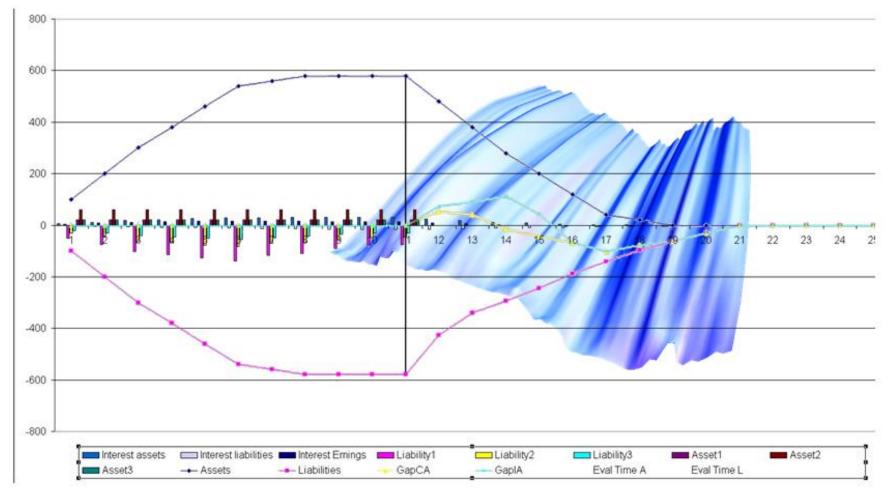
The "bow wave" of the balance sheet

Consolidation: The ball model



The "bow wave" of the balance sheet

Consolidation: The ball model



Cost reduction via canceling "waves"



» How can we achieve an optimal match between business structure, liquidity structure, and interest rate structure while taking into account their dynamics?

Photo source: www.Rudis-Fotoseite.de / pixelio.de

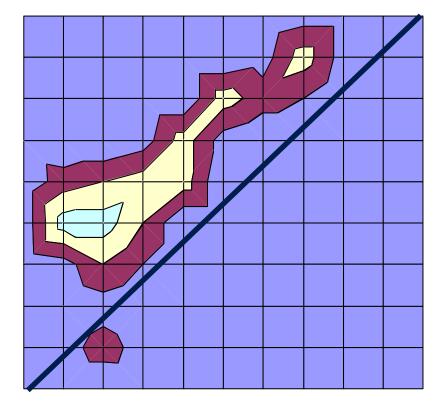
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Photo source: FotoHiero / pixelio.de

Cost reduction via canceling "waves"





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Photo source: FotoHiero / pixelio.de

The costs of the crisis

Financial Market Stabilization Fund guarantees of up to 400bn Euros recapitalize or purchase assets for up to 80bn Euros

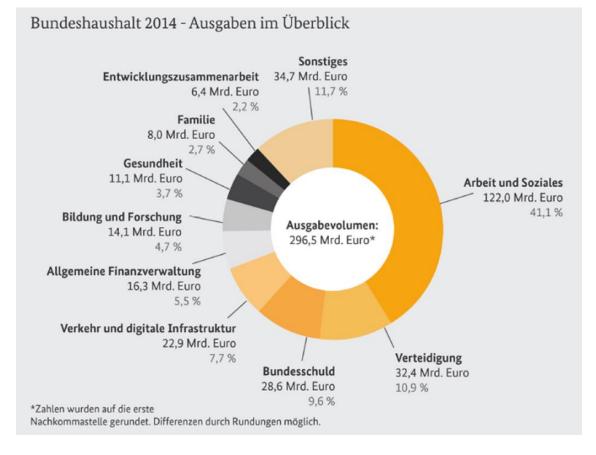
Accumulated losses of the SoFFin:

»	2009:	4.3 billion Euros
»	2010:	4.8 billion Euros
»	2011:	13.1 billion Euros
»	2012:	23 billion Euros
»	2013:	21.5 billion Euros
»	2014:	21.9 billion Euros

Equity recapitalizations (30.06.2012) :

- » Aareal Bank AG: 0.3
- » Commerzbank AG: 6.7
- » Hypo Real Estate: 9.8
- » WestLB AG: 3.0





Source: SoFFin Jahresberichte, http://www.fmsa.de/de/fmsa/soffin/Berichte/index.html

Source: Bundesministerium für Finanzen, Auf den Punkt - Bundeshaushalt 2014, August 2014



