Precision Physics and Antimatter

Part 3 Penning traps basics

Questions to be addressed

- So far theoretical description of an ideal Penning trap
    - How does this compare to reality – e.g. can we reproduce theory?
    - What are effects that could affect the measurement?
    - How can handle these effects?
    - How do we detect the ions?

Reminder

- We want to do precise and accurate measurements of particle properties

$$g/2 = 1.001\,159\,652\,180\,85\,(76)\;[0.76\ \text{ppt}]$$

➡ Isolated particles
under well defined conditions
with long observation times (precision)
in an “perfect vacuum”

- Need to introduce fields which interact with particle and measure the coupling constant

$$\omega_c = \frac{q}{m} B$$

Requirements

Requirements of the experiment:

- Extremely homogeneous magnetic field (2-4 T)
- **Single, cold** ion
- Long storage times (months)
- “Perfect“ vacuum

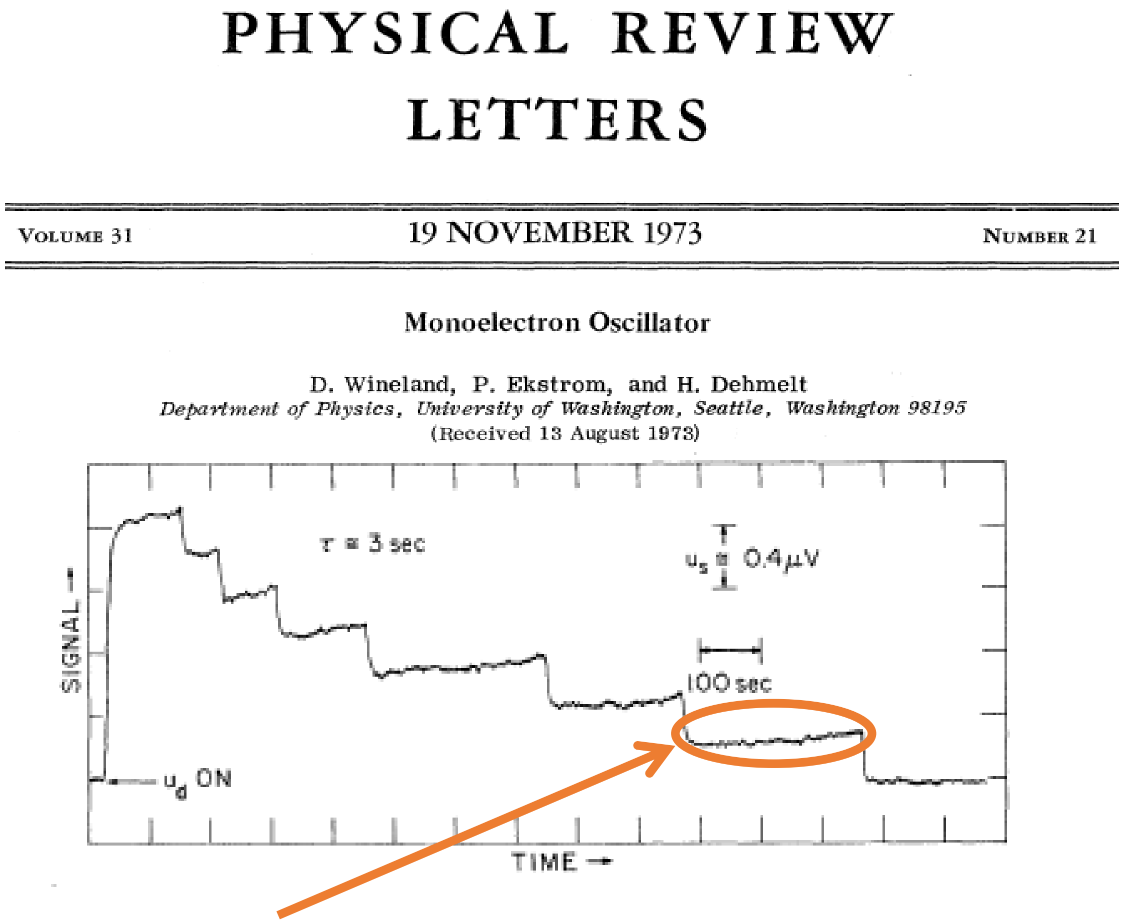
➡ **Non-destructive frequency measurements on single ions in Penning traps**

- Compensation of residual imperfections with same particle
- Vacuum better than **10^{-16} mBar**
    - less than 20 gas atoms in the trap volume !

