PAST PRESENT AND FUTURE CHALLENGES IN THE DETERMINATION OF THE STRUCTURE OF THE PROTON

LECTURE I

Maria Ubiali

University of Cambridge

08th April 2019 **Grad Days 2019**

Scope of the lectures

➡ Give an overview on our understanding on the structure of the proton: from Feynman parton model to modern QCD picture

➡ Introduce basic concepts and techniques behind the determination of the structure of the proton from experimental data

➡ Wealth of ingredients involved: perturbative QCD, experimental measurements, statistical and mathematical problems, higher order predictions, phenomenology tools, machine learning.

➡ Discuss PDF-related phenomenology at the Large Hadron Collider and beyond

➡ Discuss current frontiers and challenges

Disclaimer: these lectures are far from providing a complete picture of the topic. You can find complementary information in excellent lectures on PDFs from W. Giele, G. Salam, A. Martin, P. Nadolsky, S. Forte, D. Stump, W. Melnitchouk, D. Stump, A. Guffanti, J. Rojo … at recent graduate schools

References

- G. Ridolfi “Notes on deep-inelastic scattering and the Parton model”
- S. Forte lectures
- Ellis, Stirling and Webber “QCD and collider physics”
- G. Dissertori, I. Knowles, M. Schmelling “Quantum Chromo Dynamics”
- J. Gao, L. Harland-Lang, J. Rojo - *Phys.Rept. 742 (2018) 1-121*
- S. Forte, G. Watt - *Ann.Rev.Nucl.Part.Sci. 63 (2013)*
- S. Forte, *Acta Phys.Polon. B41 (2010) 2859*
- E. Perez, E. Rizvi, *Rep.Prog.Phys. 76 (2013) 046201.*
- A. Accardi, et al., *Eur. Phys. J. C76 (8) (2016) 471*
- A. De Roeck, R. S. Thorne, *Prog.Part.Nucl.Phys. 66 (2011) 727*
- http://pdg.lbl.gov/2017/reviews/rpp2017-rev-structure-functions.pdf

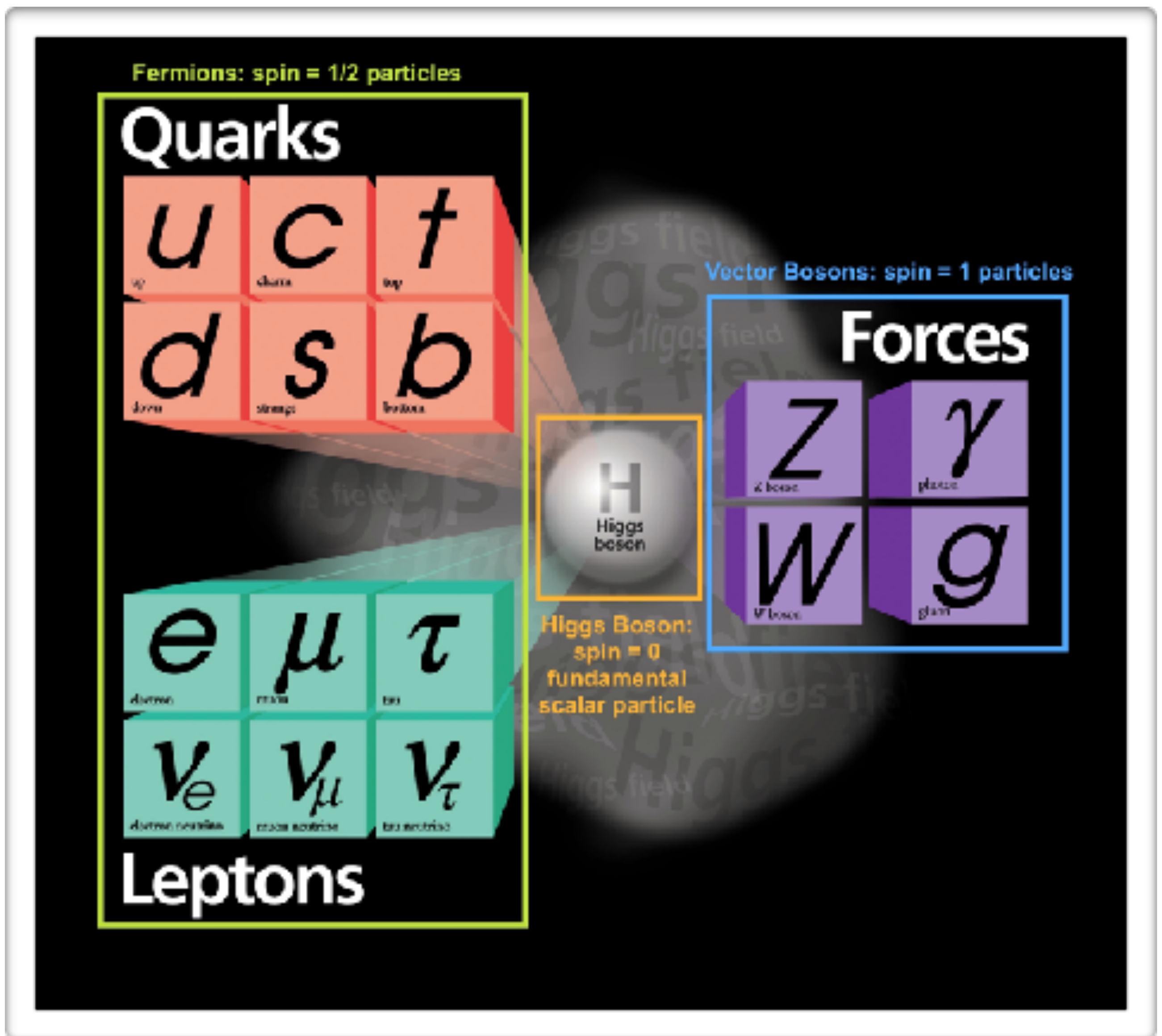
List of references complemented by specific references during the lectures

Outline

- **<u>First lecture</u> (Today)**
  - Motivation: the big picture
  - Parton Model and QCD
  - Collinear Factorisation
- Second lecture (Tuesday)
  - Experimental Data
  - Disentangling proton's components
  - Statistics and Methodology
- Third lecture (Wednesday)
  - Fits and methodology
  - The NNPDF approach
- Fourth lecture (Thursday)
  - New frontiers and challenges

Standard Model of particle physics

- Standard Model (SM) of particle physics one of the greatest triumph of Quantum Field Theories in the past century
- SM remarkably successful theory: no convincing deviations so far from its predictions



$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

Spontaneous EW Symmetry breaking

Quantum
Chromo
Dynamics

$$SU(3)_c \times U(1)_{\rm e.m.}$$

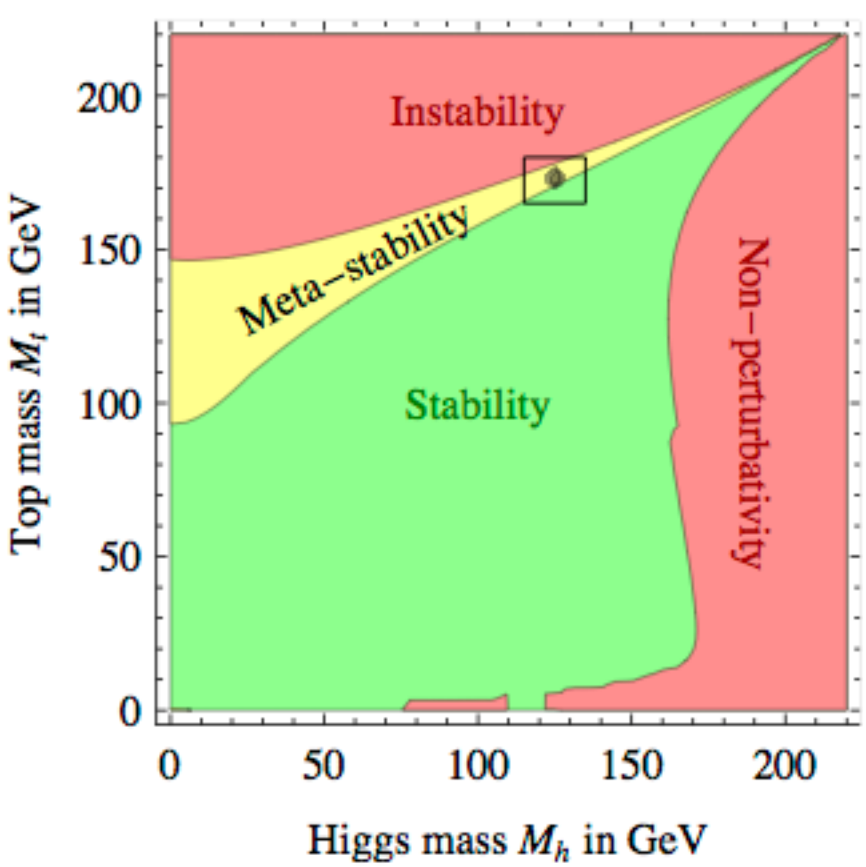
Some compelling questions

Higgs Boson

Hierarchy problem:
Huge gap between EW scale (10^2 GeV)
and Gravitational scale (10^{19} GeV)

Elementary or composite?
Additional Higgs bosons?

Is the vacuum of the universe stable?



Degrassi et al, 2012

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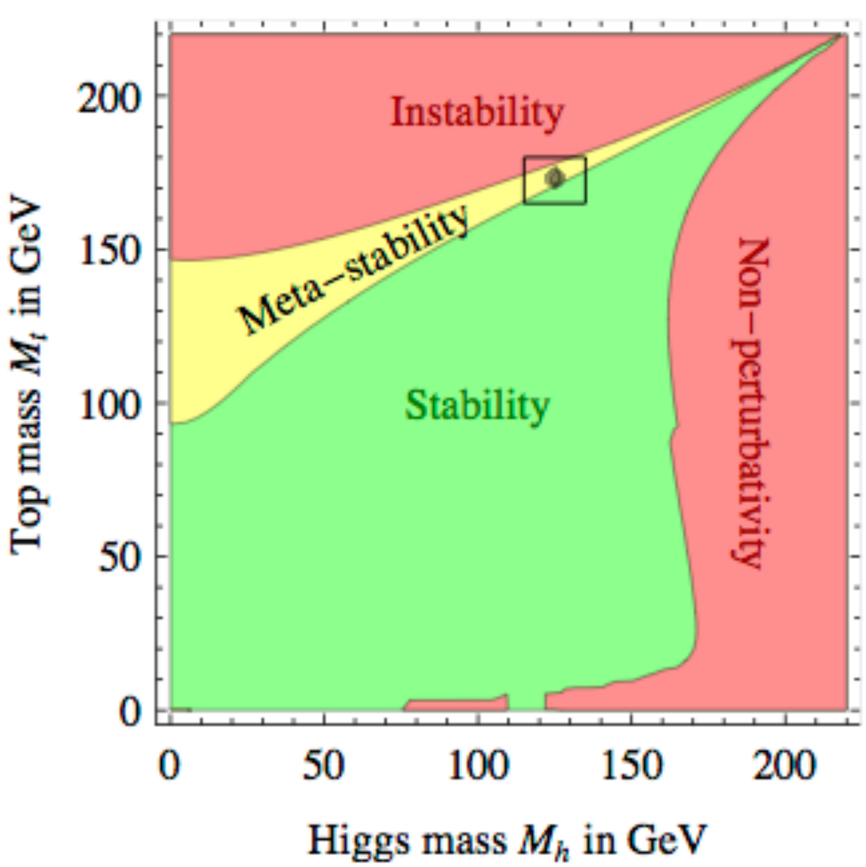
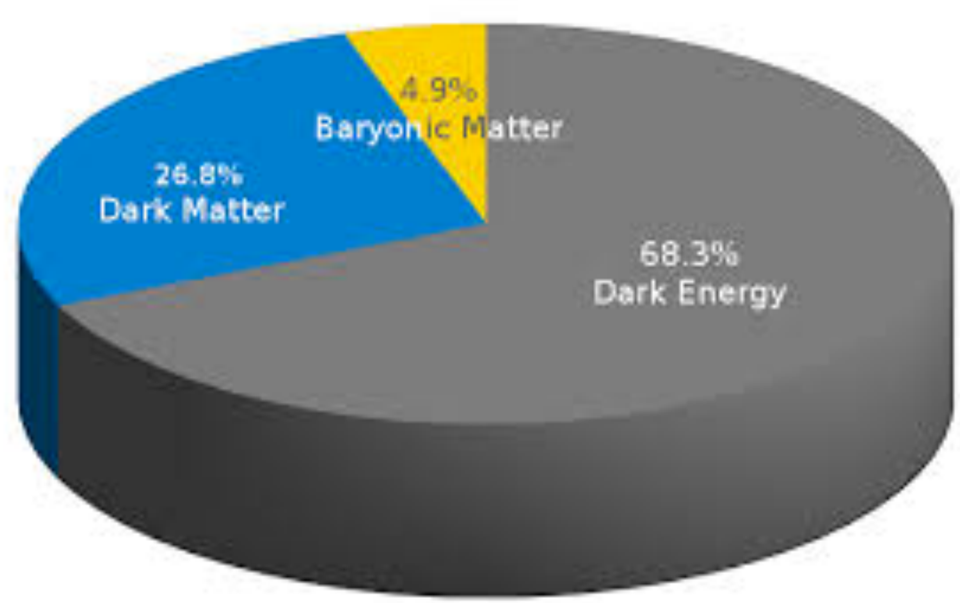


Degrassi et al, 2012

Dark Matter

Weakly-interacting massive particle?
Sterile neutrino?
Extremely light particle (axion)?
Alternative explanations?

Interaction with SM?
Self-interacting?



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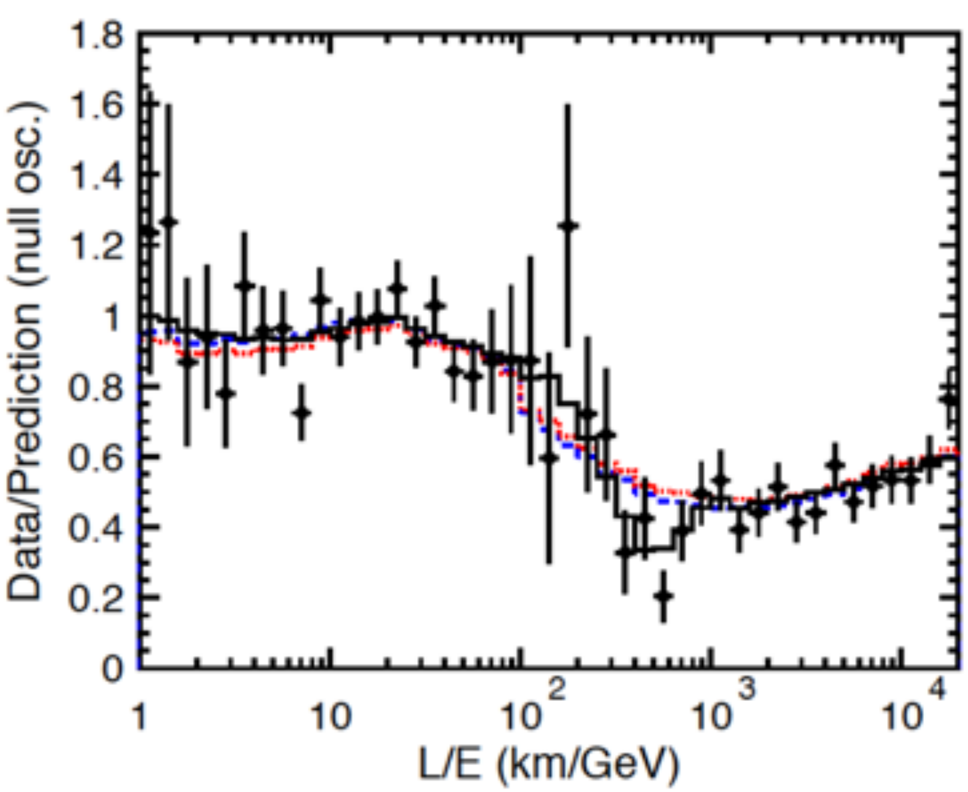


Super-Kamiokande observation of neutrino oscillations, 2004

Quarks and leptons

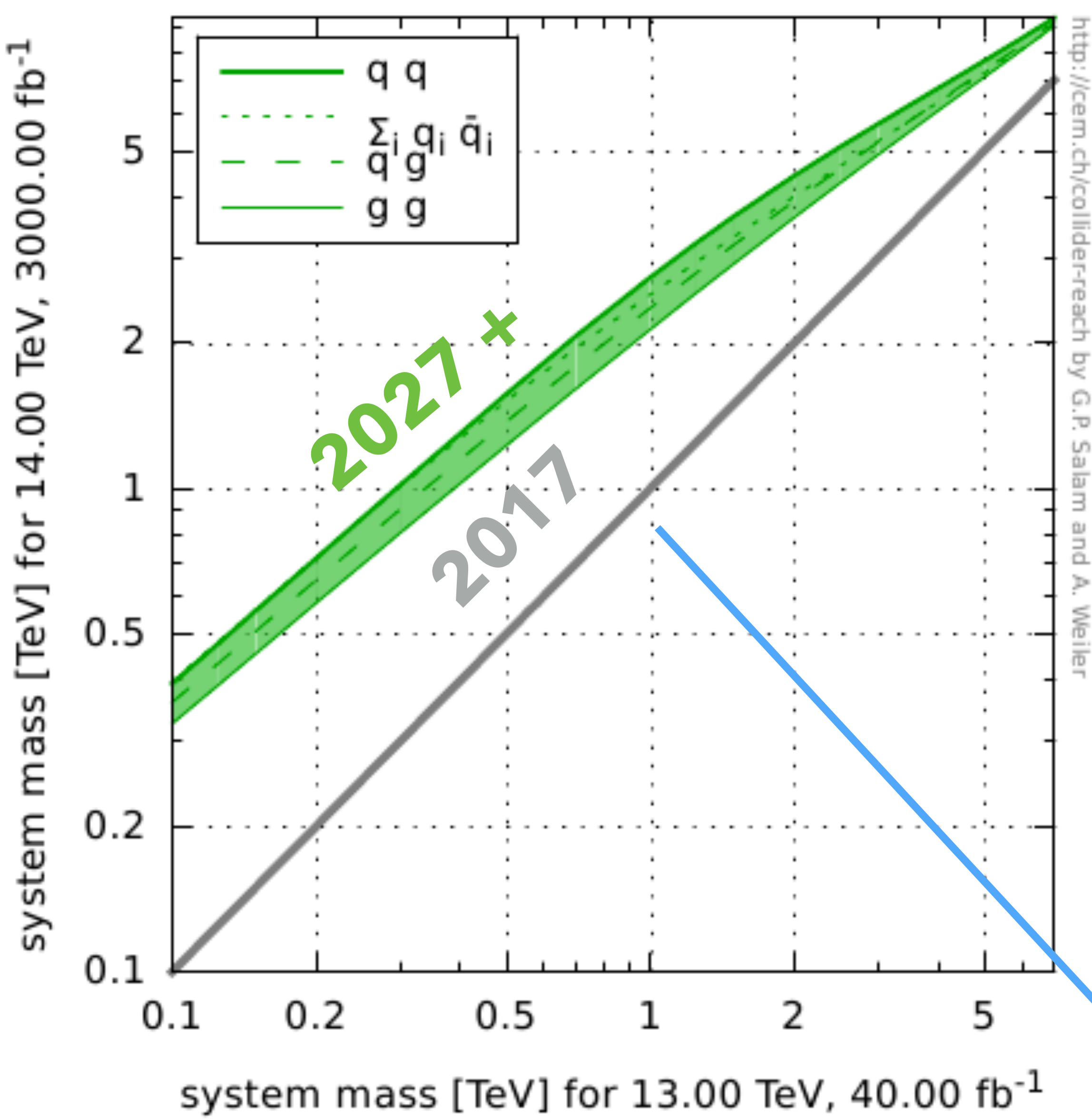
Why 3 families? Can we explain mass & mixings?