100 dollars

Source: Die Welt / August 2011

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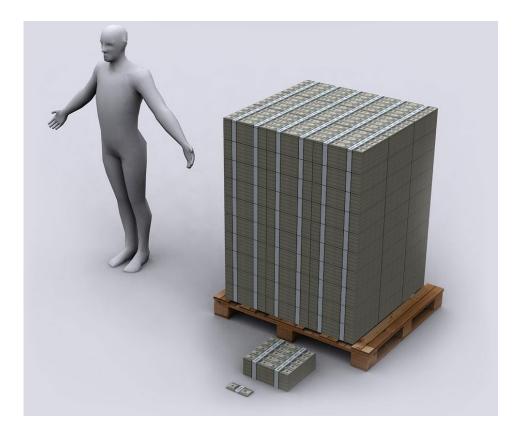
2017-10-09 | From Physics to Finance | The costs of the crisis (2/11)



10.000 dollars – average years income world wide



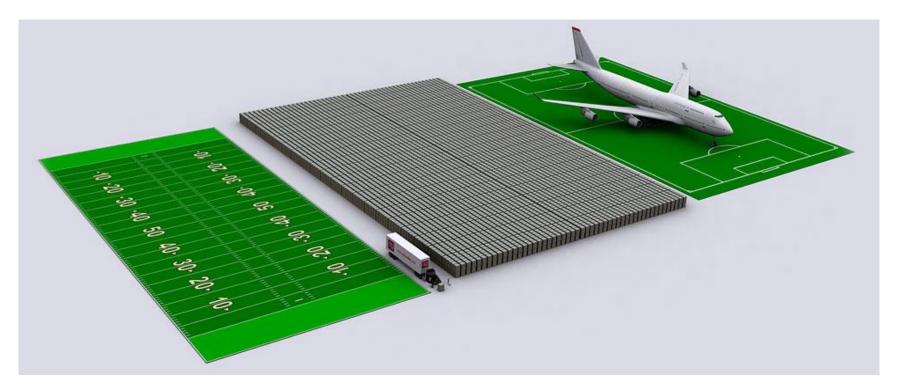
1 million dollars



100 million dollars – This amount can be transported on a europallet



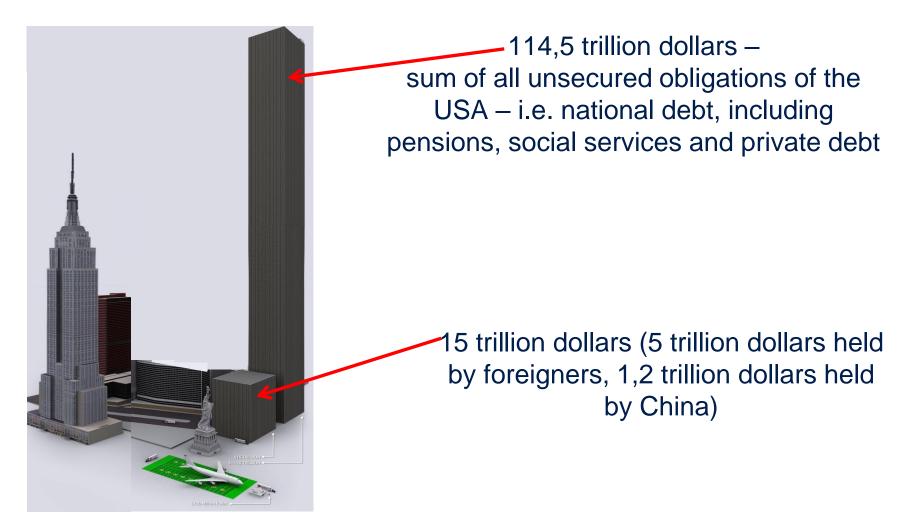
1 billion dollars – 10 europallets, not easy to transport



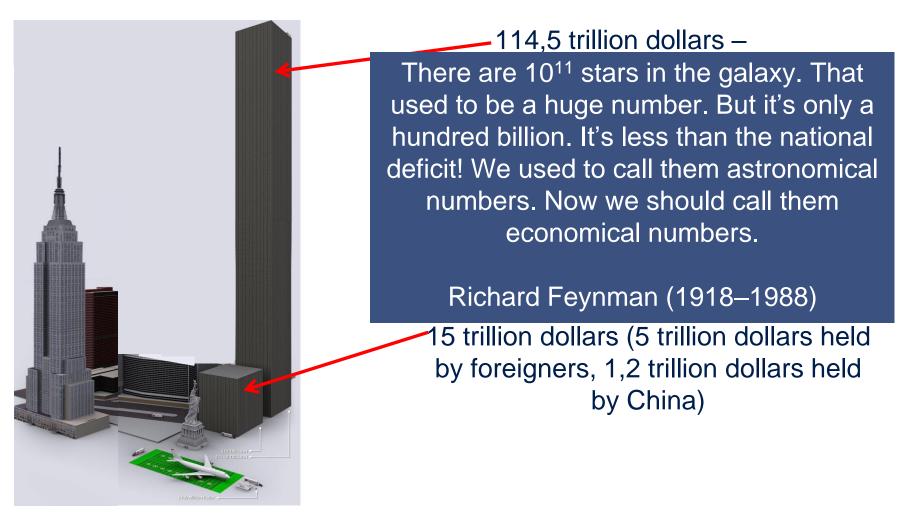
1 trillion dollars – in comparison to an American Football field or a Boeing 747