DAS SCHLIMMSTE AM IONENKÄFIG IST DIE KÄLTE UND DIE EINSAMKEIT

[P. Evers, Wundersame Welt der Atomis]

Single Ion



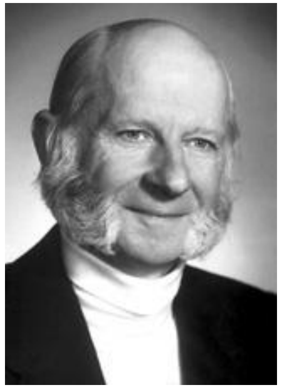
PHYSICAL REVIEW LETTERS

VOLUME 31 19 NOVEMBER 1973 NUMBER 21

Monoelectron Oscillator

D. Wineland, P. Ekstrom, and H. Dehmelt
Department of Physics, University of Washington, Seattle, Washington 98195
(Received 13 August 1973)

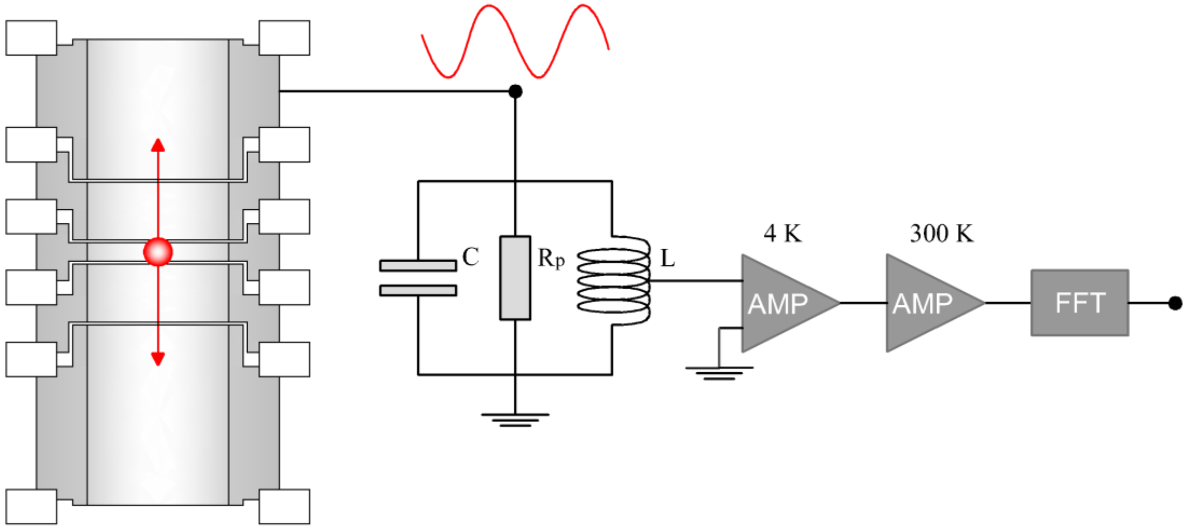
First Single Trapped ion – it was an electron



Hans G. Dehmelt , Nobel prize 1989

D. Wineland , Nobel prize 2012

Non-Destructive Frequency Detection



Oscillating particle induces image currents in trap electrodes

Image current detection:

- Non destructive – long observation times – precise information about trapped system
- Real time observation of particle manipulation

Details

Equation of motion:

$$\ddot{z} + \gamma_z \dot{z} + \omega_z^2 z = f(t)/m$$

$$m\dot{z}\ddot{z} + m\gamma_z \dot{z}^2 + m\omega_z^2 \dot{z} z = 0$$

First Integral:

$$\frac{1}{2} m\dot{z}^2 + \frac{1}{2} m\omega_z^2 z^2 = -\int dt \ \ m\gamma_z \dot{z}^2$$