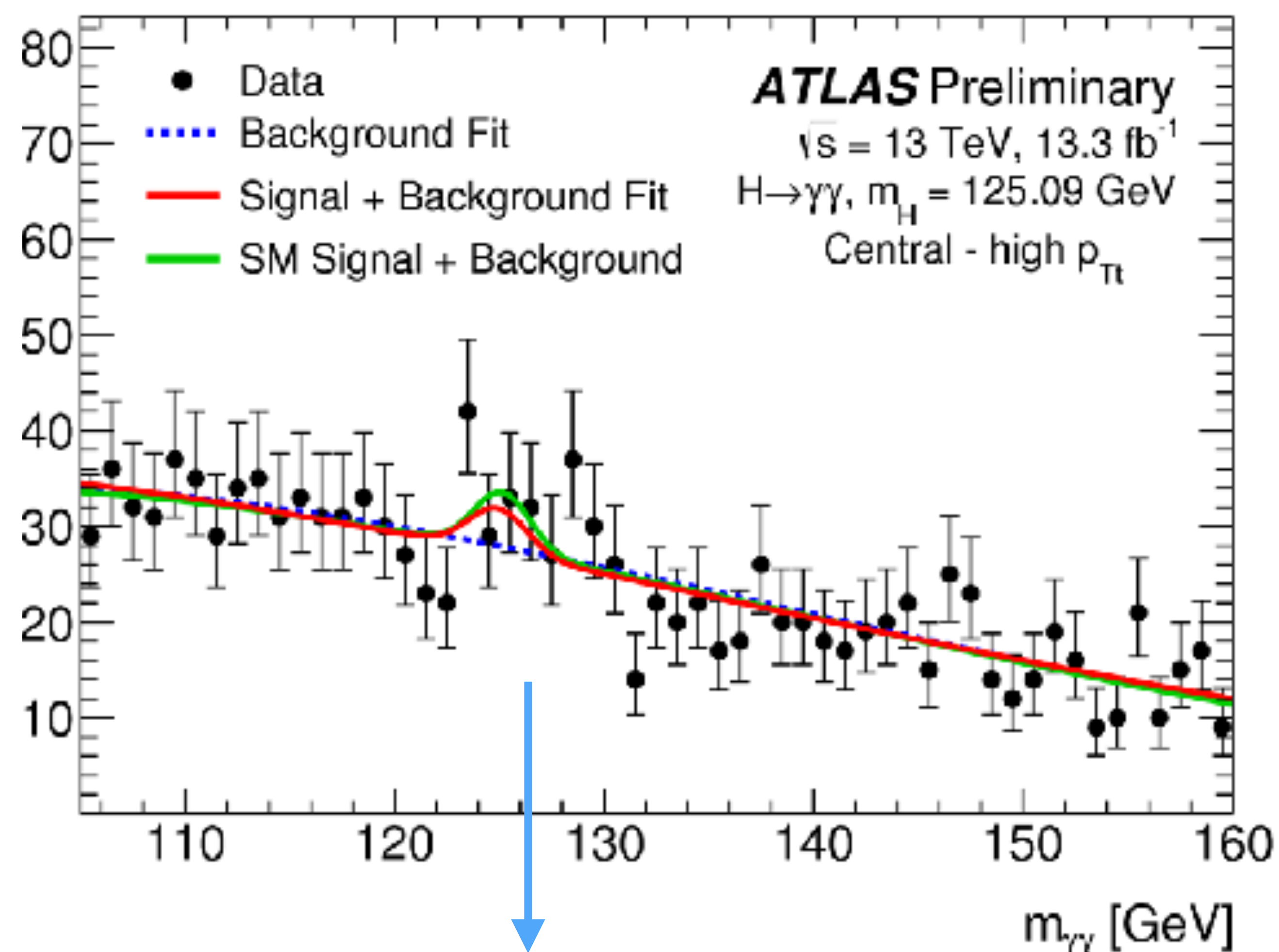
Are neutrinos Majorana or Dirac fermions?

Origin of matter-antimatter asymmetry in the Universe?

A unique opportunity

- The Large Hadron Collider at CERN most powerful accelerator ever built
- Extremely successful Run I (7-8 TeV) and great performance at Run II (13-14 TeV)
- As luminosity increases, stronger probe on known processes (Higgs, Flavour anomalies…) & larger mass reach





Higgs rediscovery at Run II

With LHC upgrade in ten years
mass reach increase by ~2

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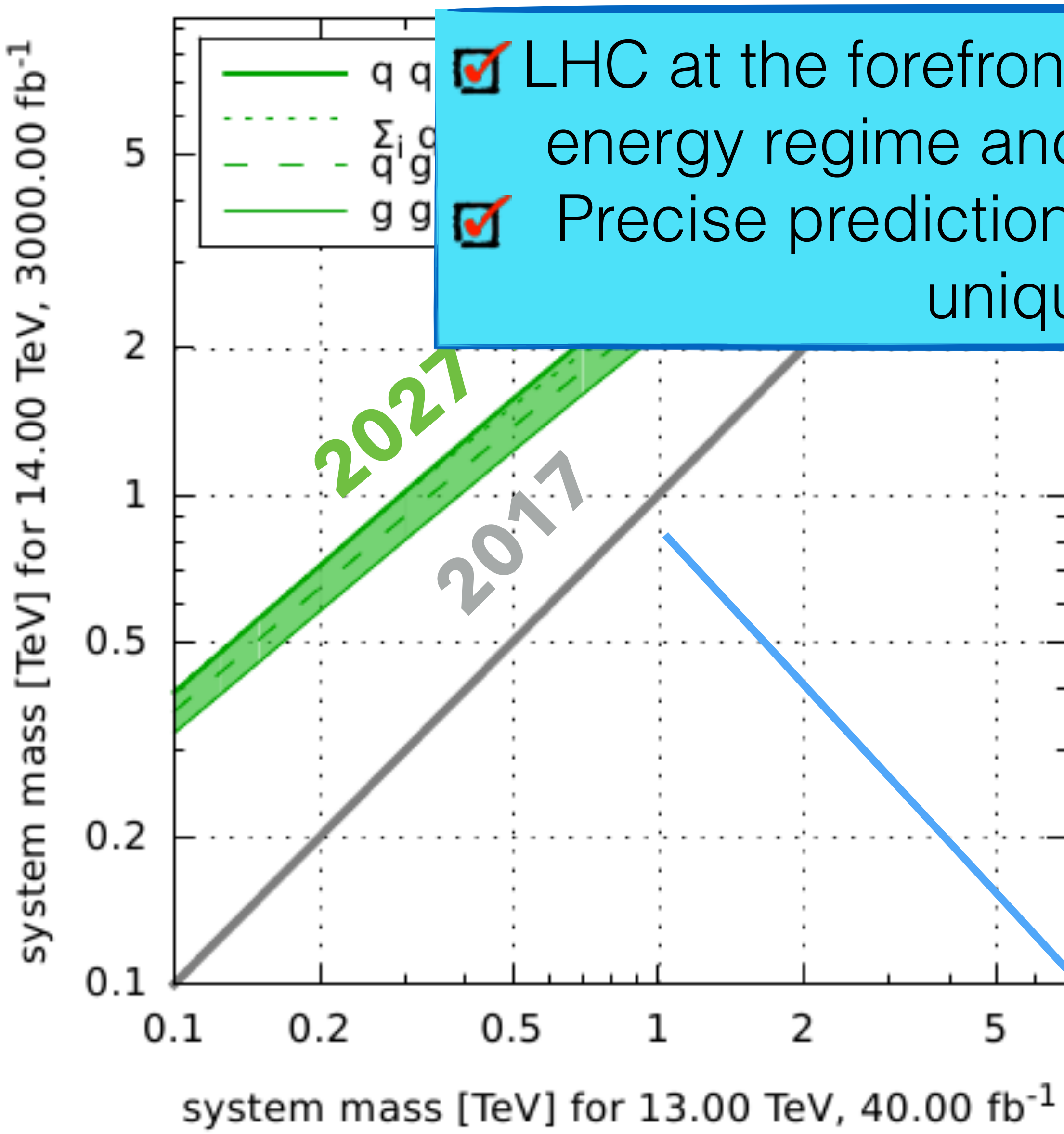
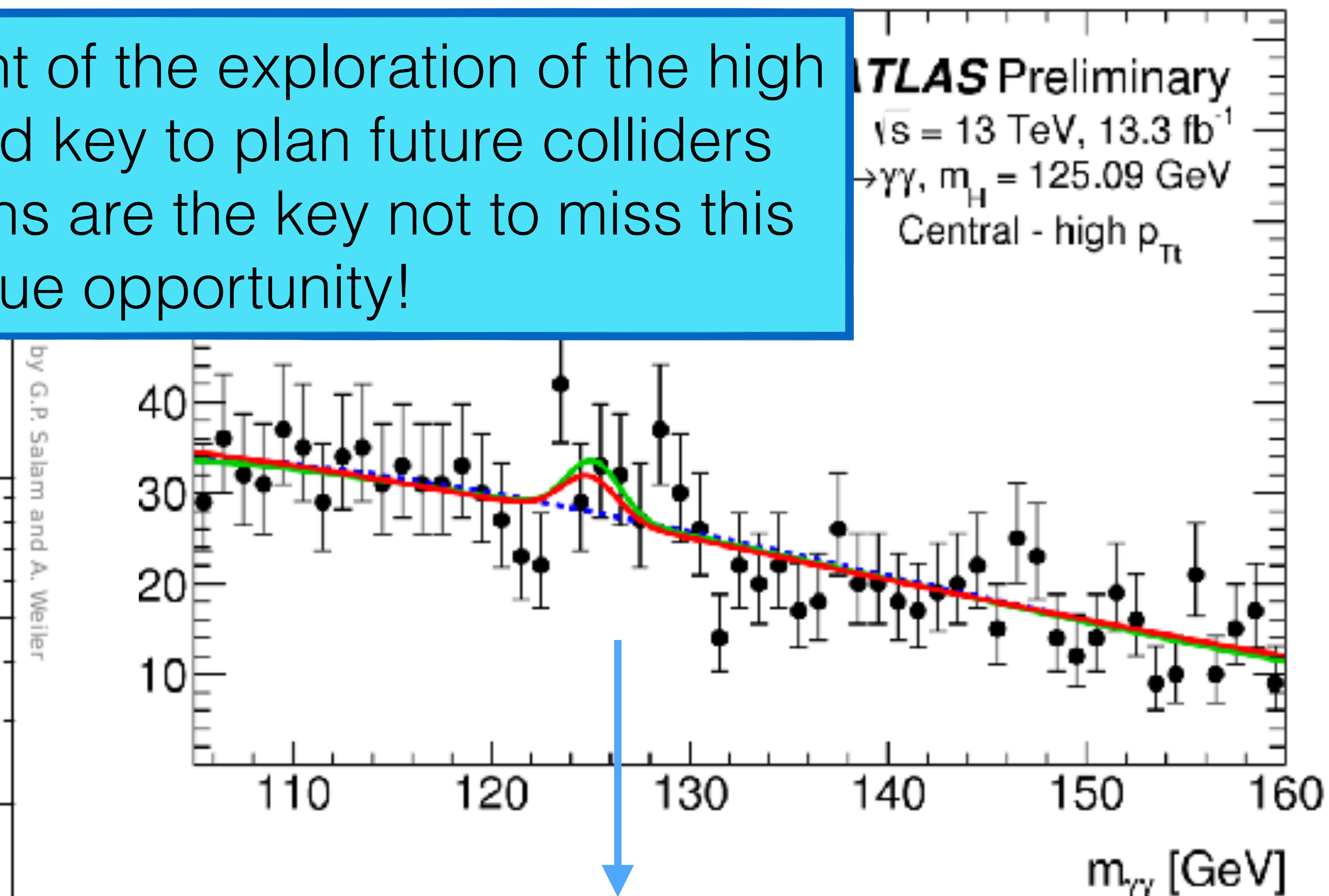


☑ LHC at the forefront of the exploration of the high energy regime and key to plan future colliders

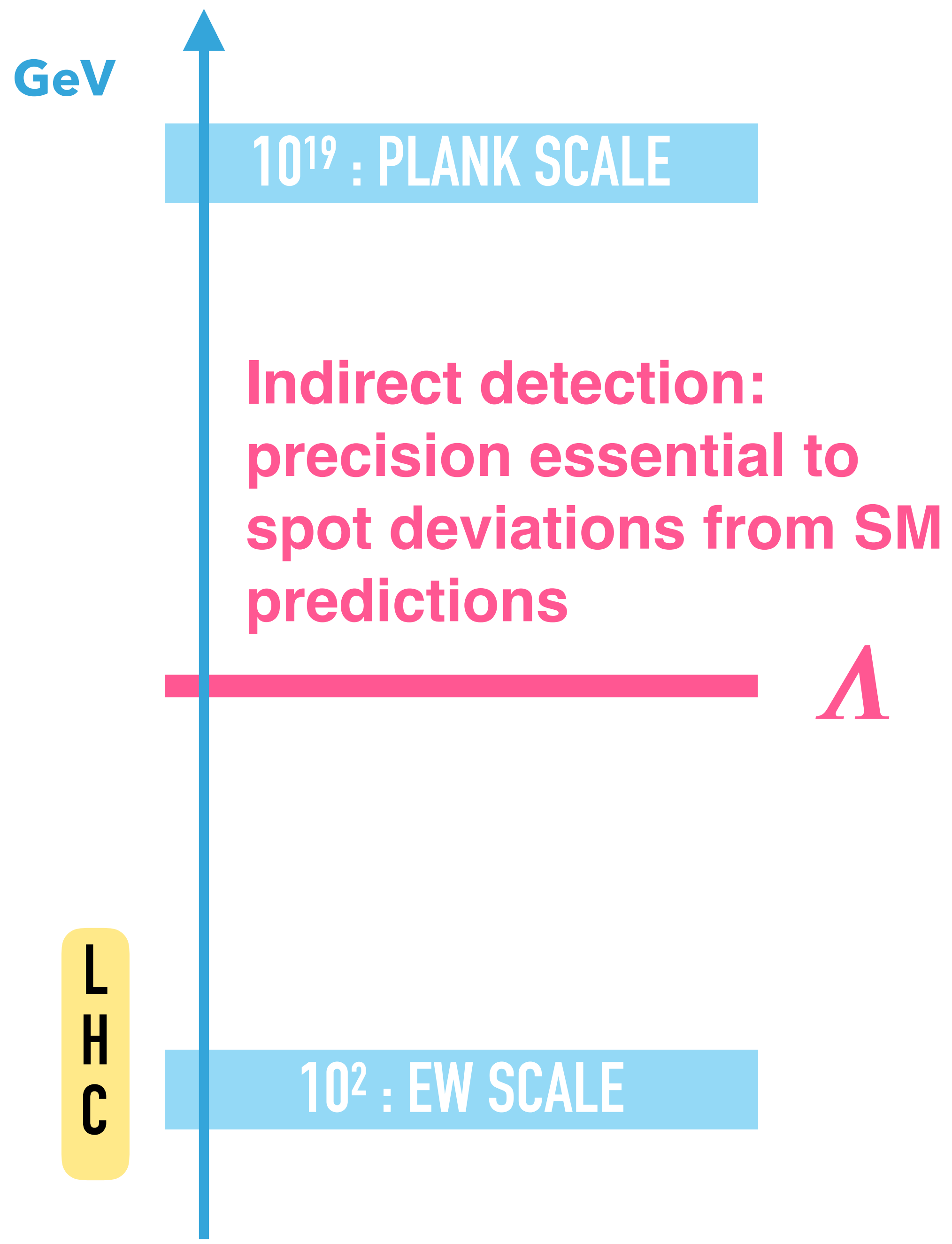
☑ Precise predictions are the key not to miss this unique opportunity!

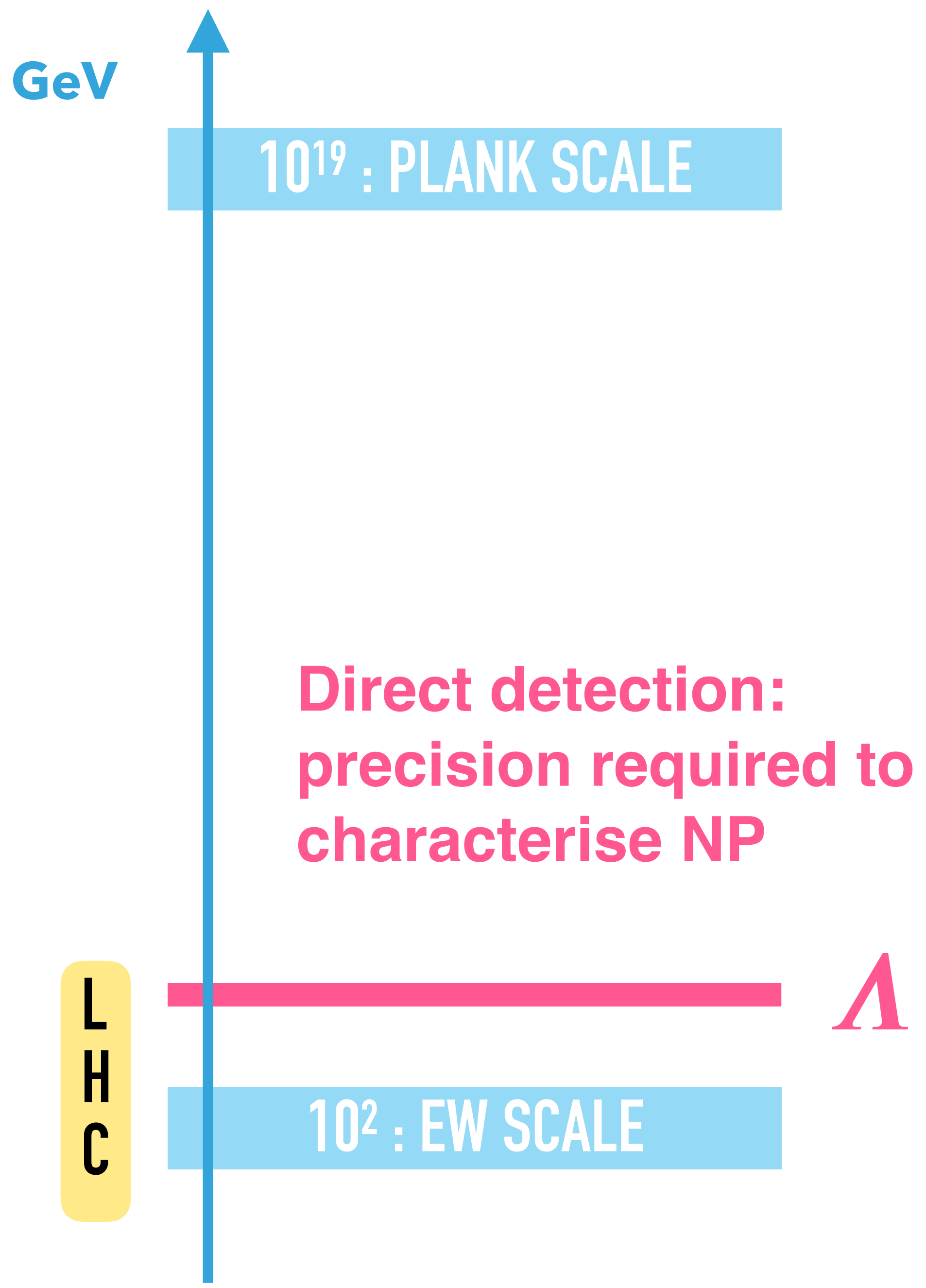


Higgs rediscovery at Run II

With LHC upgrade in ten years mass reach increase by ~2

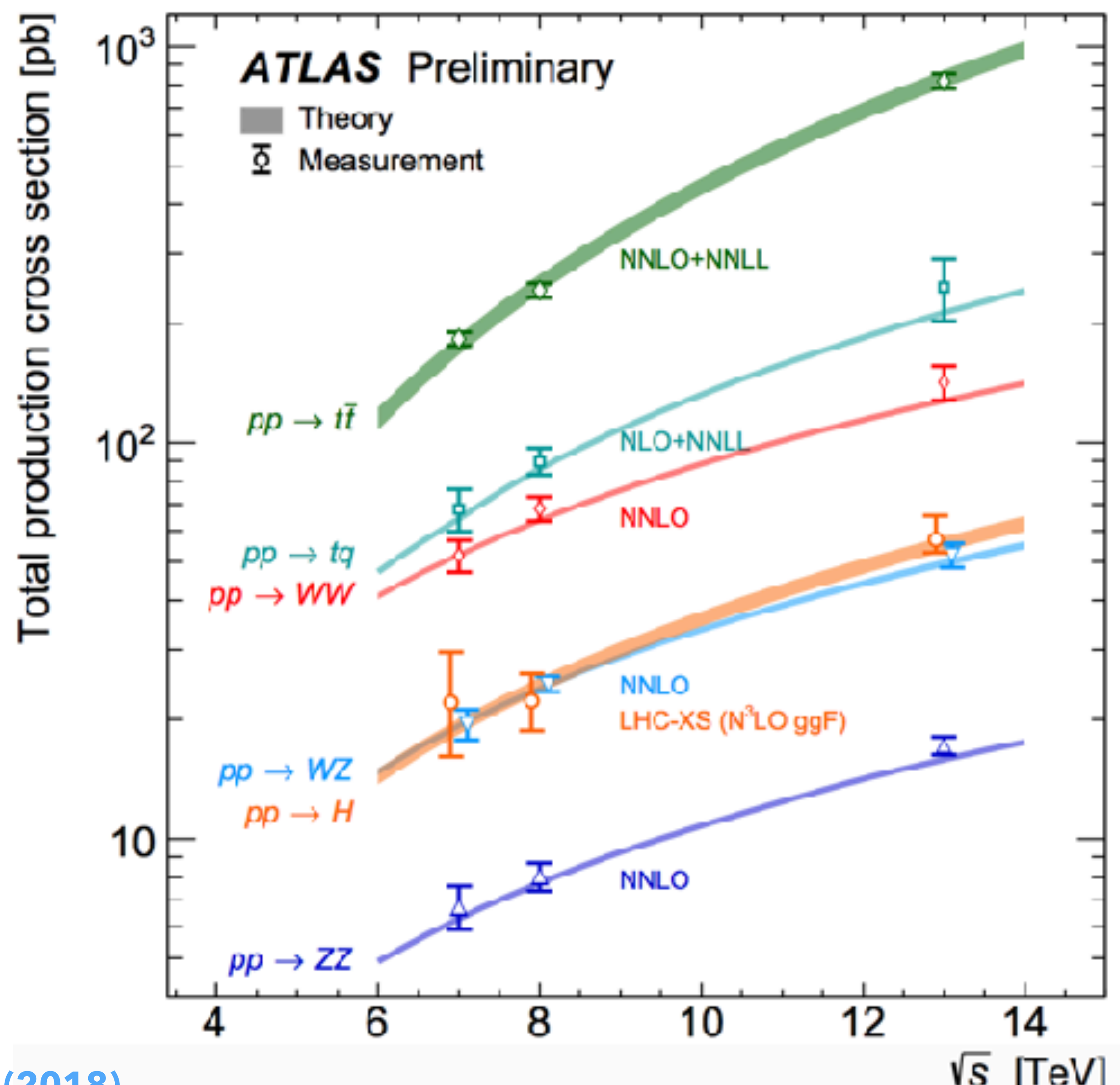
A new precision era in particle physics





A new precision era in particle physics

- Is precision physics really possible/necessary at hadron colliders?
  At the LHC a paradigm shift took place: discovery ➜ discovery through precision
- Theoretical predictions to catch up with precision of experimental data



ATLAS SM summary plot (2018)