15 trillion dollars – represents the forecasted national debt of the USA at the end of 2011

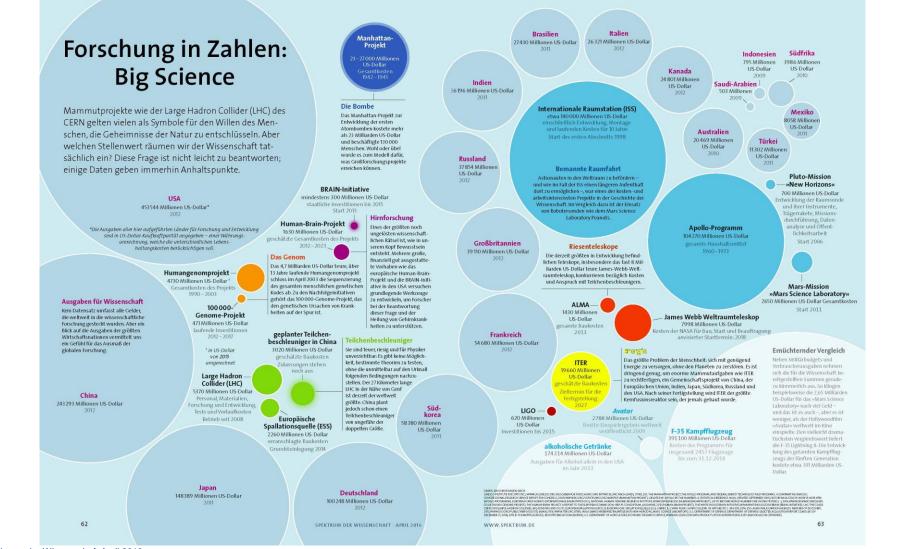


Source: Die Welt / August 2011



Source: Die Welt / August 2011

Cost of science (budget)

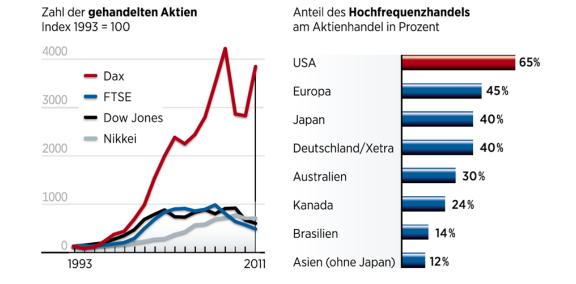


Spektrum der Wissenschaft April 2016

Is the financial complexity manageable?

High frequency trading

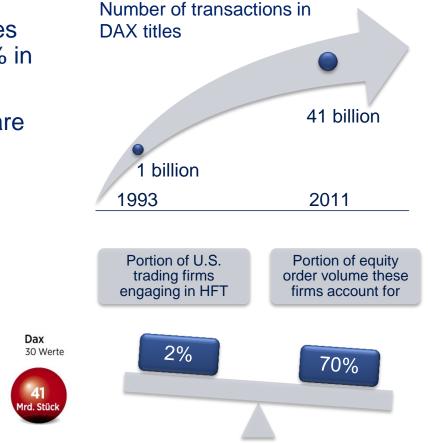
- » HFT incorporates proprietary trading strategies carried out by computers
- » Electronic exchanges were first authorized by the U.S. Securities and Exchange Commission in 1998
- » Execution times have fallen from several seconds in the year 2000 to milliseconds on modern systems



Source: Handelsblatt 2012

Volume of high frequency trading

- » Portion of HFT in U.S. equity trades has increased from less than 10 % in 2000 to over 70% in 2010
- » About 40% of Xetra transactions are carried out by HFT systems



 FTSE

 100 Werte

 343

 Mrd. Stück

 Mrd. Stück

 Handelsblatt

Zahl der 2011 gehandelten Aktien

Source: Handelsblatt 2012

Rasante Beschleunigung

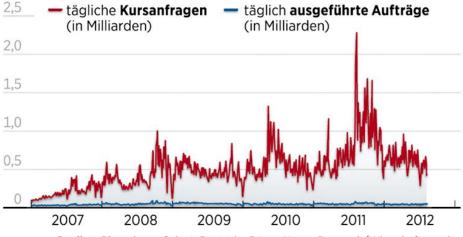
Nikkei 225 Werte

Role of high frequency trading in the crisis

- In 2010 the Dow Jones Index experienced its largest oneday point decline in history #Plash Crash"
- The U.S. Securities and Exchange Commission and the Commodity Futures Trading Commission concluded in a joint investigation that the actions of HFT firms largely contributed to volatility during the crash.