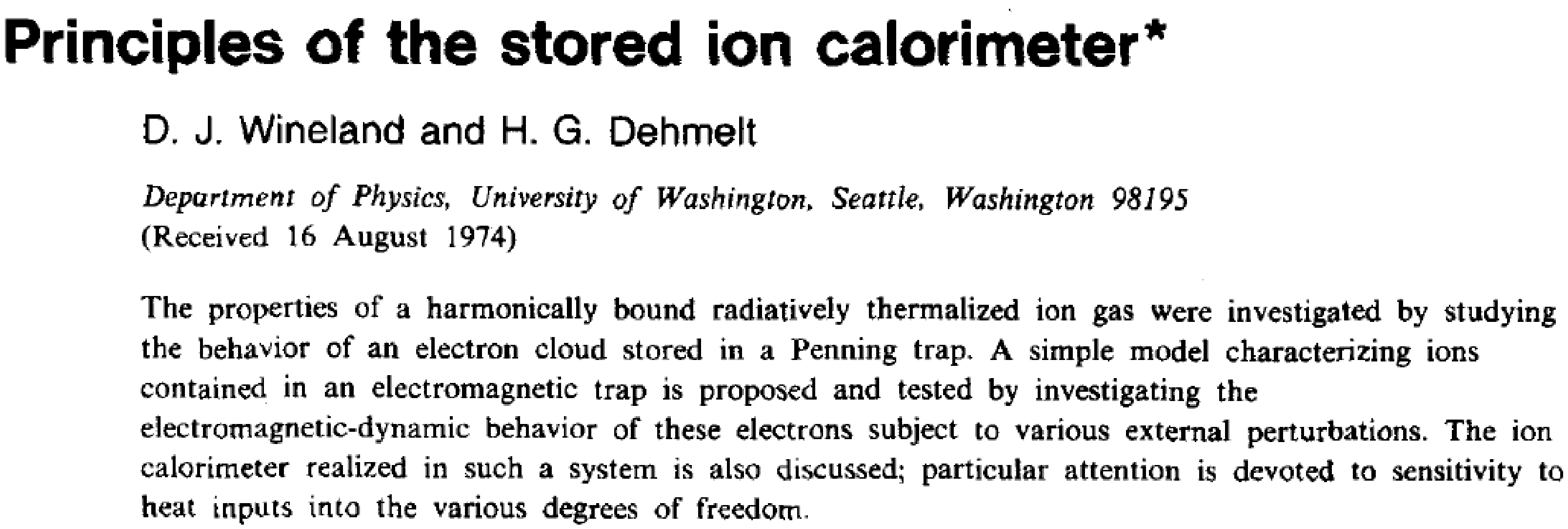
Dissipated Power:

$$P = R_p I_p^2 = m\gamma_z \dot{z}^2$$

Cooling Time Constant:

$$\gamma_z = \frac{R_p}{m} \frac{q^2}{D^2} \Leftrightarrow \tau_z = \frac{m}{R_0} \frac{D^2}{q^2}$$

And another point of view



Principles of the stored ion calorimeter*

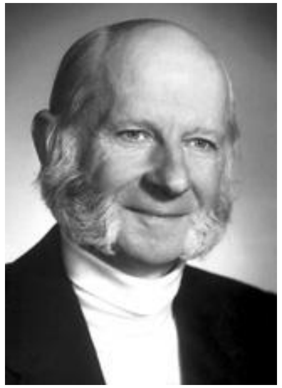
D. J. Wineland and H. G. Dehmelt

Department of Physics, University of Washington, Seattle, Washington 98195

(Received 16 August 1974)

The properties of a harmonically bound radiatively thermalized ion gas were investigated by studying the behavior of an electron cloud stored in a Penning trap. A simple model characterizing ions contained in an electromagnetic trap is proposed and tested by investigating the electromagnetic-dynamic behavior of these electrons subject to various external perturbations. The ion calorimeter realized in such a system is also discussed; particular attention is devoted to sensitivity to heat inputs into the various degrees of freedom.

Journal of Applied Physics, Vol. 46, 2 (1974)



Hans G. Dehmelt , Nobel prize 1989

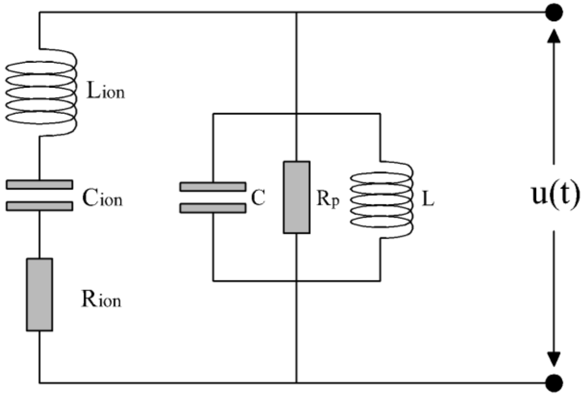
Hans G. Dehmelt , Nobel prize 2012

And another point of view

$$m\frac{D^2}{q^2}\frac{d}{dt}I_p + R_p I_p + m\omega_z^2\int dt\,\frac{D^2}{q^2}I_p = 0$$