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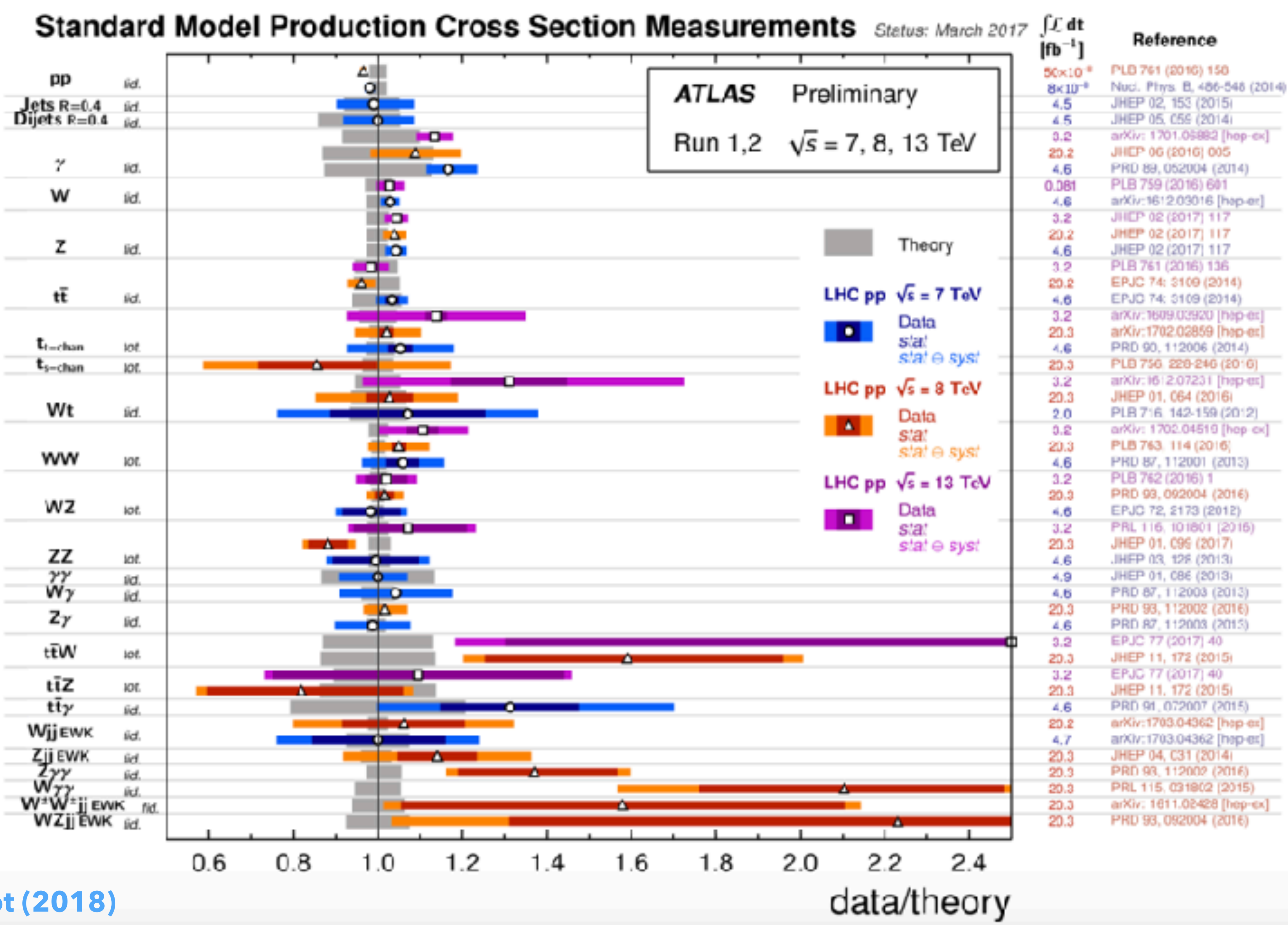
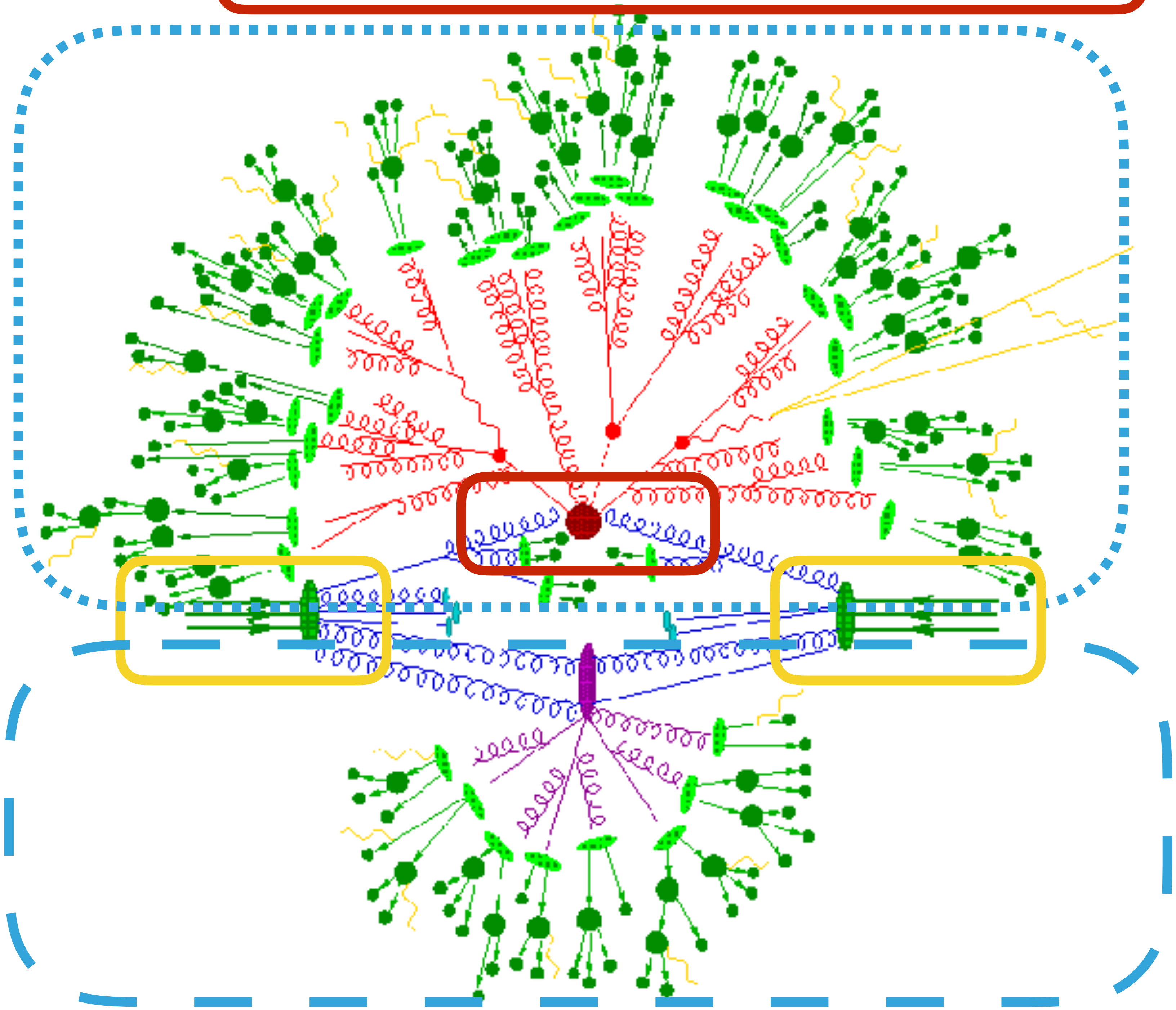


ATLAS SM summary plot (2018)

The precision ingredients

$$\sigma^{\rm th} = \hat{\sigma}[\mathcal{O}(1) + \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha_s^2) + \ldots] \otimes f_1 \otimes f_2 + \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$$



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- Parton Distribution Functions
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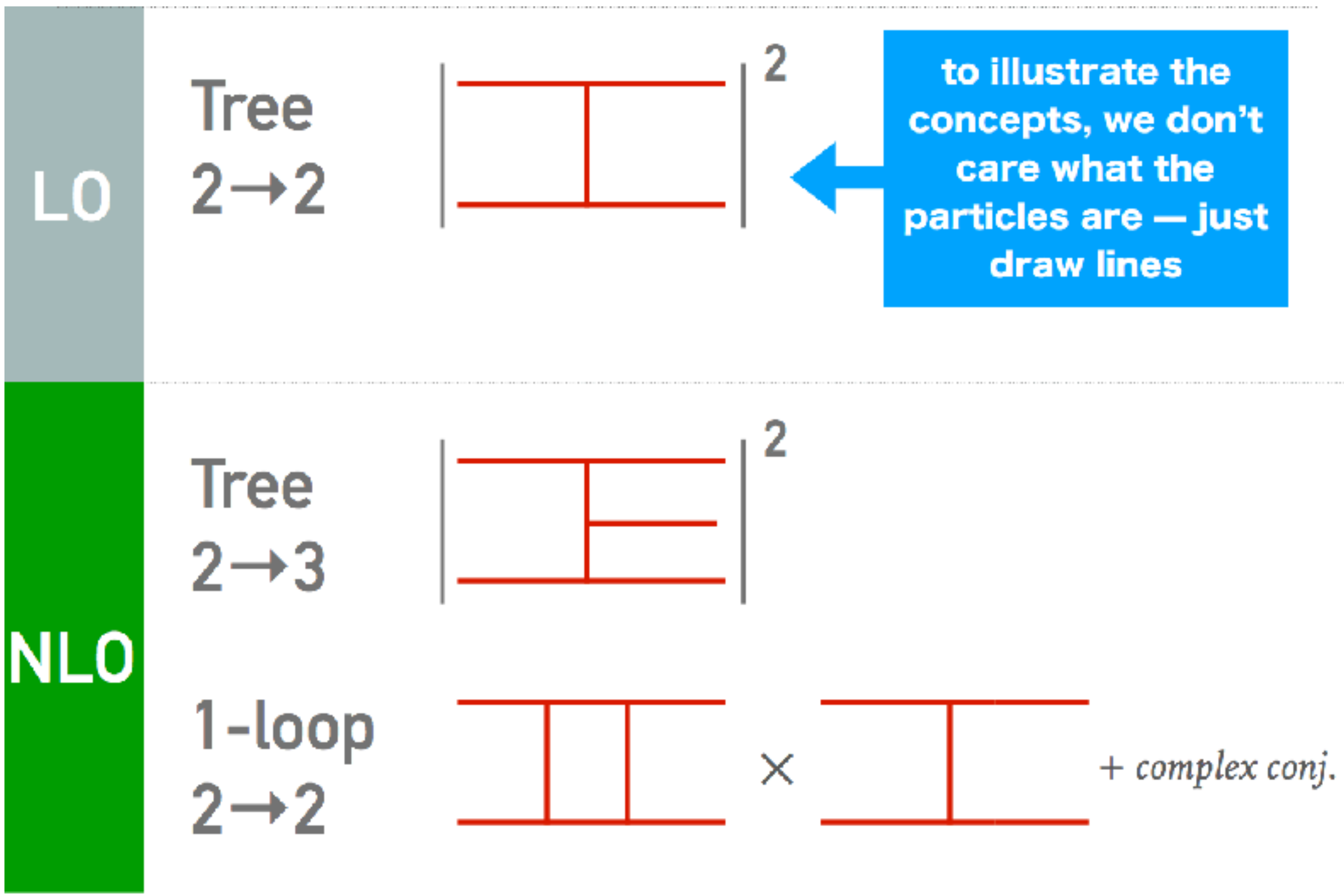
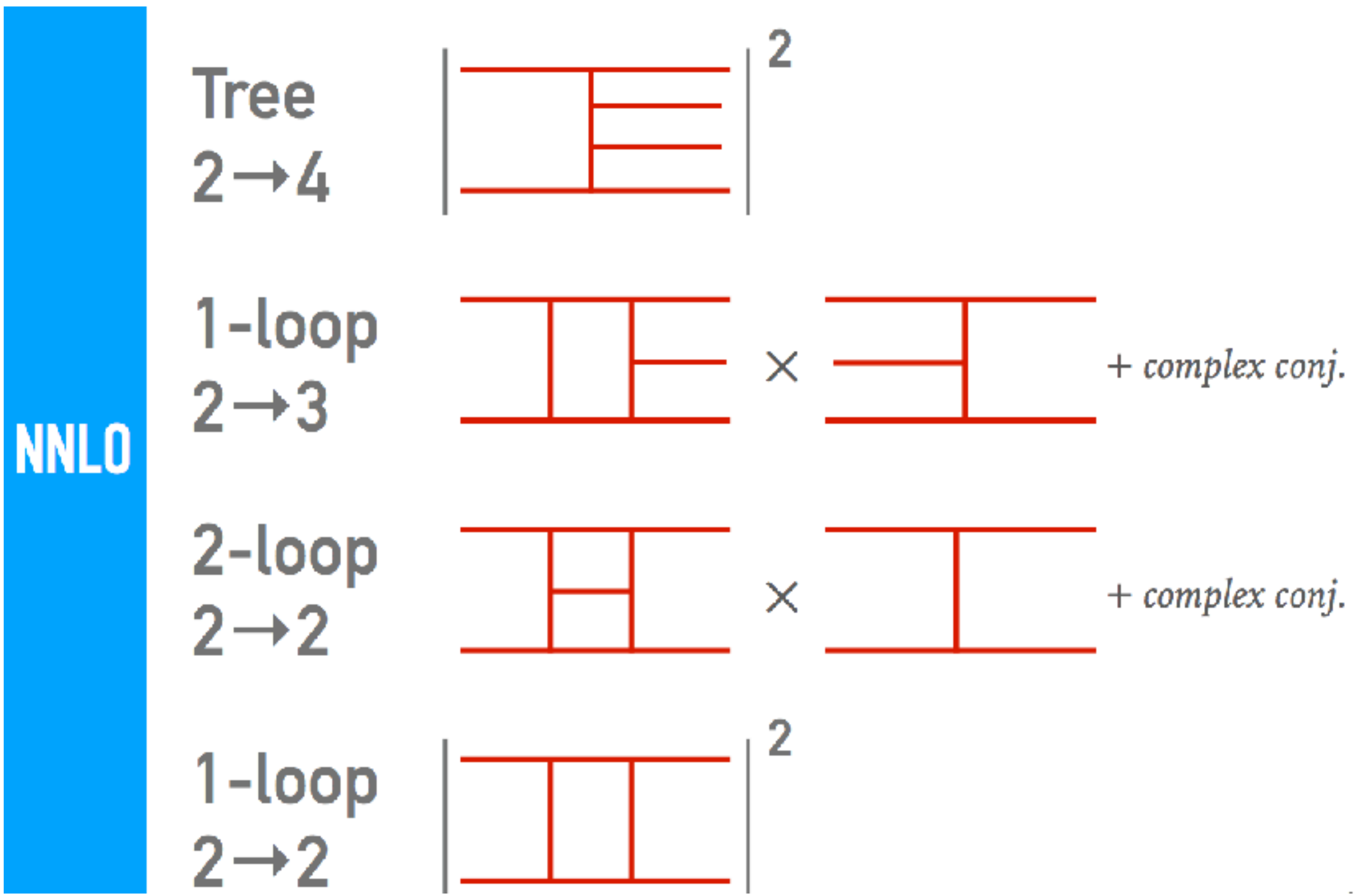
Partonic cross sections



Partonic cross sections



Partonic cross sections

Slide from Gavin Salam lectures
Quy Nhon Vietnam 2018

$$\sigma(pp \to H) = (961\,\text{pb}) \times (\alpha_s^2 + 10.4\alpha_s^3 + 38\alpha_s^4 + 48\alpha_s^5 + \cdots)$$

$$\alpha_s \equiv \alpha_s(M_H/2)$$

$$\sqrt{s_{pp}} = 13\,\text{TeV}$$

Anastasiou et al., 1602.00695 (ggF, hEFT)

pp→H (via gluon fusion) is one of only two hadron-collider processes known at N3LO

(the other is pp→H via weak-boson fusion)

The series does not converge well

(explanations for why are only moderately convincing)

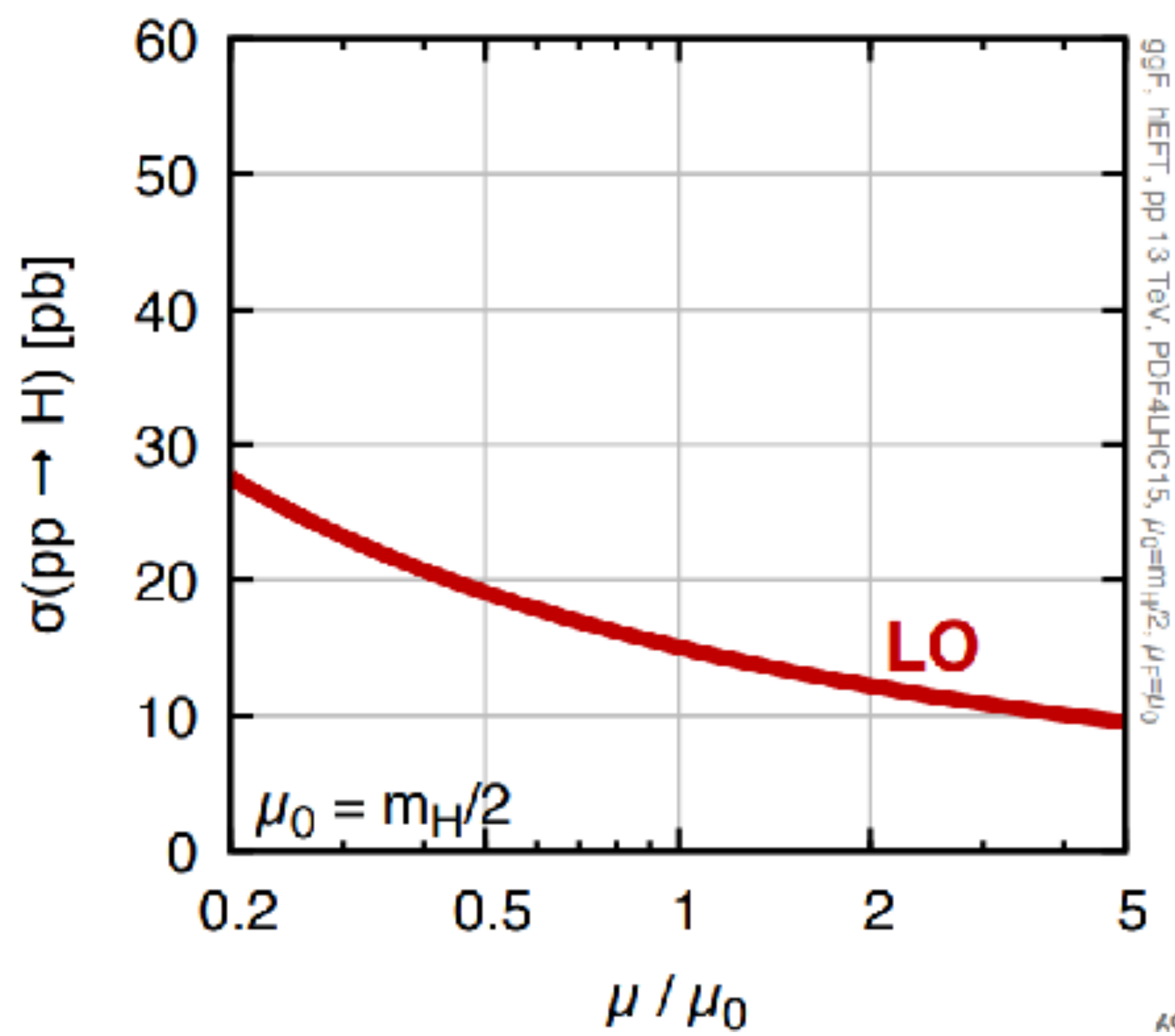
Partonic cross sections

Slide from Gavin Salam lectures
Quy Nhon Vietnam 2018

- On previous page, we wrote the series in terms of powers of $\alpha_S(M_H/2)$
- But we are free to rewrite it in terms of $\alpha_S(\boldsymbol{\mu})$ for any choice of renormalisation scale $\boldsymbol{\mu}$