Der Trick der Hochfrequenzhändler

Sie schießen massenweise Aufträge für US-Aktien in die Börsensysteme, ziehen sie dann aber blitzschnell zurück. So suggerieren sie kurstreibende Nachfrage, die aber nicht vorhanden ist. Gehandelt wird nur ein Bruchteil.



Quellen: Bloomberg, Celent, Deutsche Börse, Nanex Research/Wirtschaftswoche

Source: Handelsblatt 2012

Role of high frequency trading in the crisis

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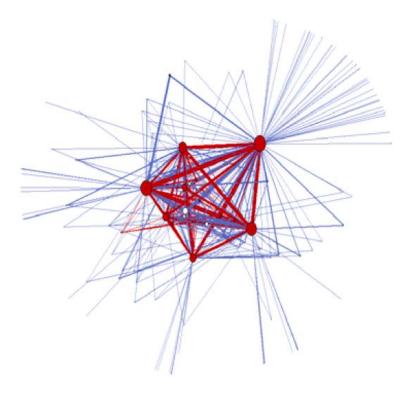
Der Trick der Hochfrequenzhändler

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Source: Handelsblatt 2012

CHAPS: Clearing House Automated Payment System CHAPS offers same-day sterling fund transfers Many flows are routed through settlement banks

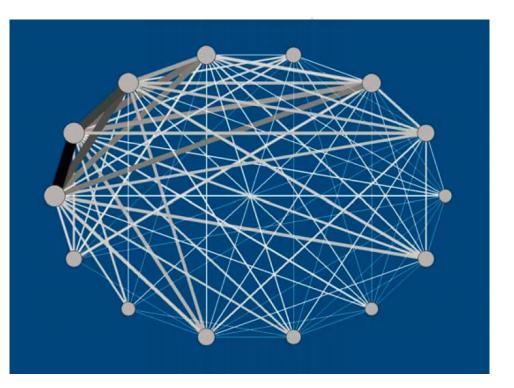


Source: Becher, Millard, and Soramäki, The network topology of CHAPS Sterling, Bank of England, Working Paper 355

CHAPS: Clearing House Automated Payment System

CHAPS offers same-day sterling fund transfers Many flows are routed through settlement banks

- The settlement banks form a complete network
- » 4 settlement banks account for almost 80% of the payments, measured by value or volume!



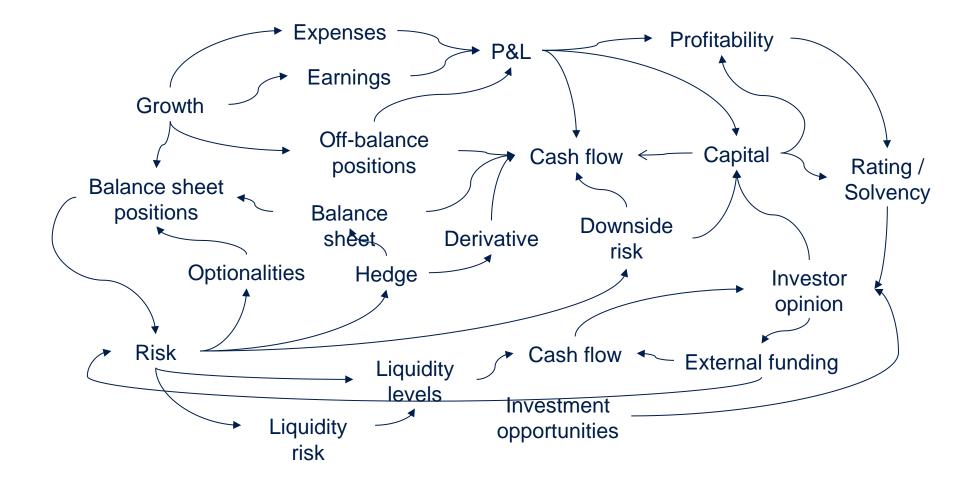
Source: Becher, Millard, and Soramäki, The network topology of CHAPS Sterling, Bank of England, Working Paper 355

CHAPS: Clearing House Automated Payment System CHAPS offers same-day sterling fund transfers Many flows are routed through settlement banks