Describes a series tuned circuit with equivalent quantities:

$$l_p = m\frac{D^2}{q^2} \quad \text{and} \quad c_p = \frac{1}{m\omega_z^2}\frac{q^2}{D^2}$$



Particle acts as a perfect conductor and shorts everything in parallel.

And another point of view

$$m\frac{D^2}{q^2}\frac{d}{dt}I_p + R_p I_p + m\omega_z^2 \int dt\ \frac{D^2}{q^2} I_p = 0$$

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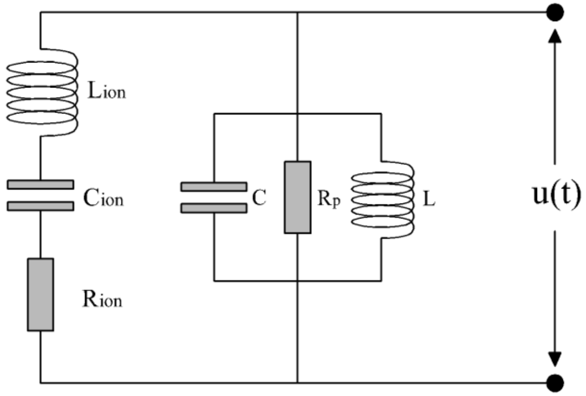
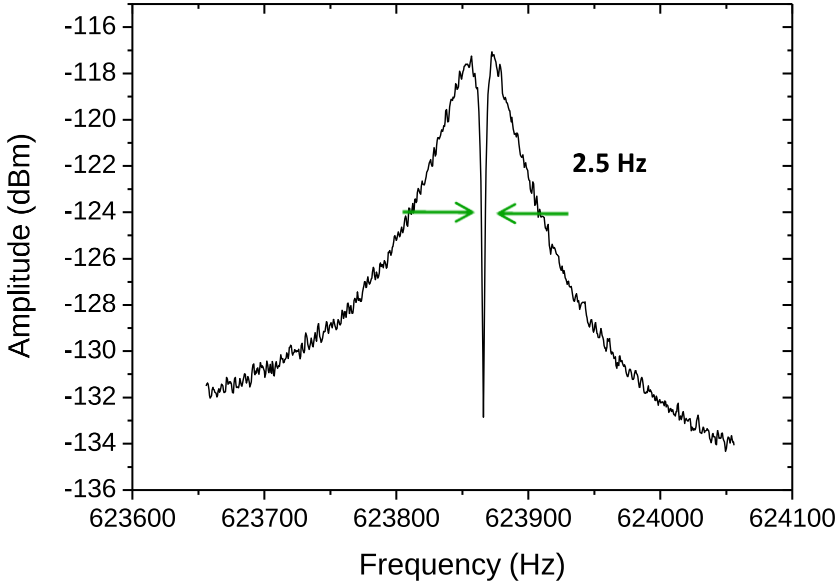


Particle acts as a perfect conductor and shorts everything in parallel.

Single proton can be stored for a year



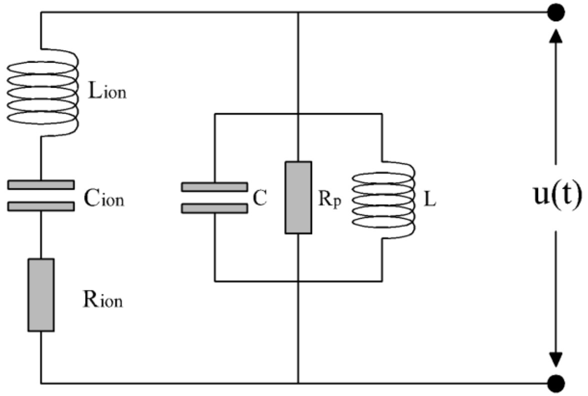
High Sensitivity High SNR

On resonance:

Inductance compensates capacitance

Detection system looks effectively like parallel resistor

What determines this resistor ?