LO

$$\sigma(pp \to H) = \sigma_0 \times \alpha_s^2(\mu)$$



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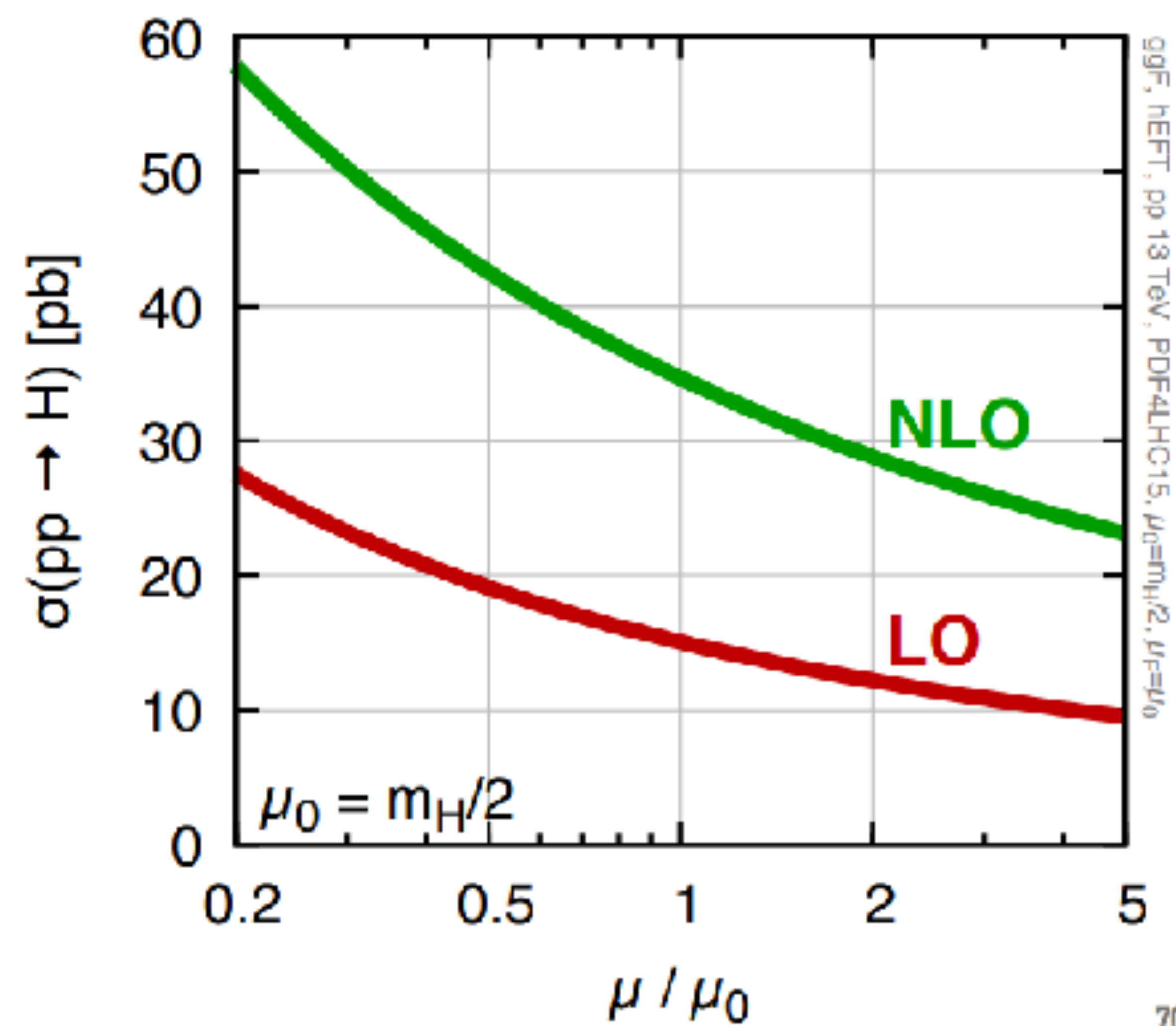
NLO

$$\sigma(pp \to H) = \sigma_0 \times \Big(\alpha_s^2(\mu) + (10.4 + 2b_0 \ln \frac{\mu^2}{\mu_0^2})\alpha_s^3(\mu)\Big)$$



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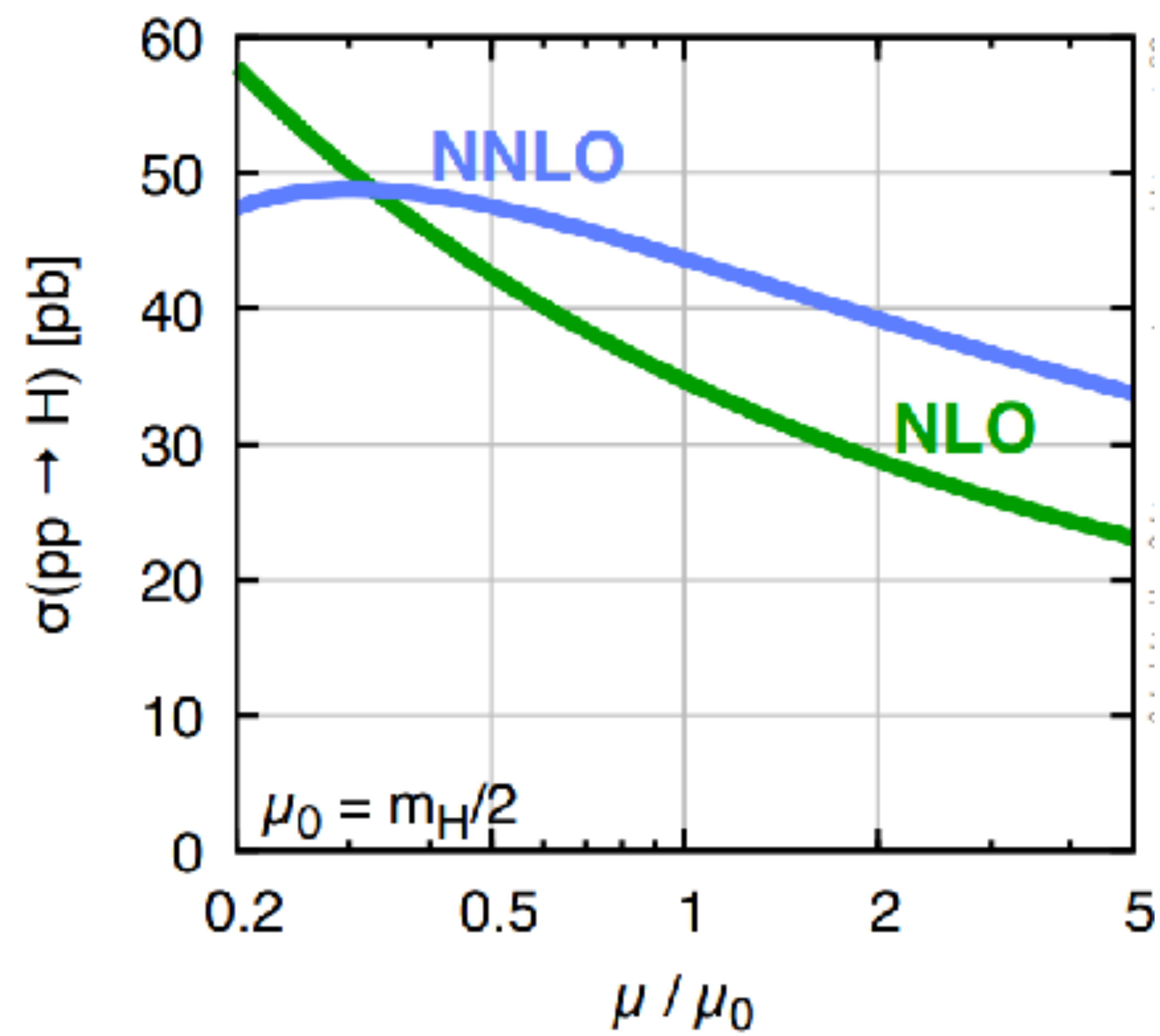
NNLO

$$\sigma(pp \to H) = \sigma_0 \times \left(\alpha_s^2(\mu) + (10.4 + 2b_0 \ln \frac{\mu^2}{\mu_0^2})\alpha_s^3(\mu) + c_4(\mu)\alpha_s^4(\mu)\right)$$



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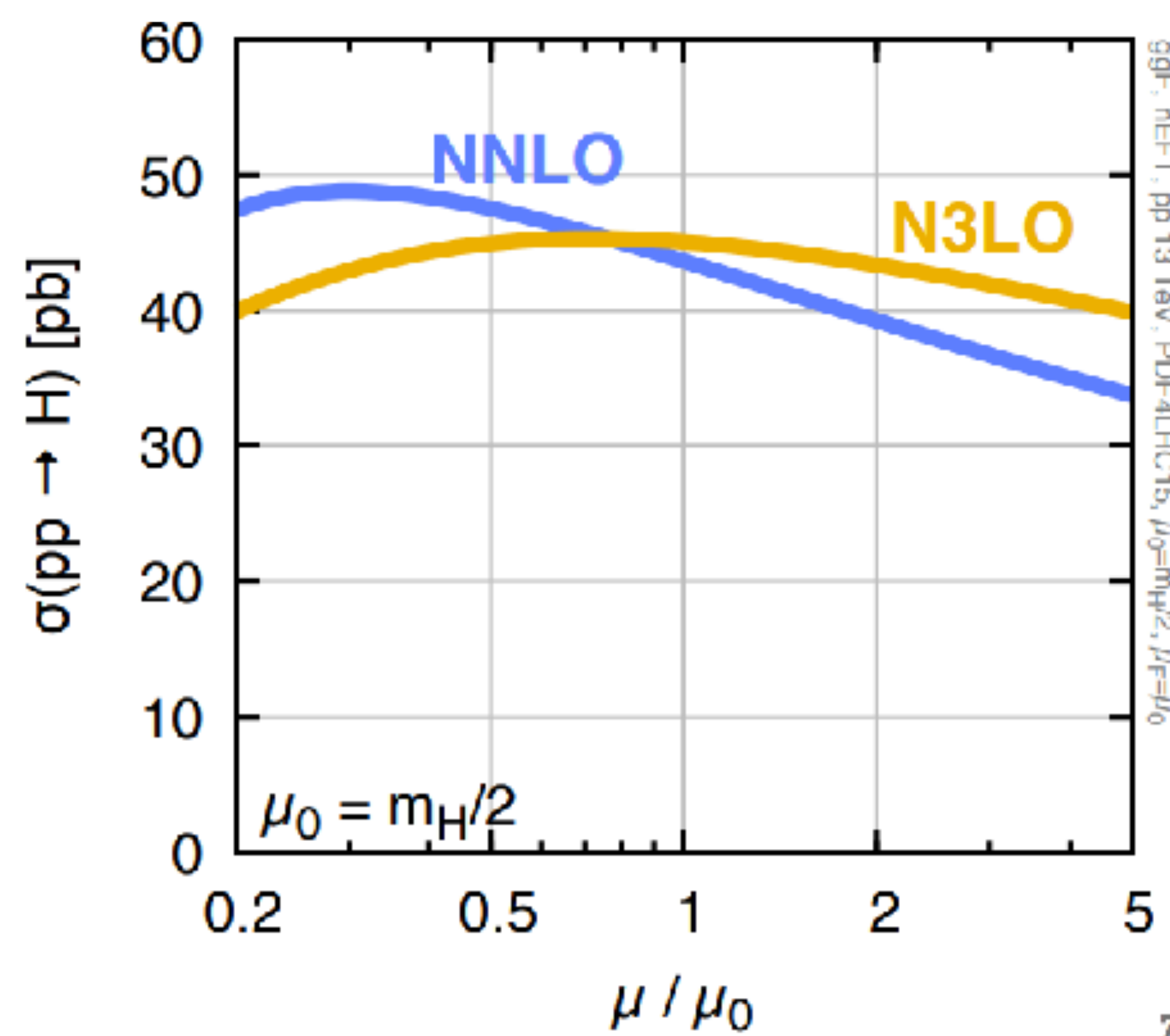
N3LO

$$\sigma(pp \to H) = \sigma_0 \times \Big(\alpha_s^2(\mu) + (10.4 + 2b_0 \ln \frac{\mu^2}{\mu_0^2})\alpha_s^3(\mu) + c_4(\mu)\alpha_s^4(\mu) + c_5(\mu)\alpha_s^5(\mu)\Big)$$



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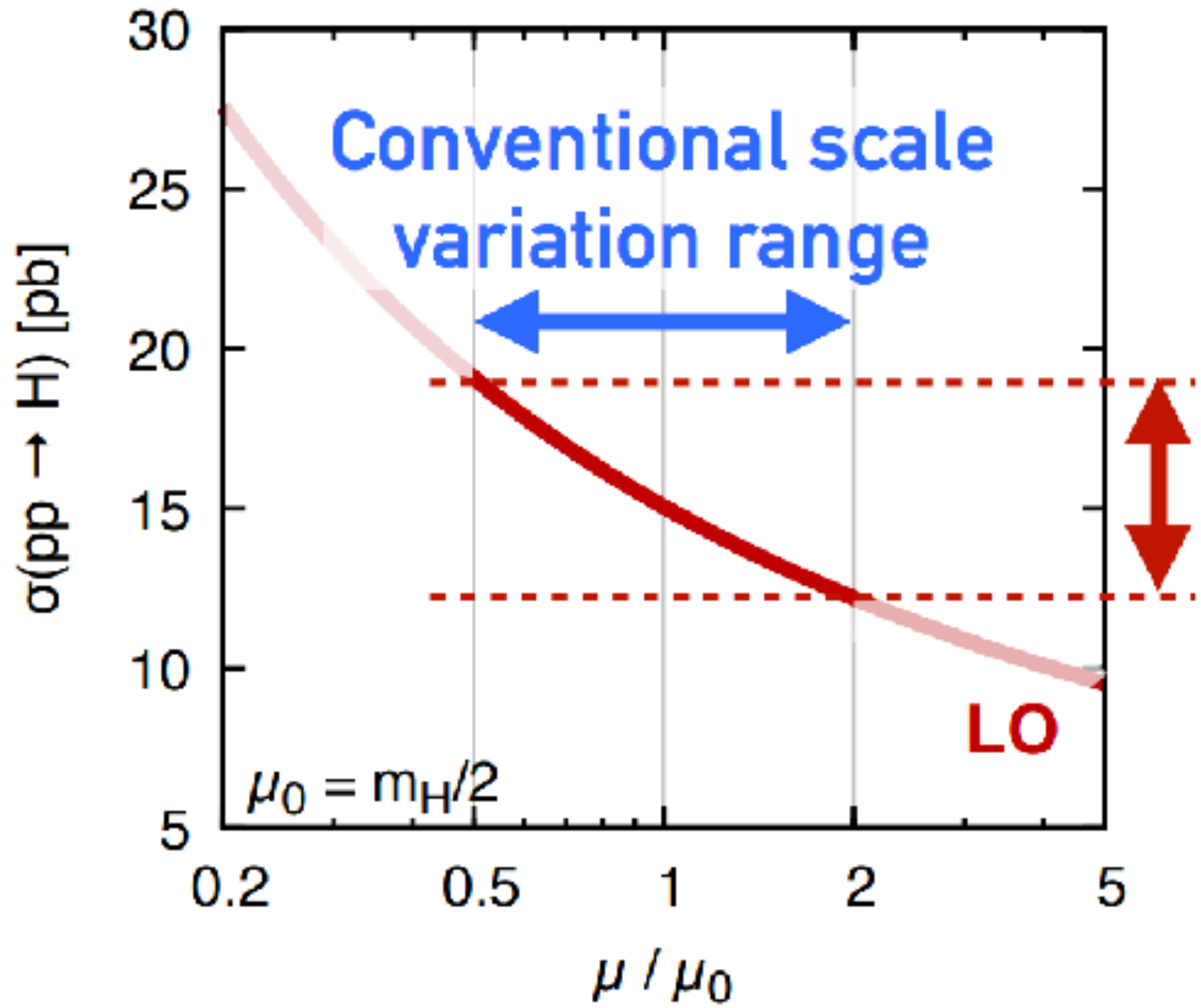

Here, only the renorm. scale μ has been varied. In real life you need to change renorm. and factorisation scales.

Convention: "theory uncertainty" (i.e. from missing higher orders) is estimated by change of cross section when varying μ in range 1/2 → 2 around central value

Partonic cross sections

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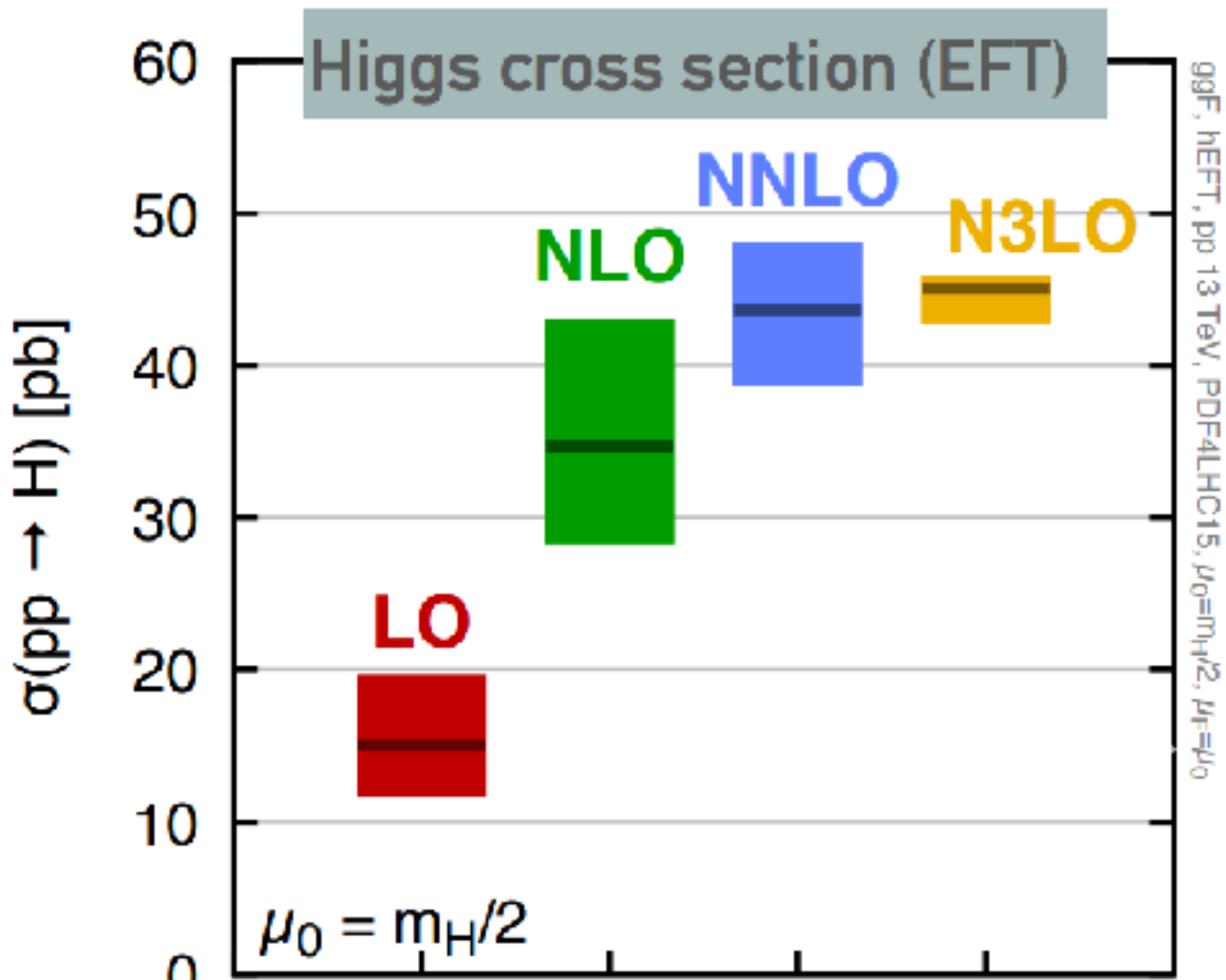
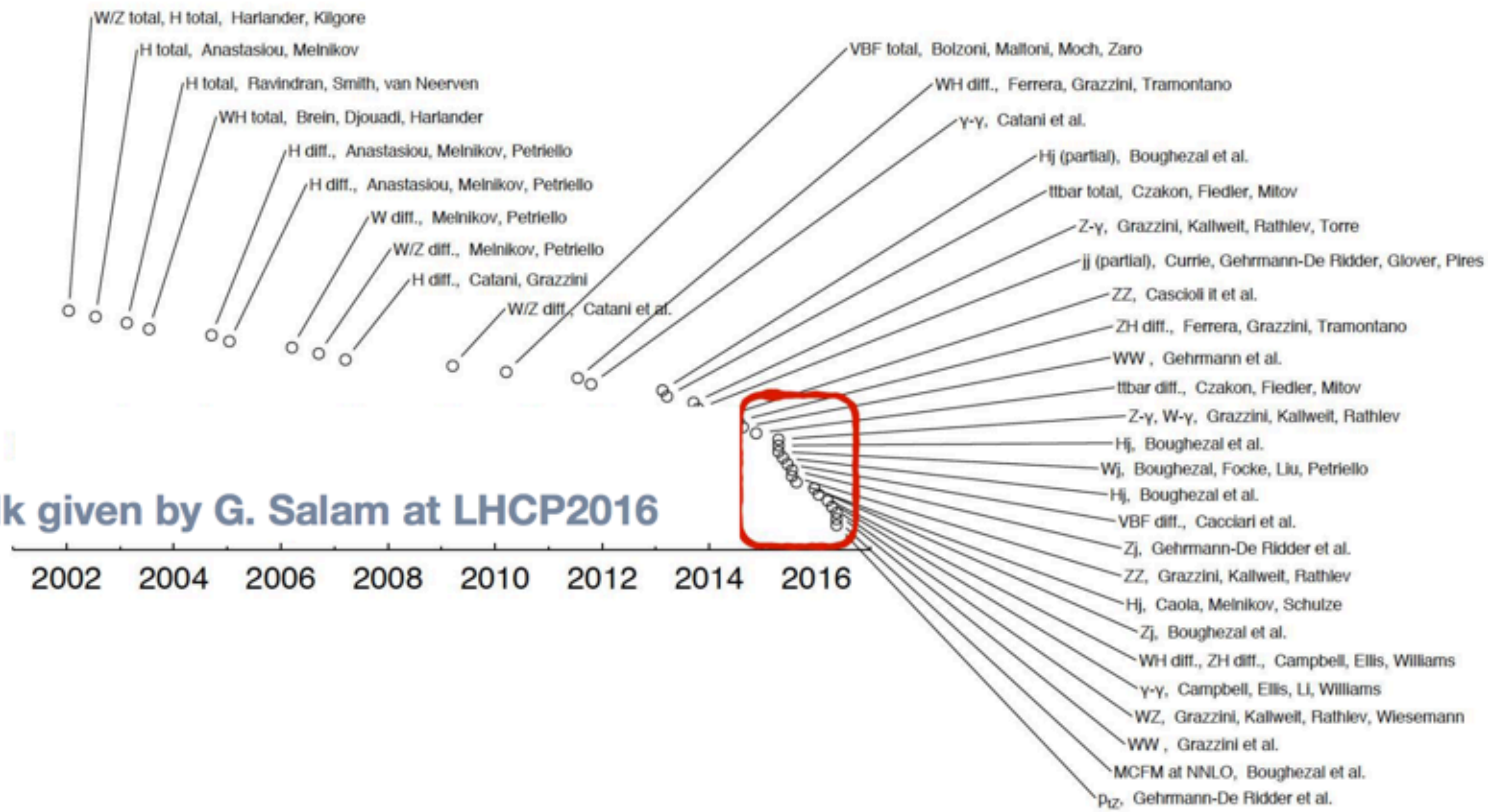


Here, only the renorm. scale $\mu(=\mu_R)$ has been varied. In real life you need to change renorm. and factorisation (μ_F) scales.