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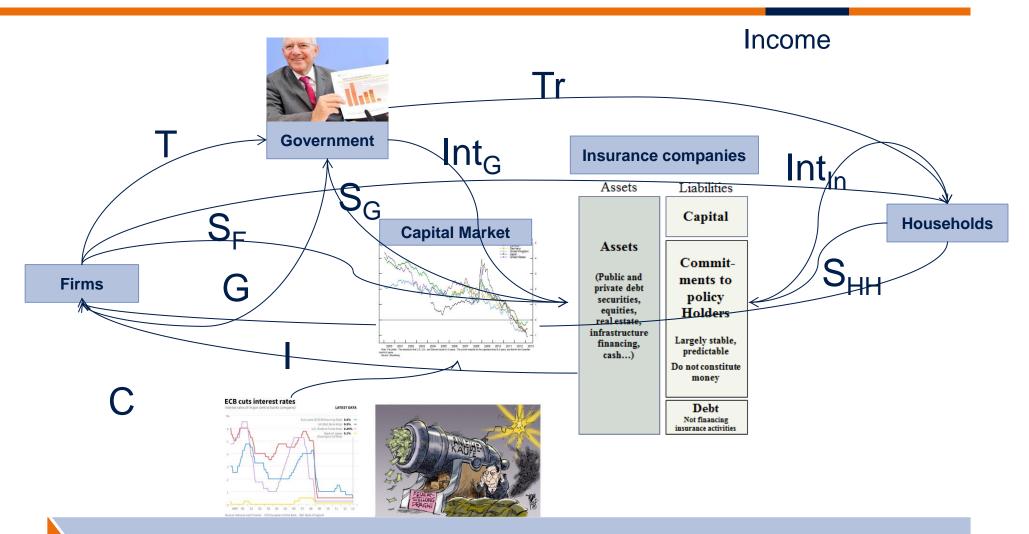
Source: Becher, Millard, and Soramäki, The network topology of CHAPS Sterling, Bank of England, Working Paper 355

Economics and banking – a complex network of dependencies



From: Managing Liquidity in Banks, R. Duttweiler, 2009

Complexity and networks: Economy and insurance companies



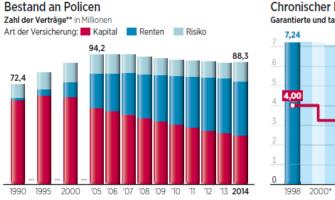
Insurance companies form a vital part of the macroeconomic flow chart

2017-10-09 | From Physics to Finance | Is the financial complexity manageable? (9/23)

Complexity and networks: All of us are affected



Lebensversicherung in Zahlen - die Luft wird dünner

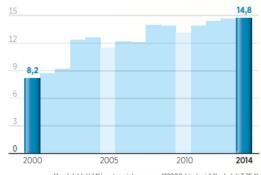


Source: bild, Handelsblatt

Chronischer Renditeschwund Garantierte und tatsächliche Zinsen von Lebenspolicen, in Prozent



Vorzeitige Ausschüttungen Entwicklung des Stornovolumens in Mrd. Euro



Handelsblatt | *Hauptversicherungen; **2000: bis Juni 4 %, ab Juli 3,25 % Foto: blickwinkel | Quellen: GDV, Morgen & Morgen

Complexity and networks: All of us are affected

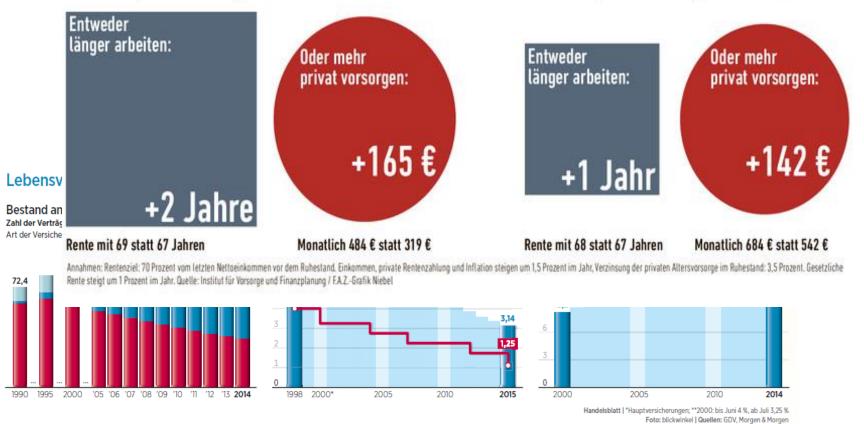
Sinkende Zinsen schmälern die Altersvorsorge – was tun?