$$R_p = 2\pi \nu_z Q L$$

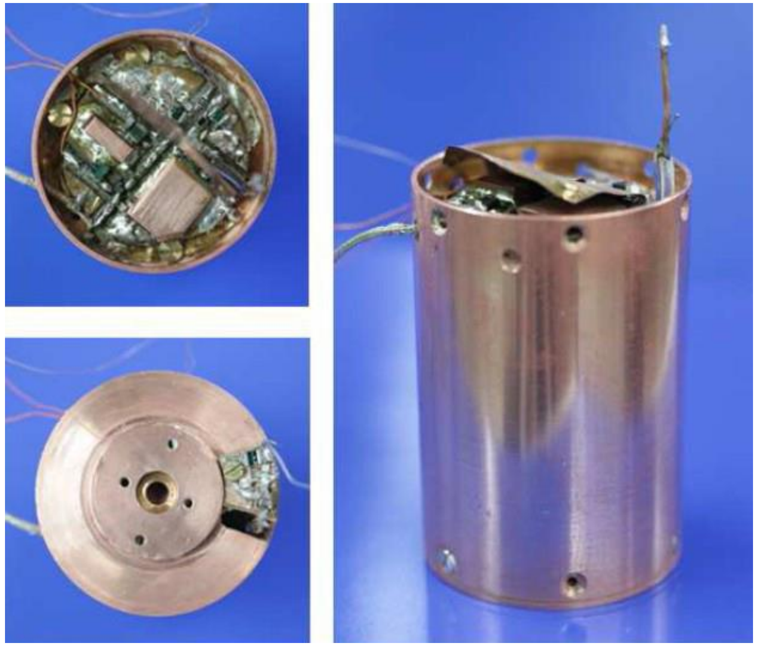
$$\sqrt{\frac{4k_BT\left(\frac{\kappa^2R_p}{1+\kappa^2R_p/R_{in}}\right)+\left(\frac{\kappa^2R_p}{1+\kappa^2R_p/R_{in}}\right)^2 i_n^2\kappa^4}{e_n^2}}$$

Cold electronics (4K)

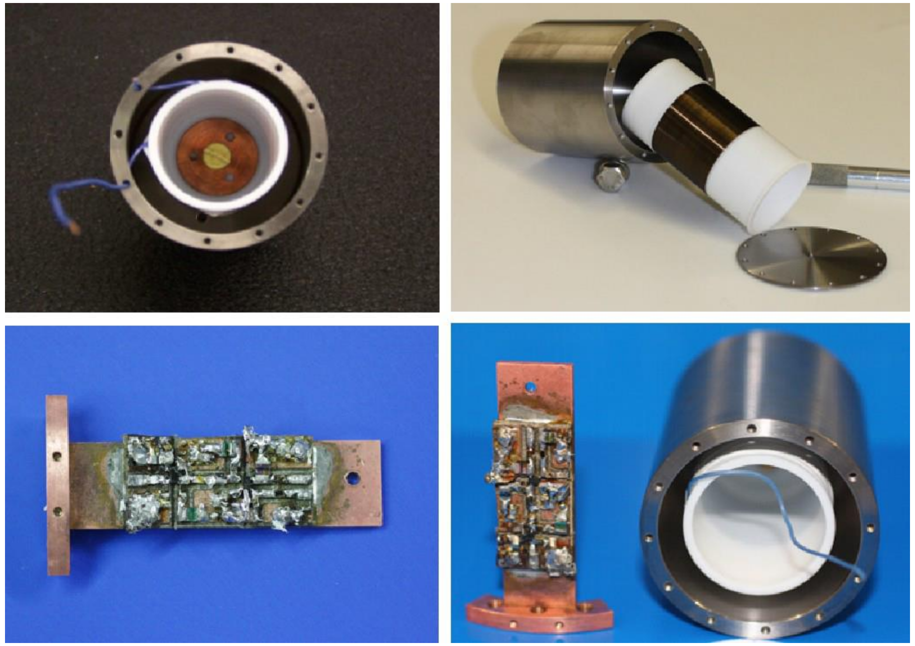
Low noise amplifiers

Superconducting material useable (NbTi)

How does it look like?