Scale dependence as the <u>theory uncertainty</u> or <u>Missing Higher Order Uncertainty (MHOU)</u>

Convention: "theory uncertainty" (i.e. from missing higher orders) is estimated by change of cross section when varying μ in range 1/2 → 2 around central value

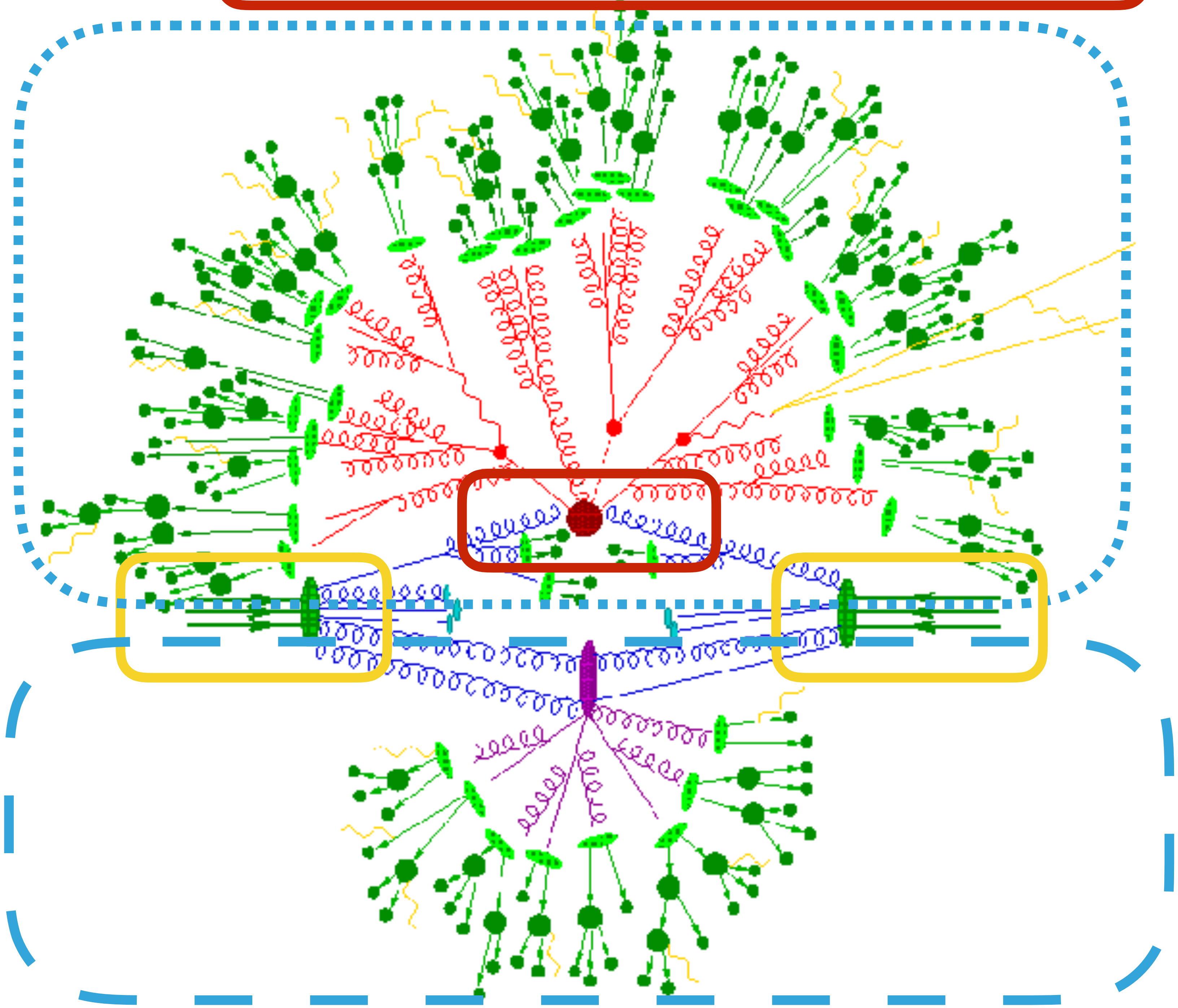
Partonic cross sections



- LO: almost all processes
- NLO: most processes (automated calculations)
- NNLO: all 2 → 1, most 2→2 (explosion of calculations in the past few years)
- N3LO: Higgs gluon fusion and Higgs via vector boson fusion

The precision ingredients

$$\sigma^{\rm th} = \hat{\sigma}[\mathcal{O}(1) + \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha_s^2) + \ldots] \otimes f_1 \otimes f_2 + \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$$



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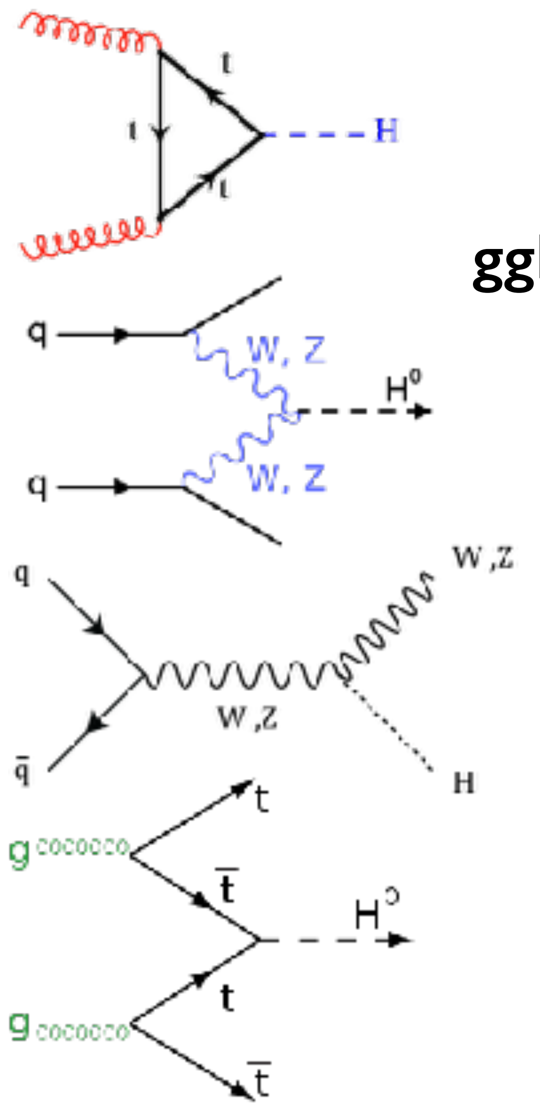
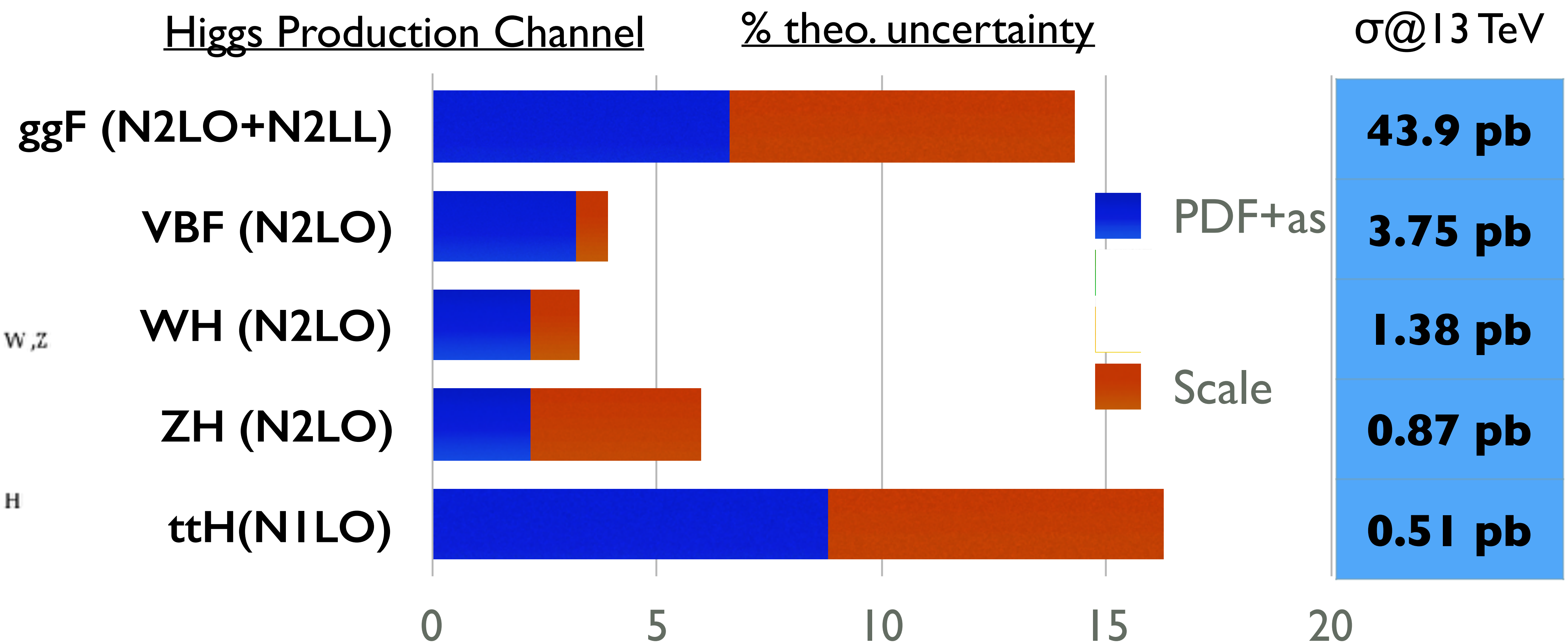
PDF uncertainties

Yellow Report 3 (2013)



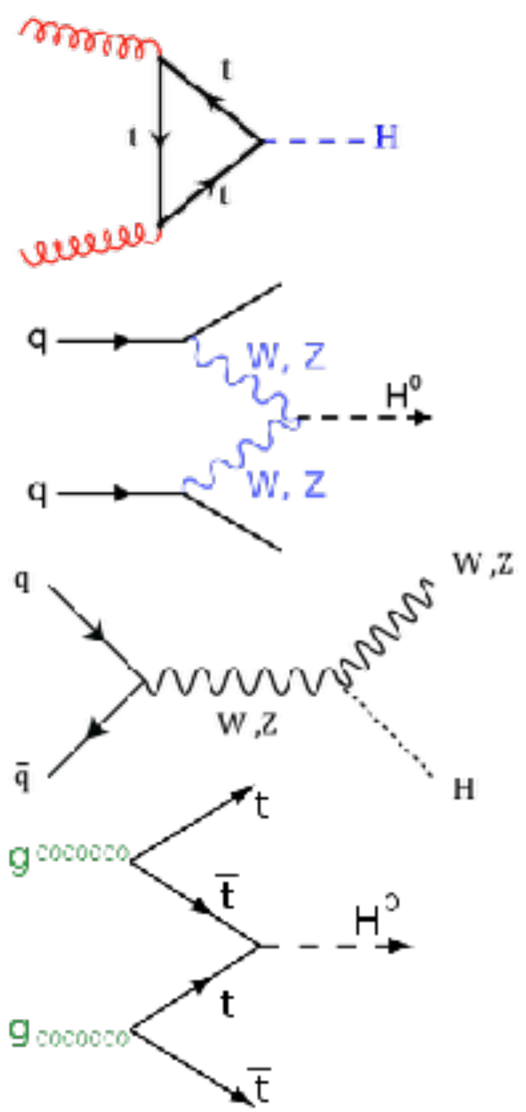


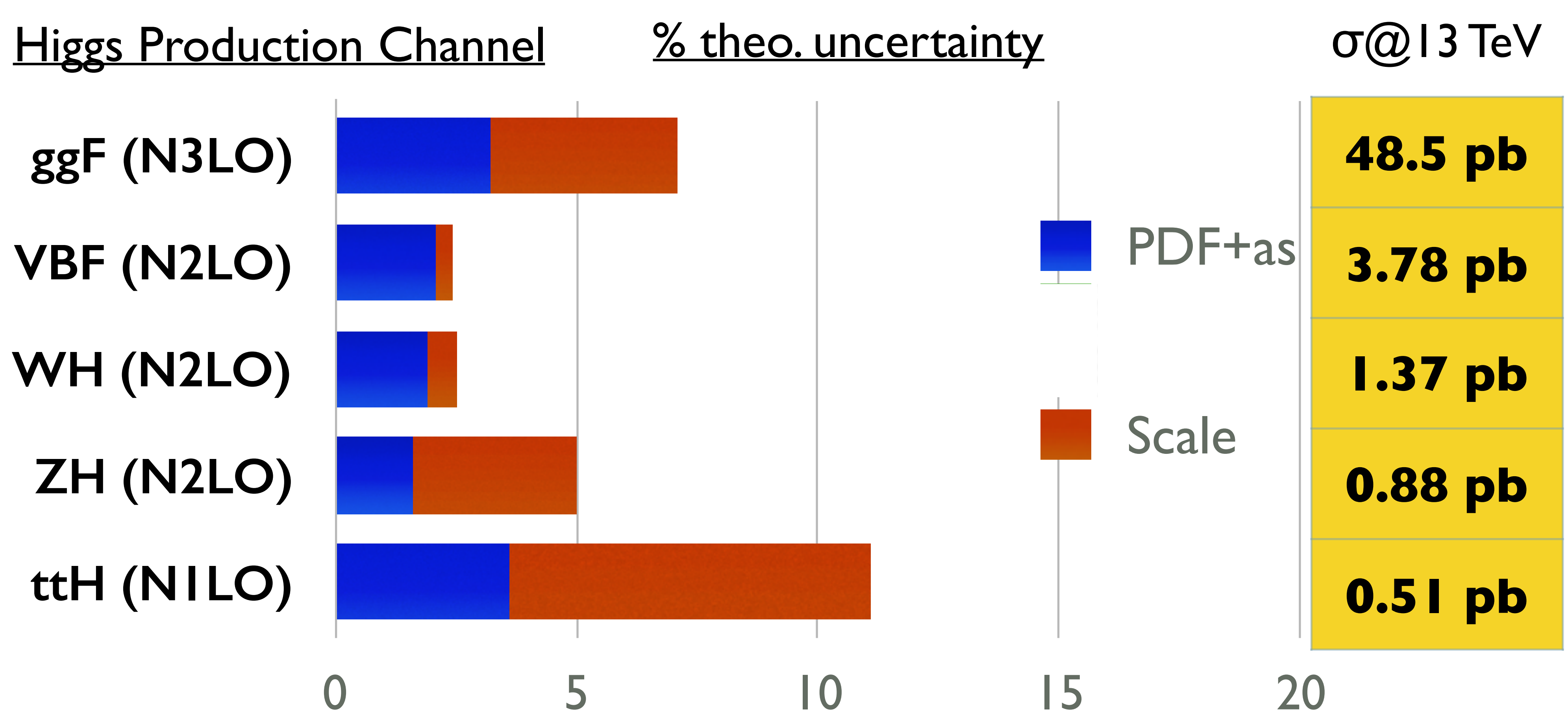
Higgs physics

PDF uncertainties limiting factor in the accuracy of theoretical predictions

PDF uncertainties

Yellow Report 4 (2016)