Beispiel 1: 30-Jähriger

Einkommen: 70 000 € jährlich, Rentenziel: 3929 € monatlich, Zinsen für die private Altersvorsorge sinken von 5% auf 3%

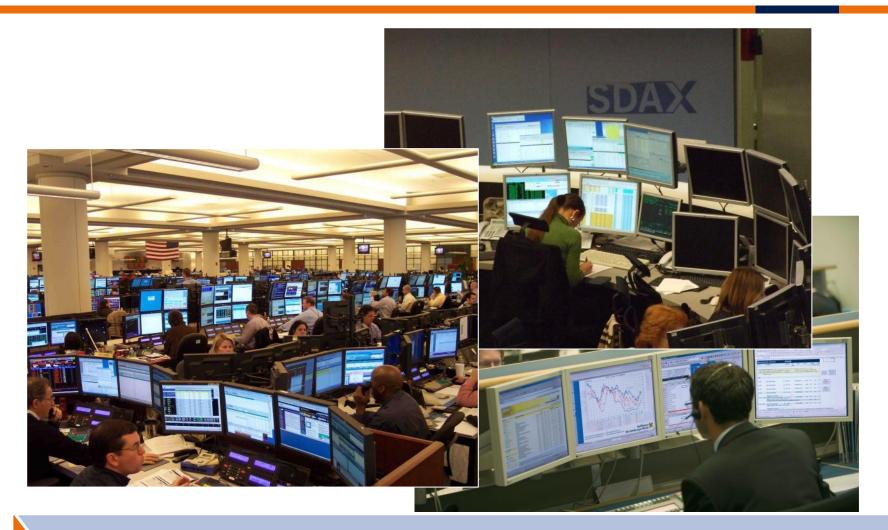
Beispiel 2: 45-Jähriger

Einkommen: 70000 € jährlich, Rentenziel: 3143 € monatlich, Zinsen für die private Altersvorsorge sinken von 5% auf 3%



Source: bild, Handelsblatt, FAZ

Collecting and processing information



Digital economy is founded on data

Photo source: en.wikipedia.org

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2017-10-09 | From Physics to Finance | Is the financial complexity manageable? (12/23)

Combating the crisis: When does financial instability become so widespread that it impairs the functioning of a financial system?

- » Need a robust measure for systemic financial stress, here: CISS = Composite Indicator of Systemic Stress
- » CISS includes 15 individual stress indicators in five segments:

Money market	Bond market	Equity market	Financial intermediaries	FX market
3M Euribor realised vola.	German 10Y Bond realised vola.	NFS stock market index realised vola.	Realised vola. equity return of bank sector index	FX rate EUR - USD realised vola.
Interest rate Spread: 3M Euribor - 3M Frech T-Bills	Yield-Spread: A-rated NFC vs. gov. Bonds (7Y)	NFS maximum cumulated index losses over 2Y window	Yield-Spread: A-rated NFC vs. A- rated FC (7Y)	FX rate EUR - GBP realised vola.
MFI emergency lending	10Y interest rate spread	Stock-bond correlation	FS equity market maximum cumulated book-price ratio (2Y-wind.)	FX rate EUR - JPY reailsed vola.

- » On basis of the raw stress indicators x_i , transformed stress indicators z_i are calculated with the following empirical CDF:
 - > $(x_{[1]}, x_{[2]}, \dots, x_{[n]})$ denotes the ordered sample with $x_{[1]} \le x_{[2]} \le \dots \le x_{[n]}$

 $z_t \coloneqq \begin{cases} \frac{i}{n} \text{ for } x_{[r]} \le x_t < x_{[r+1]}, \quad r \in \{1, 2, \dots, n-1\} \\ 1 \text{ for } x_t > x_{[n]} \end{cases} \text{ for values running from Jan. 1999 - Jan. 2002}$

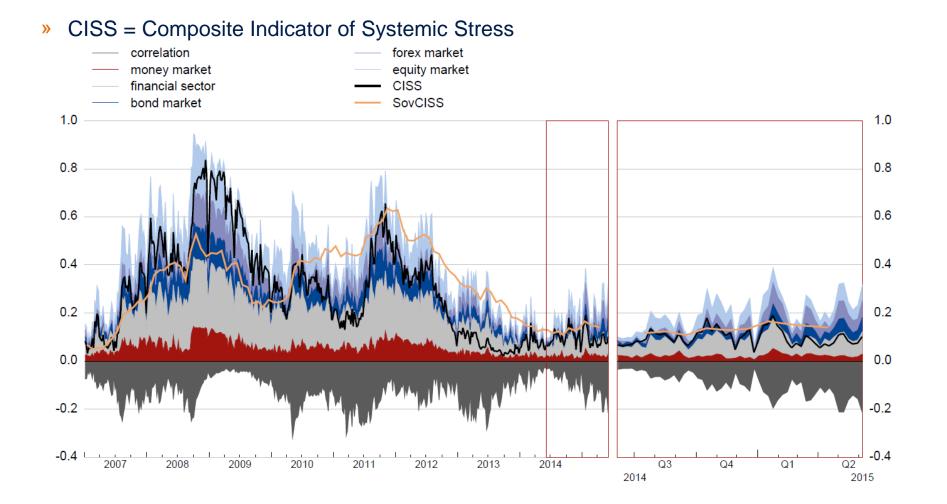
$$z_{n+T} \coloneqq \begin{cases} \frac{r}{n+T} \text{ for } x_{[r]} \le x_{n+T} < x_{[r+1]}, \quad r \in \{1, 2, \dots, n-1, \dots, n+T-1\} \\ 1 \text{ for } x_{n+T} > x_{[n+T]} \end{cases} \text{ to update CISS with near real time data}$$