Cyclotron Detector used for Proton g-factor measurement at Mainz



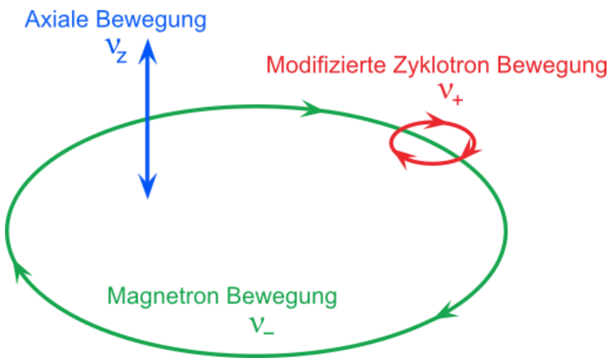
Axial Detector used for Proton g-factor measurement at Mainz

The ideal Penning trap

- Homogeneous magnetic field and electrostatic quadrupole potential

$$\vec{B} = B\vec{e}_z$$ → Radial confinement

$$\Phi(z,\rho) = V_0 c_2 \left(z^2 - \frac{\rho^2}{2} \right)$$ → Axial confinement



$$\omega_z = \sqrt{\frac{q}{m_p} 2c_2 V_0}$$

$$\omega_- = \frac{\omega_c}{2} - \sqrt{\left(\frac{\omega_c}{2}\right)^2 - \frac{\omega_z^2}{2}}$$

$$\omega_+ = \frac{\omega_c}{2} + \sqrt{\left(\frac{\omega_c}{2}\right)^2 - \frac{\omega_z^2}{2}}$$

- Magnetic field: Superconducting magnet
- Electrostatic potential: Voltages applied to trap electrodes
- If we can control these fields accurately we can do accurate measurements

$$\omega_c = \omega_- + \omega_+ = \frac{q}{m} B$$

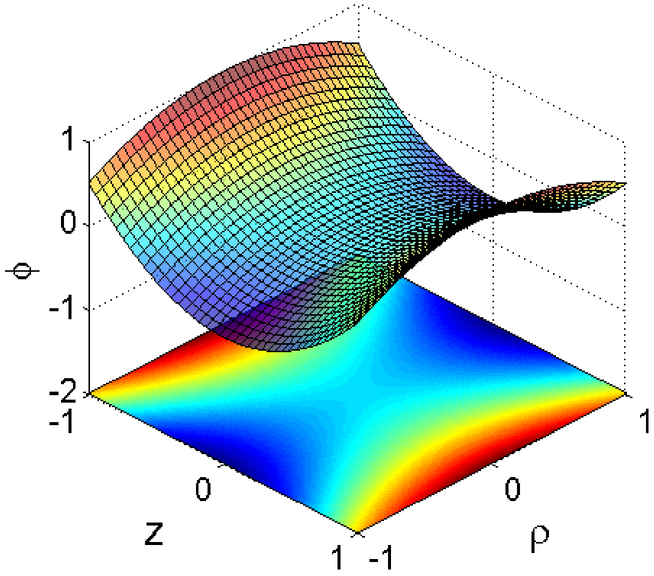
How does an ideal Penning trap look like?

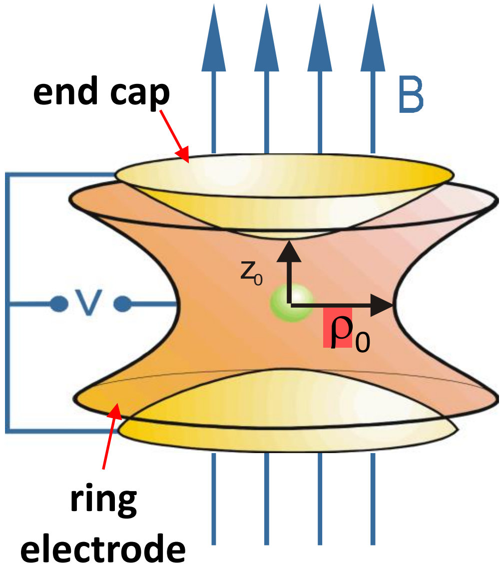
- Electrodes reflect equipotential surface