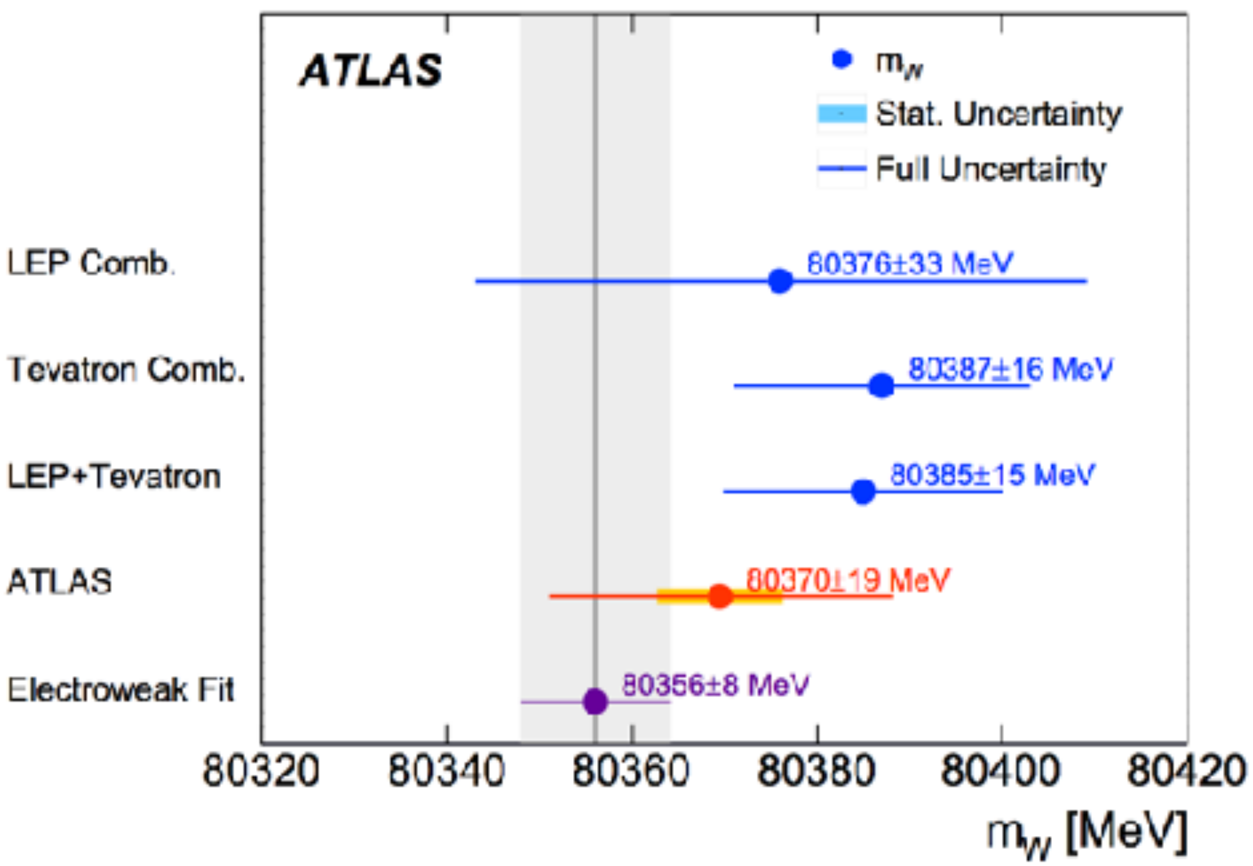


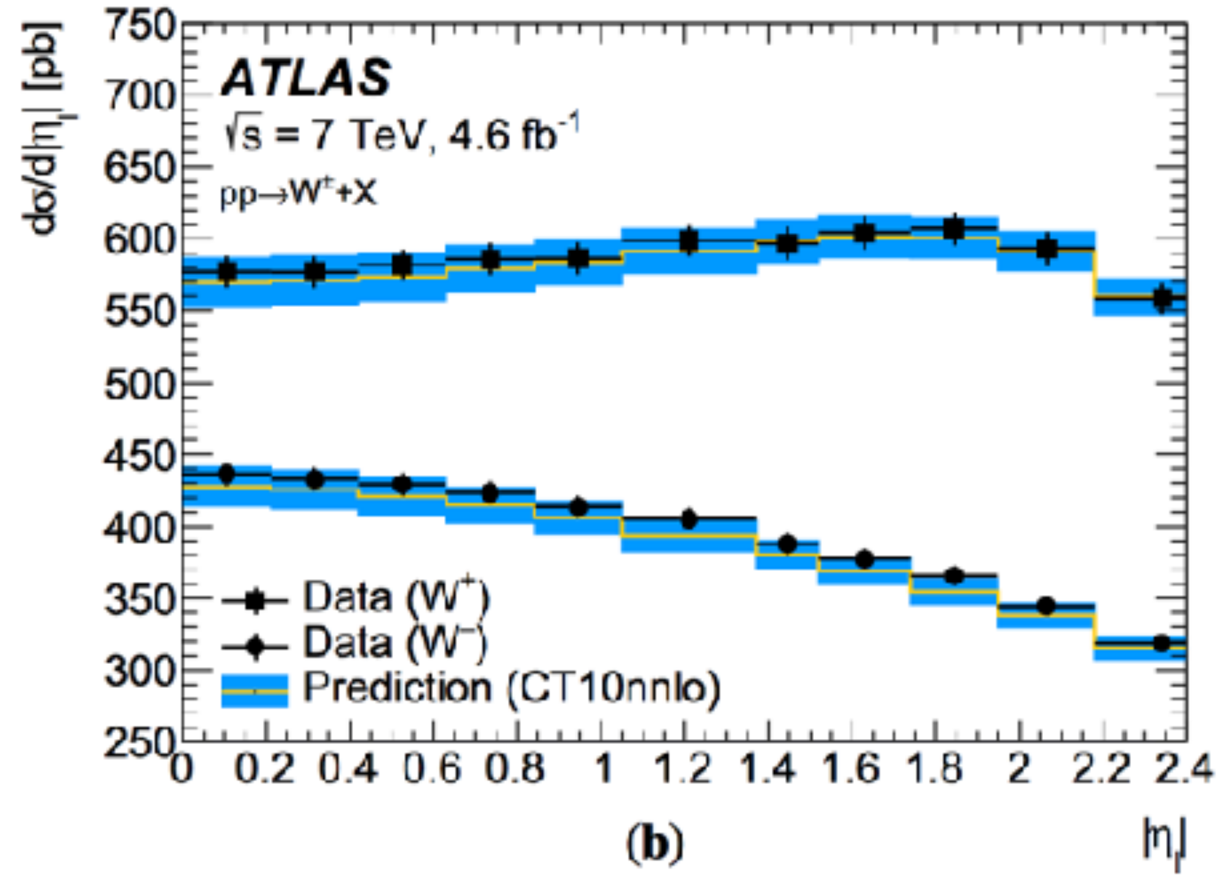


Reduced (still often dominant) PDF uncertainties

PDF uncertainties

Determination of SM parameters



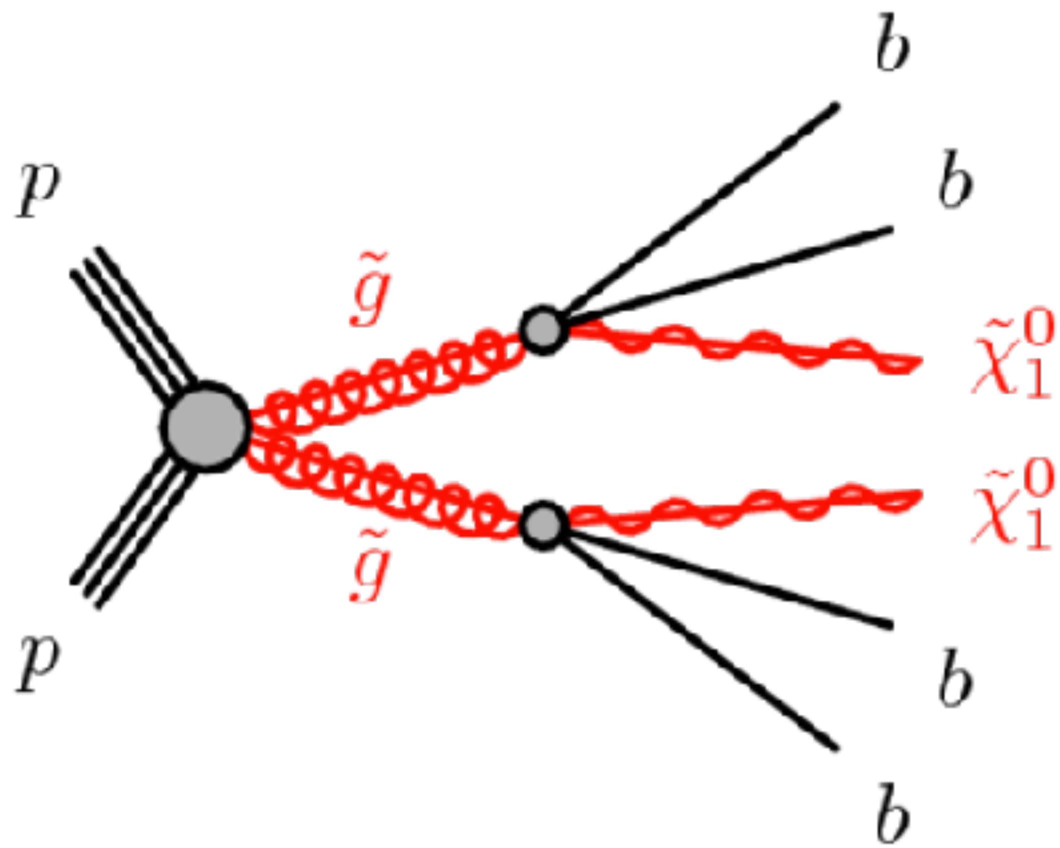


ATLAS collaboration, EPJC 78 (2018) 110

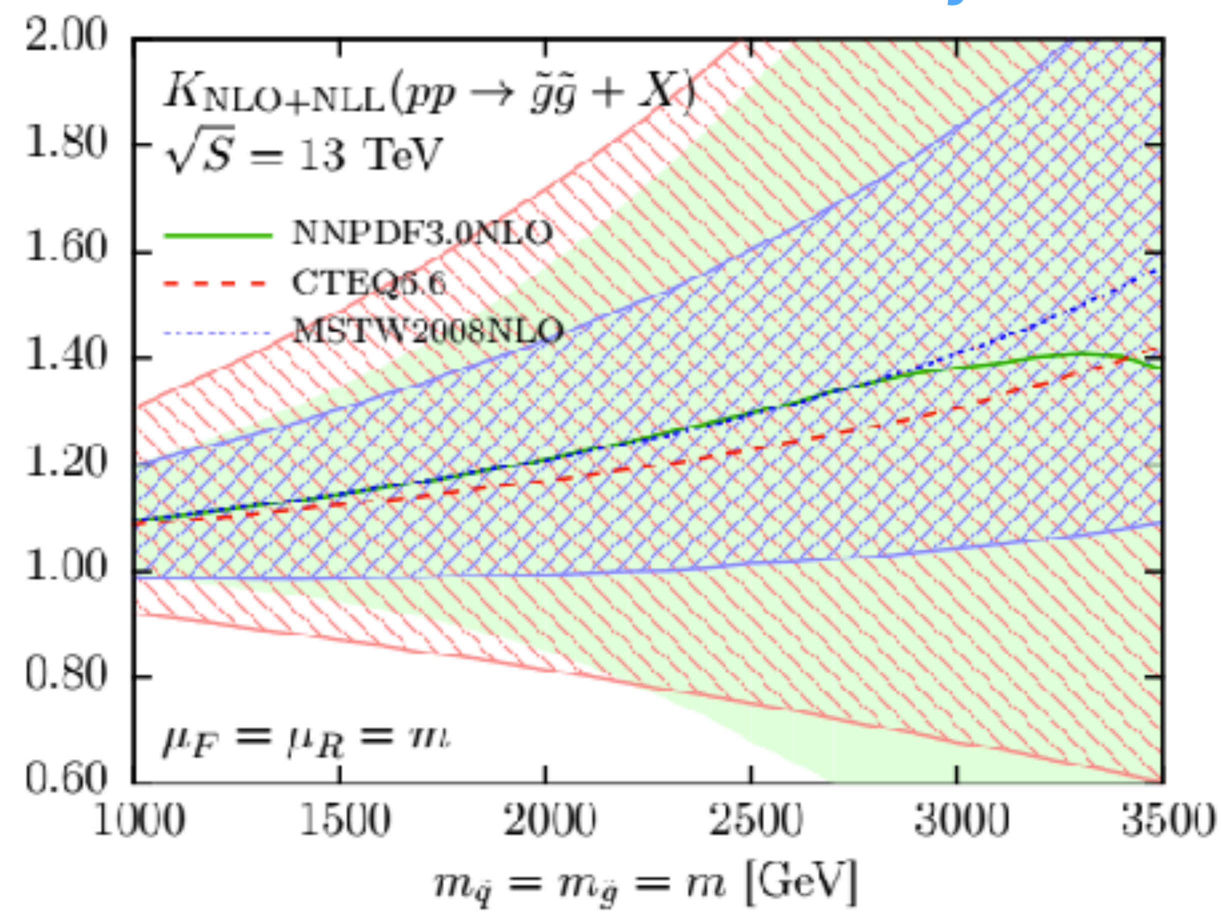
$$\eta = -\ln\tan(\theta/2)$$

Channel	$m_{W^+} - m_{W^-}$ [MeV]	Stat. Unc.	Muon Unc.	Elec. Unc.	Recoil Unc.	Bckg. Unc.	QCD Unc.	EW Unc.	PDF Unc.	Total Unc.
$W \to e\nu$	−29.7	17.5	0.0	4.9	0.9	5.4	0.5	0.0	24.1	30.7
$W \to \mu\nu$	−28.6	16.3	11.7	0.0	1.1	5.0	0.4	0.0	26.0	33.2
Combined	−29.2	12.8	3.3	4.1	1.0	4.5	0.4	0.0	23.9	28.0

PDF uncertainties



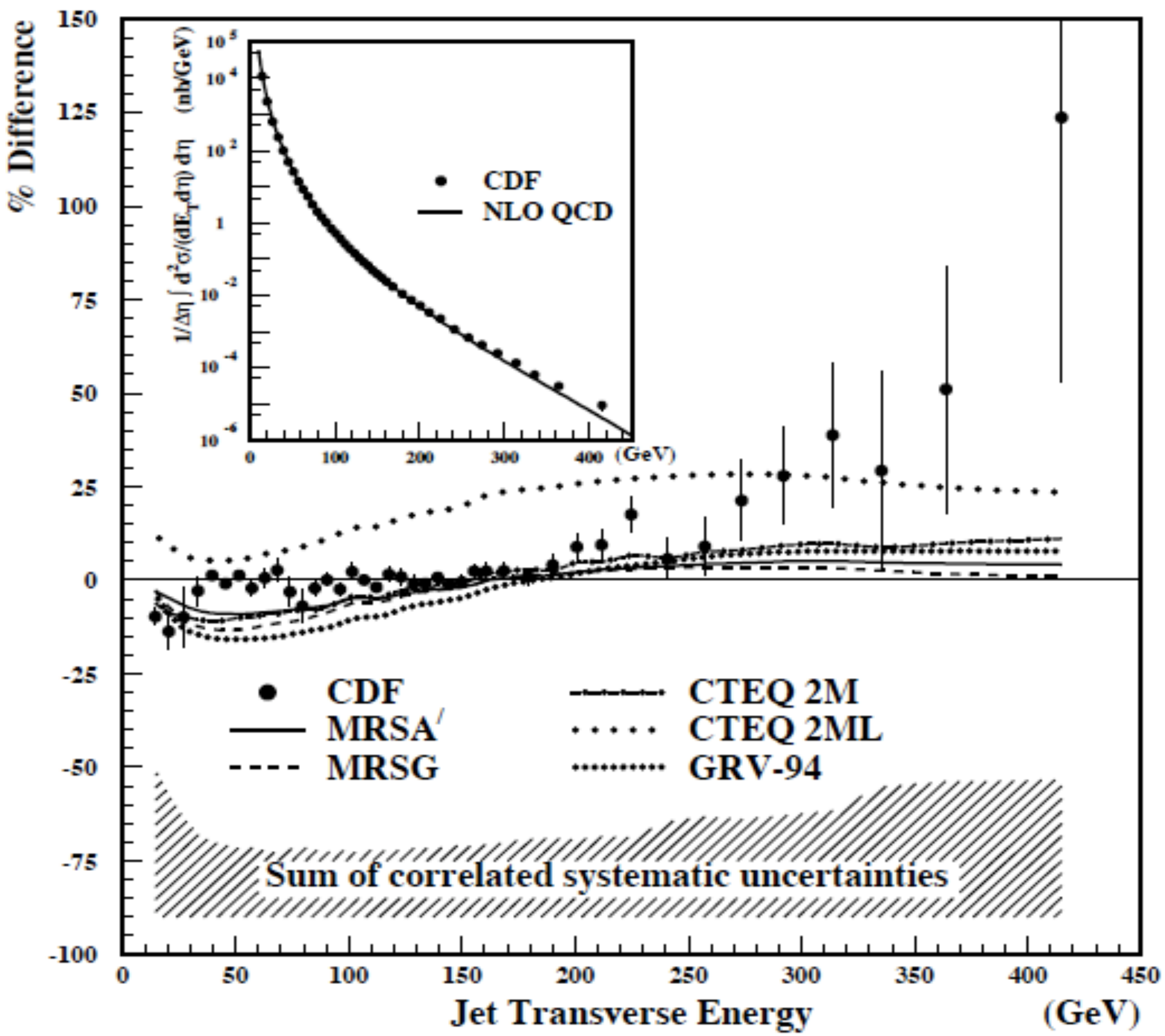
New Physics



Beenakker et al.
EPJC76 (2016)2, 53

PDF uncertainties are a limiting factor in the accuracy of theoretical predictions, both within **SM** and **beyond**

PDF uncertainties

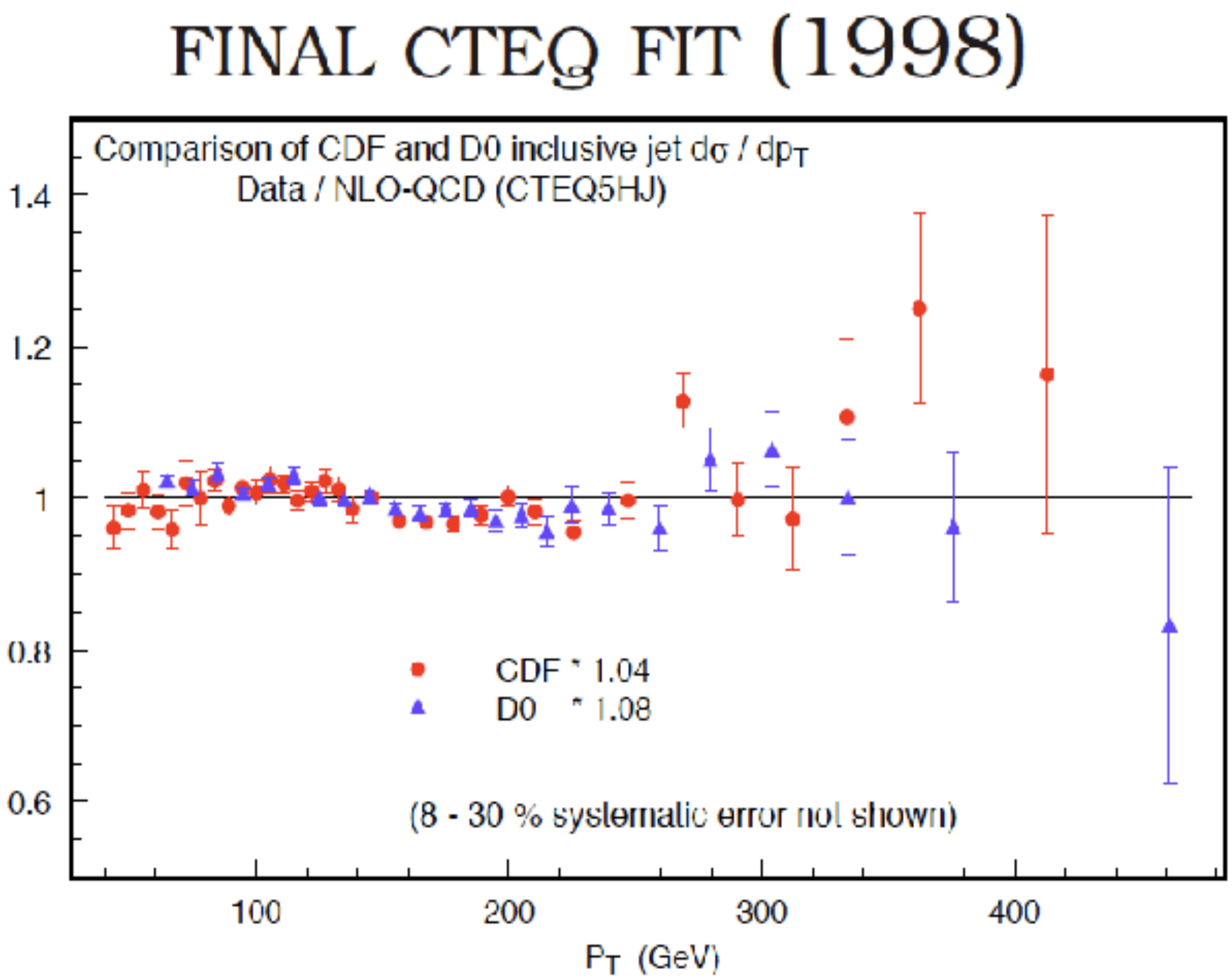


Historia magistra vitae est

Discrepancy between QCD calculations and CDF jet data (1995)

At that time there was no information on PDF uncertainties
and the theoretical prediction strongly depends on gluon shape at x>0.1

PDF uncertainties



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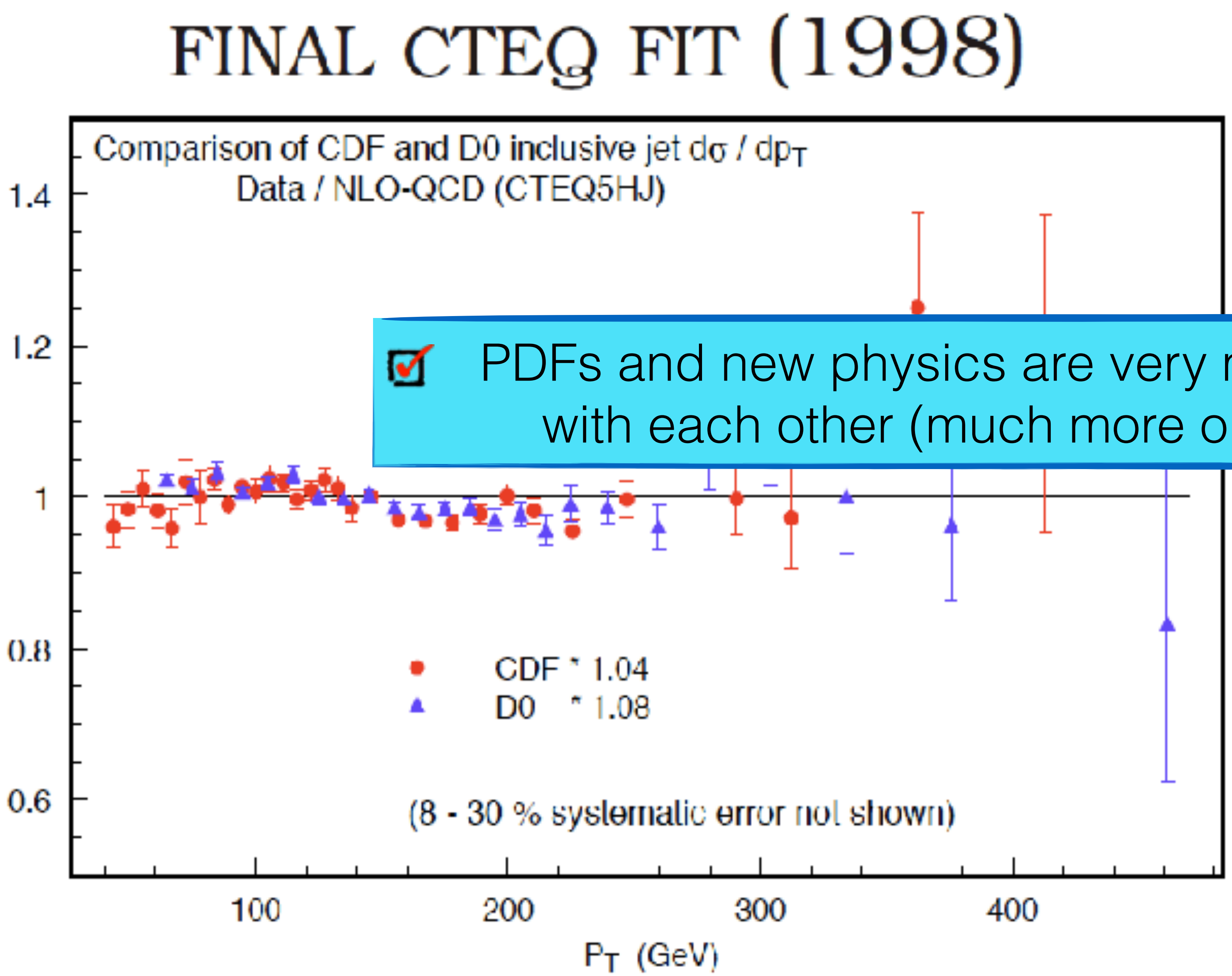
CTEQ re-performed the parton fit by including the jet data and the discrepancy was removed.