- » In every segment, the stress factors are aggregated by the arithmetic average, denoted $s_{i,t}$, $i \in \{1, ..., 5\}$.
- **>** The CISS for time t (CISS_t) is computed with methods from portfolio theory:
 - CISS_t = $\sum_{i,j} (w \cdot s_t)_i C_{t,i,j} (w \cdot s_t)_j$, with weights w = (0.15, 0.15, 0.25, 0.3, 0.15), and $(w \cdot s)_i$ the component wise multiplication
 - $\Rightarrow \text{ And the cor.-matrix } C_{t,i,j} = \begin{cases} 1 \text{ for } i = j \\ \rho_{ij,t} \text{ else} \end{cases} \text{ with } \rho_{ij,t} = \frac{\sigma_{ij,t}}{\sigma_{i,t} \sigma_{j,t}}, \sigma_{ij,t} = \lambda \sigma_{ij,t-1} + (1-\lambda) \widetilde{s_{i,t}} \widetilde{s_{j,t}}, \sigma_{i,t}^2 = \lambda \sigma_{i,t-1}^2 + (1-\lambda) \widetilde{s_{i,t}}^2, \widetilde{s_{i,t}} = s_{i,t} 0.5, \lambda \approx 0.93 \end{cases}$
- » CISS puts relatively more weight on situations where stress prevails in several market segments.

Source: European Systemic Risk Board (ESRB) Risk Dashboard, Hollo, D., Kremer, M. and Lo Duca, M., "CISS - A composite indicator of systemic stress in the financial system", Working Paper Series, No 1426, ECB, March 2012, MFI: Monetary Financial Institution, NFS: Non-Financial sector, (N)FC: (Non-)Financial corporation

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Combating the crisis: Is the financial and European debt crisis over?



Source: European Systemic Risk Board (ESRB) Risk Dashboard, Hollo, D., Kremer, M. and Lo Duca, M., "CISS - A composite indicator of systemic stress in the financial system", Working Paper Series, No 1426, ECB, March 2012

2017-10-09 | From Physics to Finance | Is the financial complexity manageable? (14/23)

Watson, we need your help!



Mensch gegen Computer: Bei der populären US-Quizshow "Jeopardy!" siegte die IBM-Maschine. Jetzt hat sie einen neuen Job

Wall Street heuert "Watson" an

Super-Computer aus der TV-Quizshow "Jeopardy" macht jetzt Banker arbeitslos

Citigroup setzt schlaue IBM-Maschine bereits für Risikoanalysen und zur Kundenberatung ein

Source: WELT KOMPAKT, March 2012

Watson, we need your help!

IBM

Watson wertet Daten von Apple-Nutzern aus

IBM will mit seinem selbst lernenden Computersystem Watson die Gesundheitsdaten von iPhone- und Apple-Watch-Nutzern analysieren – und die Ergebnisse Dritten anbieten.

VON Patrick Beuth | 14. April 2015 - 11:35 Uhr



iPhone und Apple Watch

Source: ZEIT Online

Has physics caused the crisis?

- » Risk management depends heavily on sophisticated models
- » Developed models were too complex to be understood intuitively
- Computer experts construct "financial hydrogen bombs" as already suspected by Felix Rohatyn in 1998

The main problem is: Our models have in fact become extremely complex but are still too simple to be able to incorporate the whole spectrum of variables that drive the global economy. A model is necessarily an abstraction without all details of the real world.