PDF uncertainties



☑ PDFs and new physics are very much related with each other (much more on part IV)

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The parton model and QCD

Historic overview

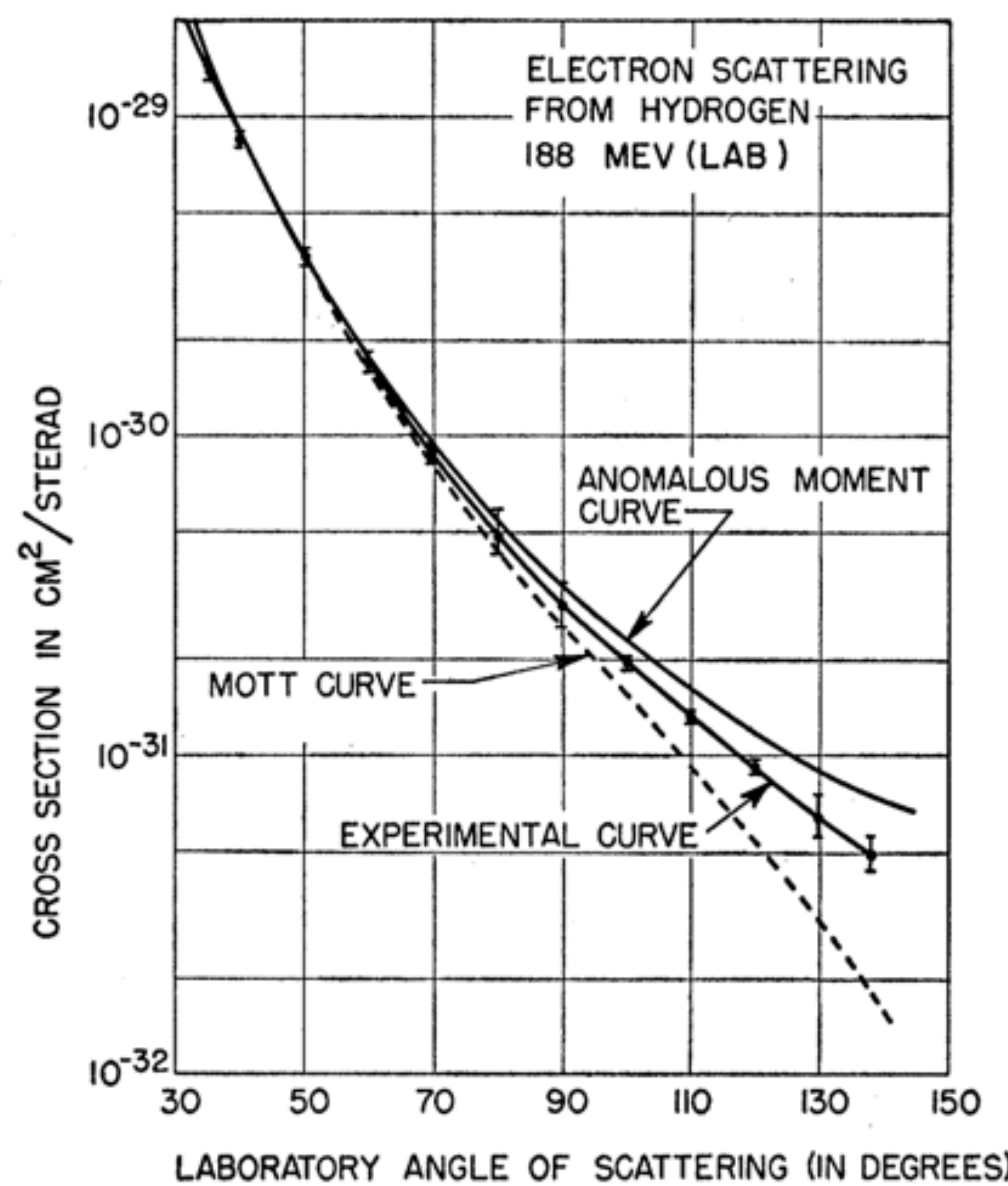
- **1955**: Hofstadter et al observed first deviations in scattering of electron off proton from simple point-like Mott scattering ➜ Finite radius of proton ~ 0.7 fm

Electron Scattering from the Proton*†‡

ROBERT HOFSTADTER AND ROBERT W. MCALLISTER

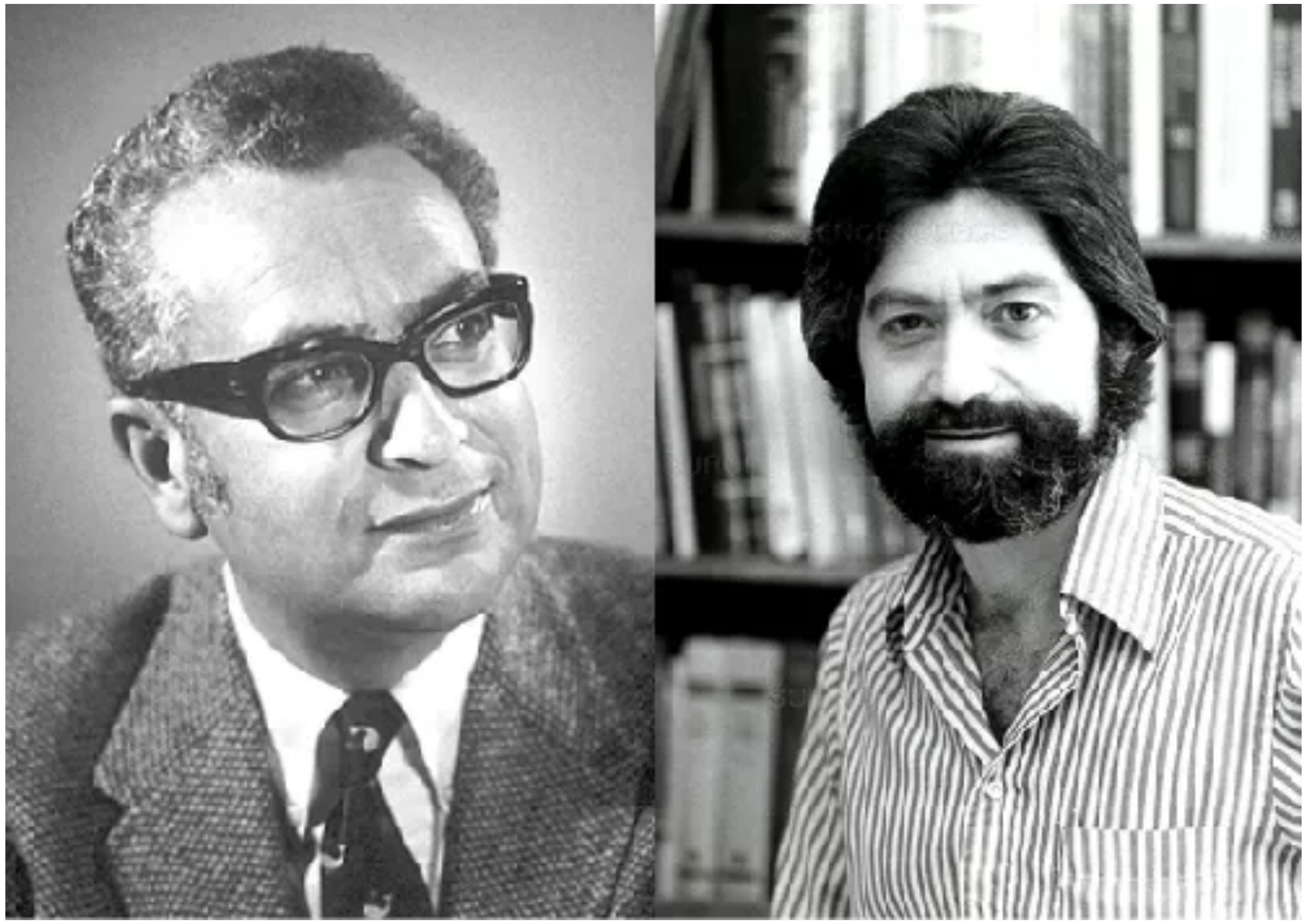
Department of Physics and High-Energy Physics Laboratory, Stanford University, Stanford, California

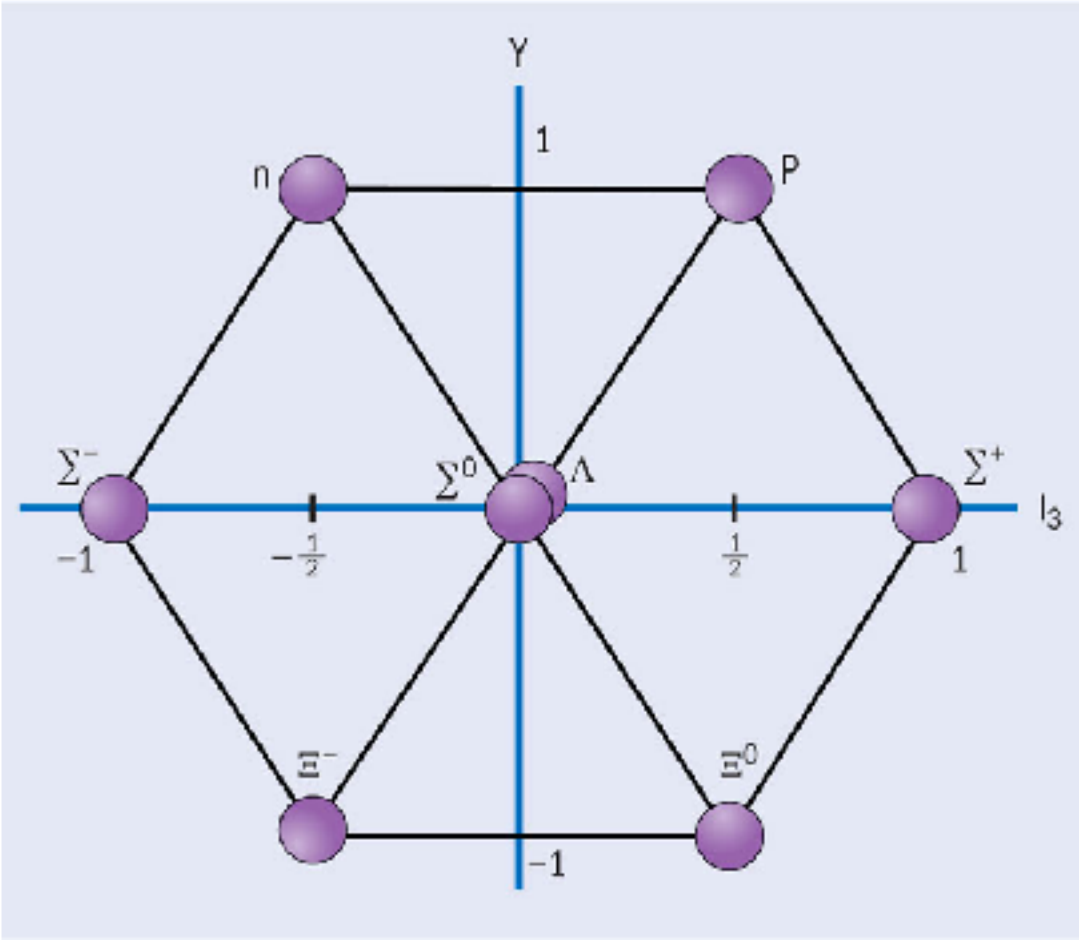
(Received January 24, 1955)



Historic overview

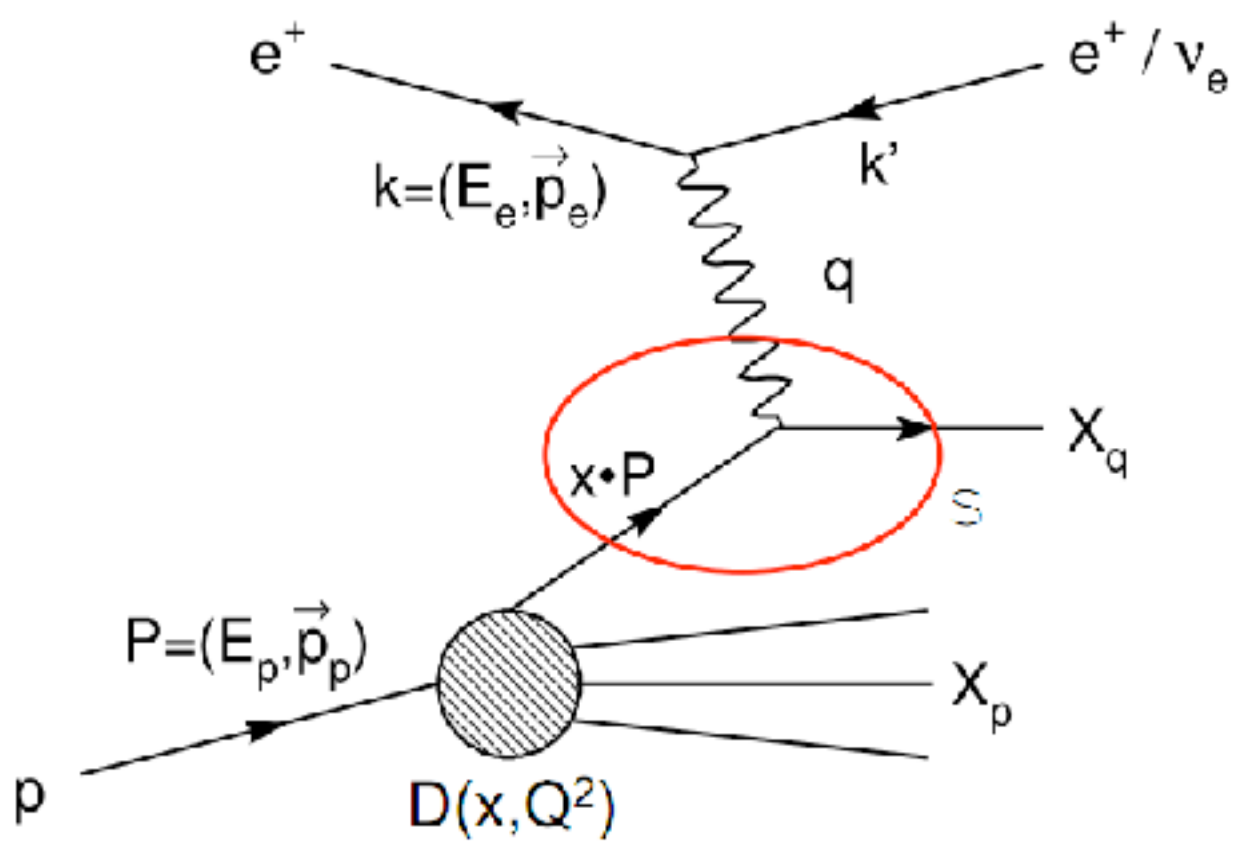
- **1964**: Zweig and Gell-Mann independently postulated existence of three aces (Zweig) or quarks (Gell-Mann) with fractional electric charge and spin–1/2 to explain proliferation of mesons and baryons in nucleon collision experiments. More of a mathematical model rather than particles! How could such objects be bound so tightly together?

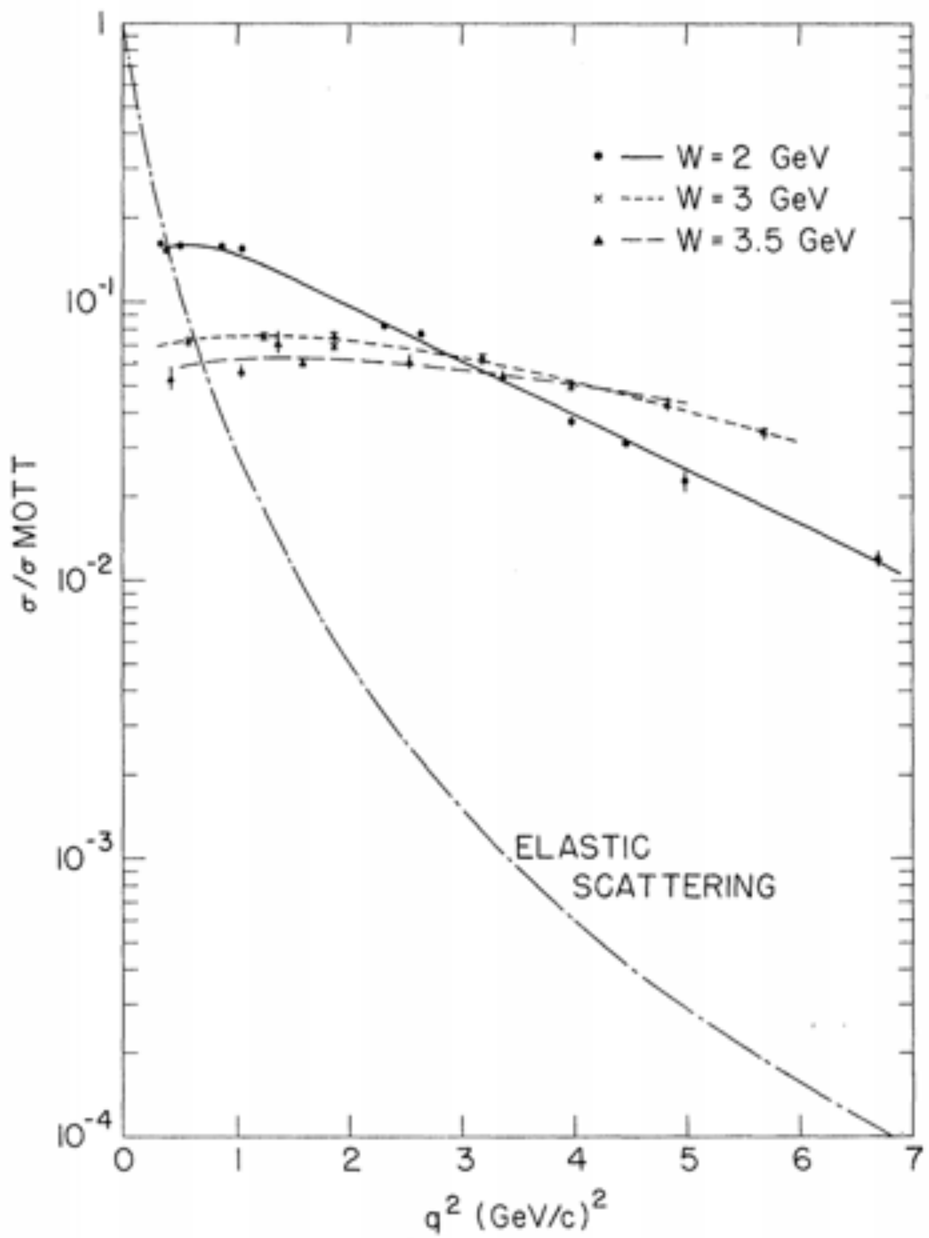




Historic overview

- **1967**: First deep-inelastic scattering experiments at SLAC 20 GeV linear accelerator gave first evidence of point-like elementary constituents which were later identified as quarks (Bjorken scaling)

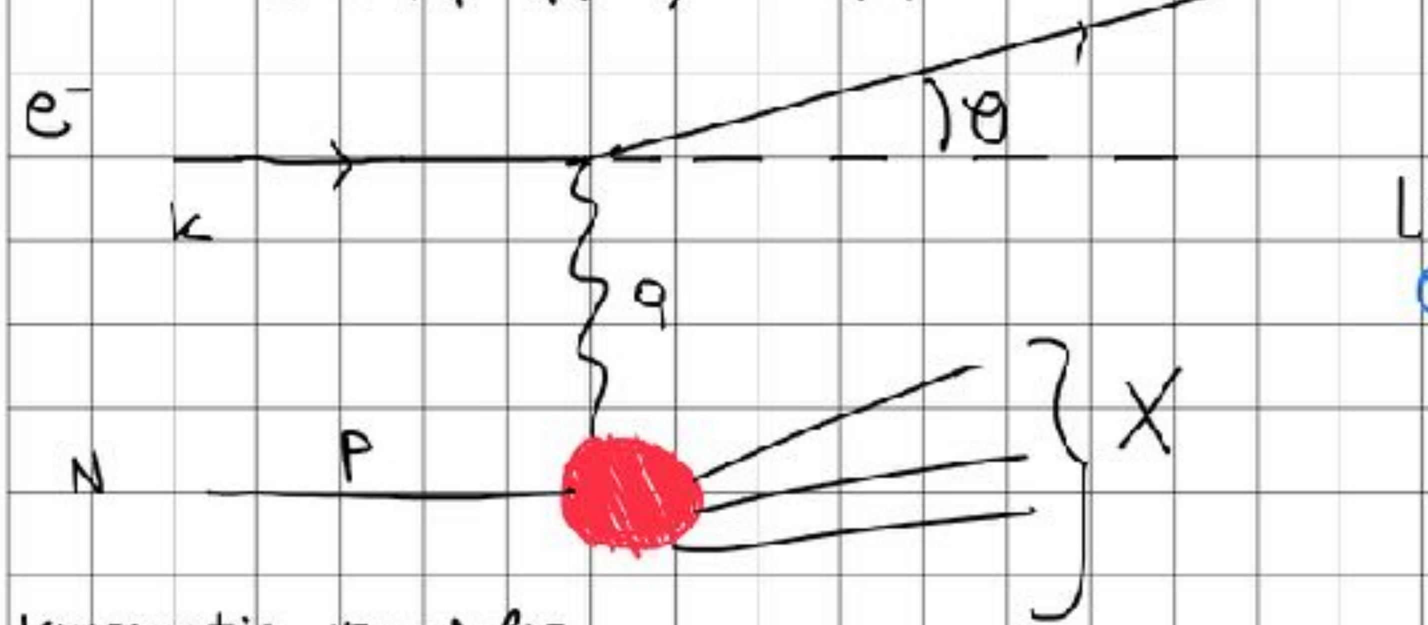




Deep Inelastic Scattering

Consider

$e^-(k) + P(p) \to e^-(k') + X$ ↳ generic hadron system



LO diagram in QED

Kinematic variables

centre-of-mass energy

$$s = (p+k)^2$$

$$q = k - k'$$

q^2 γ virtuality

For the time being consider a γ^* only because at SLAC
$s \ll M_W, M_Z$ ⇒ Z, W exchange suppressed

We will consider also W, Z when talking of HERA experiment

Deep Inelastic Scattering

Define Lorentz invariant quantities

in rest frame of p or c.o.m.

$$Q^2 = -q^2 = -(k-k')^2 = 2k\cdot k' = 2E_e E_e' (1-\cos\theta)$$

↓ ignore lepton mass

$$x_B = \frac{Q^2}{2p\cdot q}$$ Bjorken x, $x_B > 0$

$$y = \frac{p\cdot q}{p\cdot k} = \frac{E_e - E_e'}{E_e}$$ Inelasticity, $0 \leq y \leq 1$

$$M_X^2 = (p+q)^2 = \frac{Q^2(1-x)}{x}$$ Hadronic inv. mass, $x_B \leq 1$

Only two independent.

→ elastic limit $M_X^2 = M_p^2 \iff x_B = 1$ (only 1 indep.)

↓ <u>deep inelastic limit</u> $M_X^2 \gg M_p^2 \iff$ large Q^2 for fixed x

ignore proton mass $M_p \to 0$

Deep Inelastic Scattering

Formal description of scattering facilitated by "factorized" nature of the interaction. We can write the matrix element in terms of leptonic and hadronic currents as

$$\mathcal{M}(e^- p \to e^- X) = \langle e^- | J^\mu | e^- \rangle \, g_{e\gamma} \, \frac{-g_{\mu\nu}}{q^2} \, g_{h\gamma} \langle X | J^\nu | h \rangle$$

$\langle e^- | J^\mu | e^- \rangle$ ↳ QED current

$g_{e\gamma}$ ↳ e

$g_{h\gamma}$ ↳ coupling $h\gamma^*$

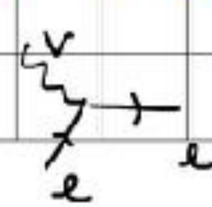
Let's generalise it to exchange of V boson γ^*, Z^*, W^* for any incoming lepton ℓ

$$\mathcal{M}(\ell p \to \ell' X) = \langle \ell | J^\mu | \ell' \rangle \, g_{\ell V} \, \frac{-g_{\mu\nu}}{q^2 - M_V^2} \, g_{hV} \langle X | J^\nu | h \rangle$$

$M_V \to$ mass of exchanged boson

$g_{\ell V}$, $g_{hV} \to$ couplings

$$g_{\ell V} = k_V \sqrt{v_{\ell V}^2 + a_{\ell V}^2}$$



$$- i e k_V \gamma_\mu (v_{\ell V} + \gamma_5 a_{\ell V})$$

Deep Inelastic Scattering

Bosons	k_V	V_{lV}	A_{lV}
γ	e_l	1	0
Z	$1/(2\sin\theta_W \cos\theta_W)$	$I_3^l - 2e_l \sin^2\theta_W$	$-I_3^l$
$W^\pm$	$V_{ll'}/(2\sqrt{2}\sin\theta_W)$	1	-1

Table 1.1: Coupling of fermions to the weak bosons. Here e_l is the electric charge measured in unit of the positron charge, I_3^l is the third component of the weak isospin, $+1/2$ for up–type quarks or neutrinos and $-1/2$ for down–type quarks or charged leptons. For charged current interactions involving quarks, the coefficients $V_{ll'}$ of the Cabibbo–Kobayashi–Maskawa matrix [18]-[19] are involved. The parameter $\sin\theta_W$ is the Weinberg mixing angle.

Deep Inelastic Scattering

The differential DIS cross section can be written as

$$d\sigma(ep \to e'X) = \frac{1}{2s} \frac{(g_{eV}\, g_{hV})^2}{(Q^2 + M_V^2)^2} L_{\mu\nu} W^{\mu\nu} (4\pi) \frac{d^3k'}{(2\pi)^3 2E'}$$

$L_{\mu\nu}$ → ② leptonic tensor

$W^{\mu\nu}$ → ③ hadronic tensor

$\frac{d^3k'}{(2\pi)^3 2E'}$ → ① final lepton phase space

① Phase space

$$\frac{d^3k'}{(2\pi)^3 2E'} = \frac{E'\, dE'\, d\cos\theta}{8\pi^2}$$

integrate over azimuthal angle φ

We can write it in terms of invariant

$$Q^2 = 2EE'(1 - \cos\theta) \Rightarrow dQ^2 = 2EE'\, d\cos\theta$$

$$2p\cdot q = \frac{s}{2E}\left(2E - E'(1 + \cos\theta)\right) = \frac{s}{2E}\left[2(E - E') + \frac{Q^2}{2E}\right]$$

$$\Rightarrow dx_B = \frac{x_B^2}{Q^2} \frac{s}{E} dE'$$

for fixed Q^2

Deep Inelastic Scattering

$$\Rightarrow \quad dx_B \, dQ^2 = \frac{2 s x_B^2}{Q^2} E' dE' d\cos\theta \quad \rightarrow$$

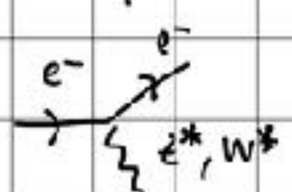
$$\frac{E' dE' d\cos\theta}{s} = \frac{Q^2}{2 s^2 x_B^2} dx_B \, dQ^2 = \frac{y^2}{2 Q^2} dx_B \, dQ^2$$

(2) Leptonic tensor

$$L_{\mu\nu} = \frac{1}{2} \langle l | J^{\dagger}_{\mu} | l' \rangle \langle l' | J_{\nu} | l \rangle$$

$$\bar{u}(k') (-ie\gamma_\mu) u(k)$$

$$\bar{v}(k) (-ie\gamma_\mu) v(k)$$



$$\bar{u}(k') \left[-ie k_v \gamma_\mu (v_{ev} + a_{ev} \gamma_5) \right] u(k)$$

$$L_{\mu\nu} \propto Tr[\not{k} \Gamma_\nu \not{k}' \Gamma_\mu] \quad (e^- \text{ or } \nu), \qquad Tr[\not{k}' \Gamma_\nu \not{k} \Gamma_\mu] \quad (e^+ \text{ or } \bar{\nu})$$

Deep Inelastic Scattering

$$\Rightarrow \quad L_{\mu\nu} = 2\left[k_\mu k'_\nu + k_\nu k'_\mu - \frac{Q^2}{2} g_{\mu\nu} \pm i\, c_{\ell v}\, \varepsilon_{\mu\nu}{}^{\rho\sigma} k_\rho k'_\sigma\right]$$

$$c_{\ell v} = \frac{2 a_{\ell v} v_{\ell v}}{v_{\ell v}^2 + a_{\ell v}^2} \qquad \hookrightarrow \pm \; \begin{matrix} e^- & \nu \\ e^+ & \bar{\nu} \end{matrix}$$

(3) Hadronic Tensor

$$W^{\mu\nu} = \frac{1}{2} \frac{1}{4\pi e^2} \sum_X \langle p | J^\mu | X \rangle \langle X | J^\nu | p \rangle \; \delta^{(4)}(p + q - P_X)$$

General form (using p^μ q^μ 4-vectors and $g^{\mu\nu}$ $\varepsilon^{\mu\nu\alpha\beta}$ isotropic tensors)

$$W^{\mu\nu} = -g_{\mu\nu} F_1 + \frac{1}{(p\cdot q)}\left[p^\mu p^\nu F_2 + i\, \varepsilon^{\mu\nu}{}_{\alpha\beta}\, p^\alpha p^\beta F_3 + (F_4 + i F_5) p^\mu q^\nu + (F_4 - i F_5) p^\nu q^\mu + F_6 q^\mu q^\nu\right]$$

with F_i dimensionless functions of x, Q^2

- Time-reversal $\Rightarrow F_5 = 0$
- Conservation of EM current $q^\mu W^{\mu\nu} = W^{\mu\nu} q_\mu = 0$

$$\Rightarrow W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x, Q^2) + \frac{\tilde{p}_\mu \tilde{p}_\nu}{(p\cdot q)} F_2(x, Q^2) + \frac{i \varepsilon_{\mu\nu\alpha\beta}\, p^\alpha q^\beta}{2(p\cdot q)} F_3(x, Q^2)$$

Deep Inelastic Scattering

$$\tilde{p}_\mu = p_\mu - \frac{(p\cdot q)}{q^2} q_\mu$$

$F_{1,2,3}$ → structure functions to be determined experimentally

If we require parity conservation (QED) ⇒ $F_3 = 0$
But if Z is exchanged $F_3 \neq 0$

$$L_{\mu\nu} W^{\mu\nu} = 2Q^2 F_1 + \frac{4(p\cdot k)(p\cdot k')}{(p\cdot q)} F_2 \mp C_{\ell\nu} \frac{F_3}{(p\cdot q)} p\cdot(k+k') Q^2$$

$$= \frac{2Q^2}{y^2} \left[y^2 F_1 + (1-y) F_2 \mp C_{\ell\nu}\, x \left(y - \frac{y^2}{2}\right) F_3 \right]$$

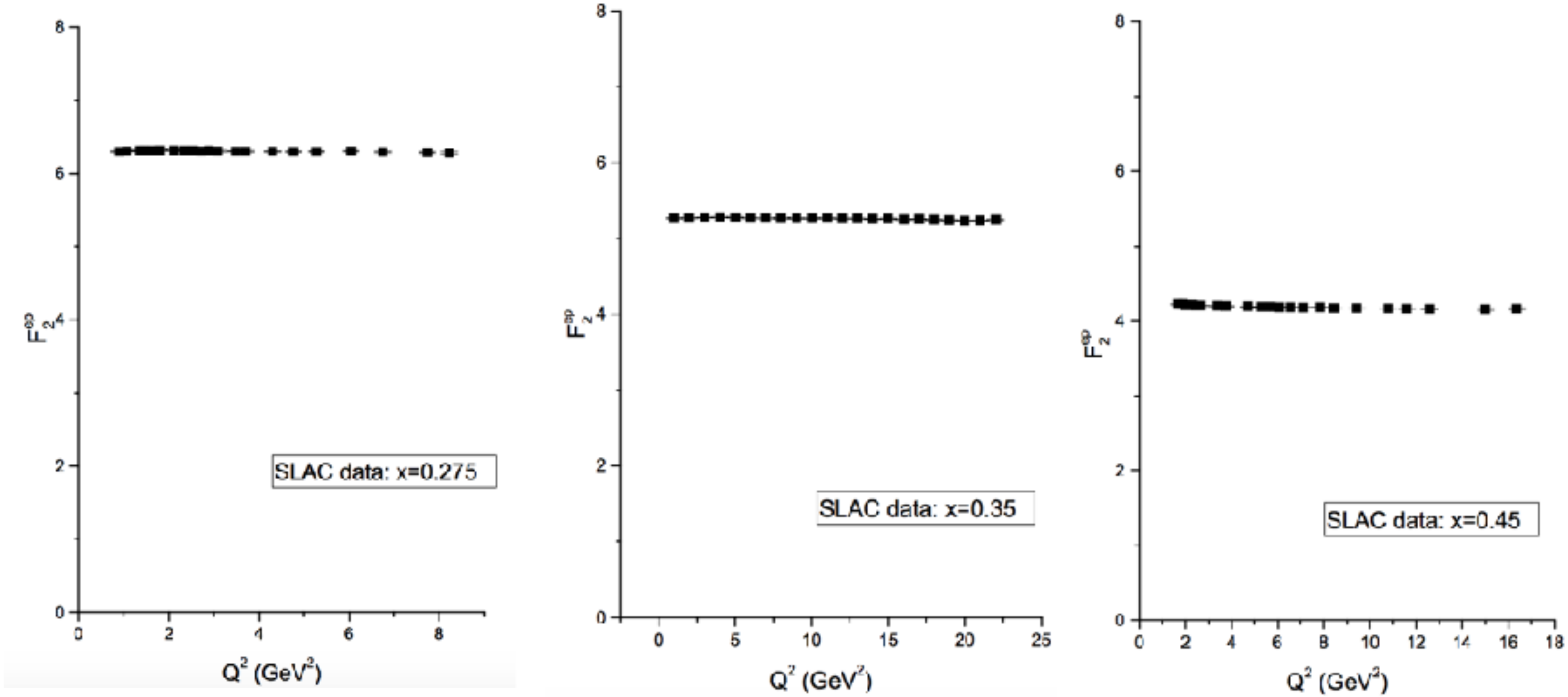
$$\frac{d\sigma}{dx\,dQ^2} = \frac{4\pi}{x} \frac{\alpha_{\ell\nu}\,\alpha_{h\nu}}{(Q^2+M_V^2)^2} \left[x y^2 F_1 + (1-y) F_2 \mp C_{\ell\nu}\, x \left(y - \frac{y^2}{2}\right) F_3 \right]$$

↓ ↓ ↓

can be determined experimentally

Deep Inelastic Scattering

The surprising results at SLAC was that F1,2 did not vanish as Q2 increased, rather they remained finite and constant and depended only on xB - Bjorken scaling (1969)



Such scaling demonstrated that the exchanged vector boson (photon) scatters off point-like objects that have no mass or scale associated. The lepton scatters off charged spin 1/2 constituents (partons) that carry a fraction x of proton momentum

The Parton Model

Basic assumption

z: fraction of proton momentum carried by parton i

$$\ast \quad d\sigma(P) = \sum_{i \in \text{partons}} \int_0^1 dz \underbrace{f_i(z)}_{\text{distribution function}} d\hat{\sigma}_i(zP)$$

Scattering off massless spin 1/2 parton i with charge e_i (in units of proton charge) and momentum $\hat{p} = zP$

$$Q(\hat{p}) + \gamma^*(q) \rightarrow Q(\hat{p}') \qquad \text{(consider } \gamma^* \text{ exchange)}$$

$$\frac{1}{2}\sum |\mathcal{M}|^2 = \frac{1}{2} e^2 \underbrace{Tr[\hat{\not{p}}\gamma^\nu \hat{\not{p}}' \gamma^\mu]}$$

$$4\left[\hat{p}^\nu \hat{p}'^\mu + \hat{p}^\mu \hat{p}'^\nu - g^{\mu\nu}\, \hat{p}\cdot\hat{p}'\right] \qquad \hat{p}' = \hat{p} + q$$

$$\Rightarrow \hat{W}^i_{\mu\nu}(\hat{p}, q) = \frac{e_i^2}{4\pi}\frac{1}{2}\int \frac{d^3\hat{p}'}{(2\pi)^3 2\hat{p}'_0}(2\pi)^4 \delta^{(4)}(\hat{p} + q - \hat{p}')\sum_{\text{spin}} \bar{u}(\hat{p}')\gamma_\mu u(\hat{p})\, \bar{u}(\hat{p})\gamma_\nu u(\hat{p}')$$

The Parton Model

$$= e_i^2 \int d^4\hat{p}' \, \delta(\hat{p}'^2) \, \delta^{(4)}(\hat{p} + q - \hat{p}') \left(\hat{p}_\mu \hat{p}'_\nu + \hat{p}_\nu \hat{p}'_\mu - g_{\mu\nu} \hat{p}\cdot\hat{p}'\right)$$

$$= e_i^2 \, \delta\left((\hat{p}+q)^2\right) \left[\hat{p}_\mu (\hat{p}+q)_\nu + \hat{p}_\nu (\hat{p}+q)_\mu - g_{\mu\nu} \hat{p}\cdot(\hat{p}+q)\right]$$

$$= e_i^2 \, \delta(2\hat{p}\cdot q + q^2) \Big[2\hat{p}_\mu \hat{p}_\nu - 2\frac{\hat{p}\cdot q}{q^2}(\hat{p}_\mu q_\nu + \hat{p}_\nu q_\mu) + 2\left(\frac{\hat{p}\cdot q}{q^2}\right)^2 q_\mu q_\nu - 2\left(\frac{\hat{p}\cdot q}{q^2}\right)^2 q_\mu q_\nu - g_{\mu\nu} \hat{p}\cdot q \Big]$$

$$= \frac{e_i^2}{2\hat{p}\cdot q} \delta\left(\frac{2\hat{p}\cdot q}{2\hat{p}\cdot q} - \frac{Q^2}{2\hat{p}\cdot q}\right) \left[2\left(\hat{p}_\mu - \frac{\hat{p}\cdot q}{q^2} q_\mu\right)\left(\hat{p}_\nu - \frac{\hat{p}\cdot q}{q^2} q_\nu\right) + (\hat{p}\cdot q)\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\right]$$

$$= e^2 \, \delta(1-\hat{x}) \left[\frac{1}{2}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) + \frac{1}{(\hat{p}\cdot q)}\left(\hat{p}_\mu - \frac{\hat{p}\cdot q}{q^2} q_\mu\right)\left(\hat{p}_\nu - \frac{\hat{p}\cdot q}{q^2} q_\nu\right)\right]$$

with $\hat{x} = \dfrac{Q^2}{2\hat{p}\cdot q}$

The Parton Model

Applying (*) to DIS cross section we get

$$\frac{4\pi\alpha^2 L_{\mu\nu} W^{\mu\nu}(P,q)}{Q^4} \frac{y^2}{2Q^2} dQ^2 dx = \frac{4\pi\alpha^2}{Q^4} L_{\mu\nu} \frac{y^2}{2Q^2} dQ^2 \sum_i \int_0^1 dz\, f_i(z)\, \hat{W}^{\mu\rho}(zP,q)\, d\hat{x}$$

Keeping into account that $\hat{x} = x_B/z$, we get

$$W_{\mu\nu}(P,q) = \sum_{i \in \text{partons}} \int_0^1 dz\, f_i(z) \frac{1}{z} \hat{W}^i_{\mu\nu}(zP,q)$$

Asking $(zP+q)^2 \geq 0 \Rightarrow z \geq x_B$

$$\Rightarrow W_{\mu\nu}(P,q) = \sum_i \int_{x_B}^1 \frac{dz}{z} e_i^2 f_i \left[\frac{1}{2}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) + \frac{z}{P\cdot q}\left(P_\mu - \frac{P\cdot q}{q^2} q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2} q_\nu\right)\right] \delta(1 - x_B/z)$$

$\frac{dz}{z} \delta(1-x_B/z) \rightarrow dz\, \delta(z - x_B)$

$$\Rightarrow W_{\mu\nu}(P,q) = \sum_i e_i^2 f_i(x) \left[\frac{1}{2}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) + \frac{x}{P\cdot q}\left(P_\mu - \frac{P\cdot q}{q^2} q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2} q_\nu\right)\right]$$

The Parton Model

Compared to general formula we get

$$F_1(x, Q^2) = \frac{1}{2} \sum_i e_i^2 f_i(x)$$

$$F_2(x, Q^2) = \sum_i e_i^2 \, x \, f_i(x)$$

Explicit scaling! Also note that in PARTON MODEL

$$F_2(x) = 2x \, F_1(X)$$ Callan – Gross Relation typical of spin 1/2

Indeed F_1 → absorption of transversally polarized virtual photon

$F_L = F_2 - 2xF_1$ → " " longitudinally polarized " "

$F_L \ll F_1$ confirms spin 1/2 of partons

proof $\varepsilon^\mu \varepsilon^{*\nu} W_{\mu\nu}(p,q) \propto \sigma(p + \gamma^* \to X)$

Take longitudinally polarized photon

The Parton Model

- long. polarized γ

$$\varepsilon = \varepsilon_L = \lambda \left(p - \frac{p \cdot q}{q^2} q \right) \qquad |\lambda|^2 = \frac{q^2}{(p \cdot q)^2}$$

such that

$$q \cdot \varepsilon_L = 0$$
$$\varepsilon_L \cdot \varepsilon_L^* = -1$$

$$\sigma_L \propto \varepsilon_L^{\mu} \varepsilon_L^{*\nu} W_{\mu\nu}(p, q) = F_1(x, Q^2) - \frac{F_2(x, Q^2)}{2x}$$

- transv. polarized γ

$$p \cdot \varepsilon_T = 0$$
$$q \cdot \varepsilon_T = 0$$
$$\varepsilon_T \cdot \varepsilon_T^* = -1$$

$$\sigma_T \propto \varepsilon_T^{\mu} \varepsilon_T^{*\nu} W_{\mu\nu}(p, q) = F_1(x, Q^2)$$

$$\Rightarrow \frac{\sigma_L}{\sigma_T} = 1 - \frac{F_2(x, Q^2)}{2x F_1(x, Q^2)} \rightarrow = 1 \text{ parton model} \Rightarrow \sigma_L = 0$$

Note that it would be $\neq 0$ if partons were spin 0.

Exercise I: Z contribution

- Show that, in the Parton model, considering also the contribution of a virtual Z boson and its interference with the photon one obtains:

$$F_2^{\gamma,Z}(x) = x\sum_{i=1}^{n_f} c_i[q_i(x) + \bar{q}_i(x)]$$

$$F_3^{\gamma,Z}(x) = \sum_{i=1}^{n_f} d_i[q_i(x) - \bar{q}_i(x)]$$

Where

$$c_i = e_i^2 - 2e_i V_{eZ} V_{iZ} P_Z + (V_{eZ}^2 + A_{eZ}^2)(V_{iZ}^2 + A_{iZ}^2)P_Z^2$$

$$d_i = -2e_i A_{eZ} A_{iZ} P_Z + 4V_{eZ} A_{eZ} V_{iZ} A_{iZ} P_Z^2$$

$$P_Z = \frac{Q^2}{(Q^2 + M_Z^2)(4s_w^2 c_w^2)}$$

→

$$c_w = \cos\theta_w$$

$$s_w = \sin\theta_w$$

Exercise II: Paschos-Wolfenstein relation

- Show that, in the Parton model, considering a (anti)neutrino-initiated DIS process on a deuteron target – assuming SU(2) isospin symmetry $u_n(x)=d_p(x)$ and $d_n(x) = u_p(x)$ – the ratio R

$$R = \frac{\sigma_{\rm NC}(\nu) - \sigma_{\rm NC}(\bar{\nu})}{\sigma_{\rm CC}(\nu) - \sigma_{\rm CC}(\bar{\nu})}$$

NC (mediated by Z) and CC (mediated by $W^{+/-}$), assuming strange and anti-strange to be equal in the target, is independent of Parton Distribution Functions and can be used to determined the Weinberg angle θ_W

$$R = \frac{1}{2}\left(\frac{1}{2} - \sin\theta_w^2\right)$$

You may use (without deriving it) the result (and set c, cbar = 0)

$$\begin{aligned}
F_2^{W^-} &= 2x\left(u + \bar{d} + \bar{s} + c\right), \\
F_3^{W^-} &= 2x\left(u - \bar{d} - \bar{s} + c\right), \\
F_2^{W^+} &= 2x\left(d + \bar{u} + \bar{c} + s\right), \\
F_3^{W^+} &= 2x\left(d - \bar{u} - \bar{c} + s\right),
\end{aligned}$$

Wrap-up

➡ The structure of the proton has been a crucial ingredient to test and verify perturbative QCD and it is now key to the precision challenge that we are facing at the LHC

➡ Today's lecture

- ✓ Parametrisation of the proton in terms of structure functions
- ✓ Parton model picture
- ✓ (QCD - Improved parton model)
- ✓ (DGLAP evolution equations)
- ✓ (Collinear Factorisation Theorem)

Extra material

Deep Inelastic Scattering

Slide from F Olness lectures
CTEQ school 2017

$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times 1$$

$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times F\left(\frac{Q^2}{\Lambda^2}\right)$$

Λ of order of the proton mass scale

$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times \sum_i e_i^2$$

Partonic cross sections

Slide from Gavin Salam lectures
Quy Nhon Vietnam 2018

$$\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \qquad [\alpha_s \equiv \alpha_s(\sqrt{s_{e^+e^-}})]$$

$$= R_0 \left(1 + 0.32\alpha_s + 0.14\alpha_s^2 - 0.47\alpha_s^3 - 0.59316\alpha_s^4 + \cdots\right)$$

Baikov et al., 1206.1288
(numbers for γ-exchange only)

This is one of the few quantities calculated to N4LO

Good convergence of the series at every order
(at least for $\alpha_s(M_Z) = 0.118$)