When things fall apart



Vienna, May 9th, 1873

OPTICIANS

New York, October 25th, 1929

Northern Rock, September 18th, 2007

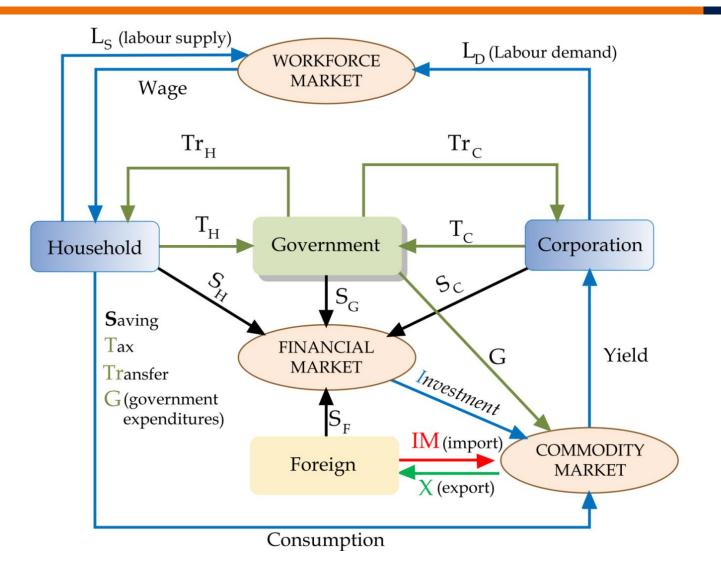


Photo source: en.wikipedia.org

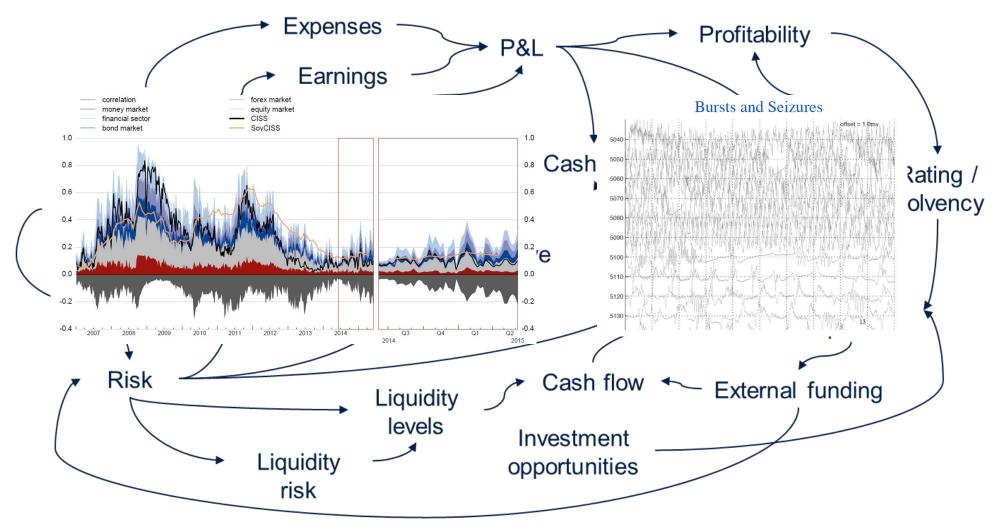


northern rock

Economics and banking – a complex network of dependencies



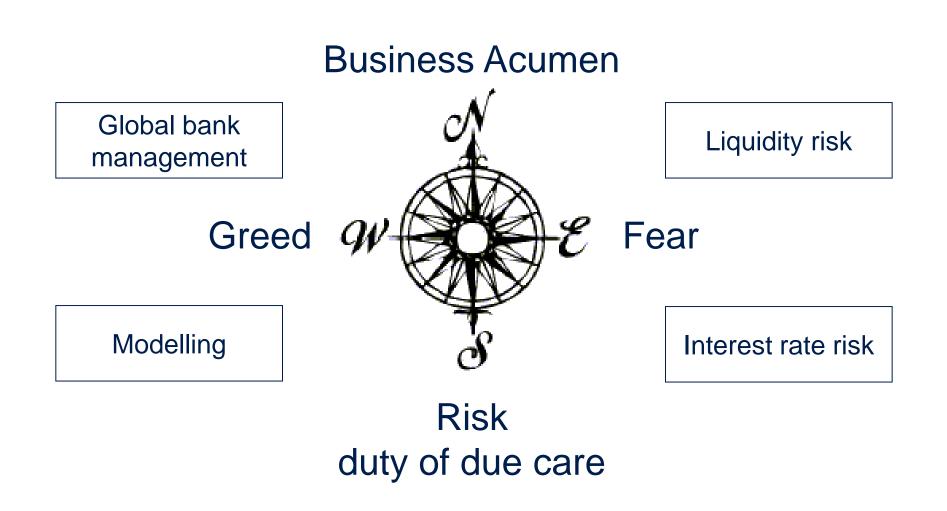
Can human brains handle the amount of information?



From: Managing Liquidity in Banks, R. Duttweiler, 2009; European Systemic Risk Board (ESRB) Risk Dashboard; Didier Sornette, Neuron, Reviews on Cognitive Architectures, Vol. 86, Number 1, Oct. 2015

2017-10-09 | From Physics to Finance | Is the financial complexity manageable? (21/23)

The four "business dimensions"



2017-10-09 | From Physics to Finance | Is the financial complexity manageable? (22/23)

Has physics caused the crisis?

- » Risk management depends heavily on sophisticated models
- » Developed models were too complex to be understood intuitively
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Physics has not caused the crisis \rightarrow

Ignoramus et ignorabimus

versus We have to know. We will know. D. Hilbert

Everything which is not forbidden is compulsory. M. Gell-Mann

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