$$\Phi(z,\rho) = V_0 c_2\left(z^2 - \frac{\rho^2}{2}\right) = Const.$$

→

$$\left(z^2 - \frac{\rho^2}{2}\right) = Const.$$

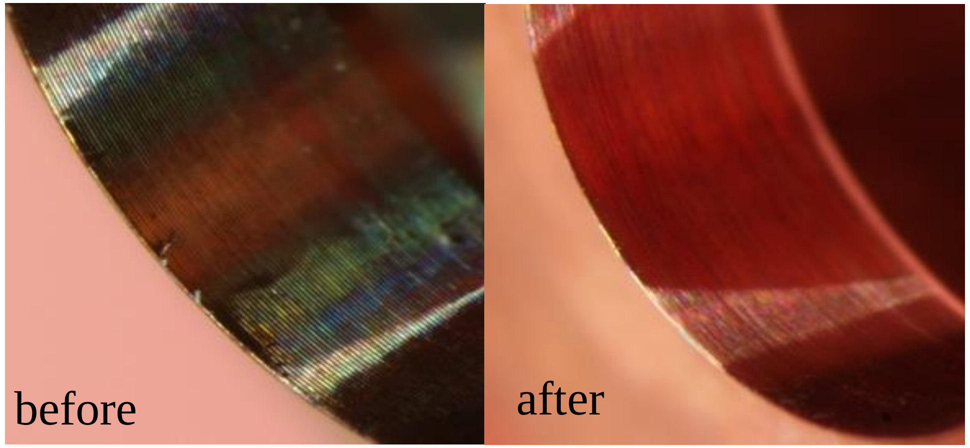


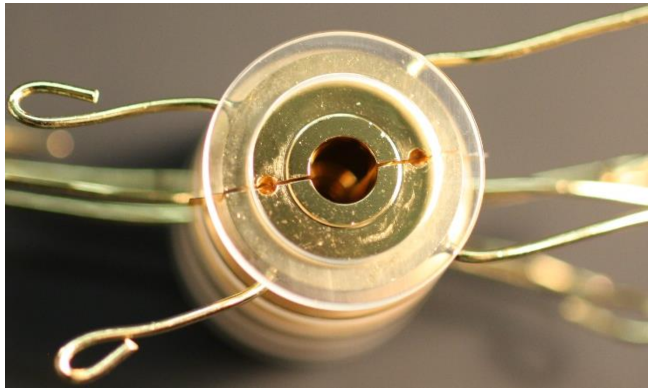


→ Equipotentials are hyperbolic surfaces

Penning traps

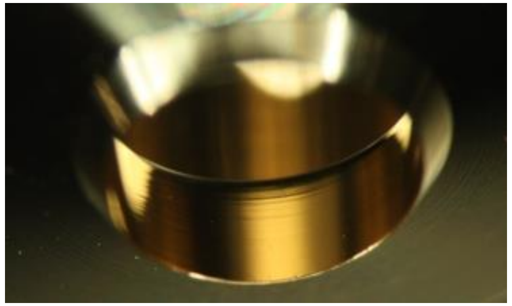
polish of the inner surfaces

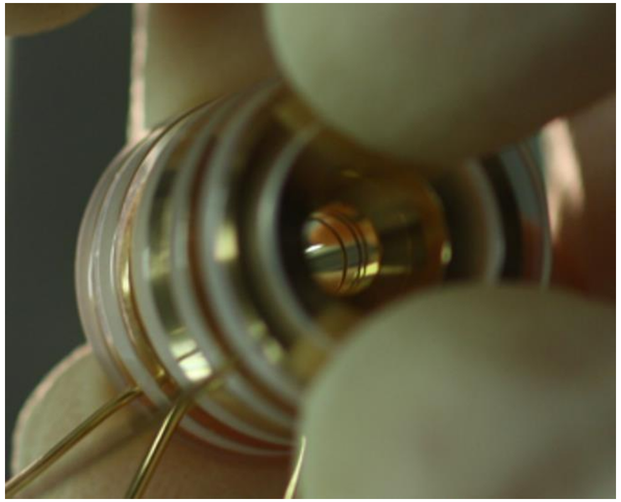




electrodes nickel plating

galvanic gold plating





Penning traps

Electrodes of a Penning trap

Superconducting Magnet

Real Penning traps I

- In reality it does not look that nice
  - Errors in construction, e.g. no ideal hyperbolic surfaces,

    Anharmonicity $\Phi(z,\rho)_z = V_0(c_2 z^2 + c_4 z^4 + c_8 z^8 ....)$

  - Misalignment: $\vec{B} \neq B\vec{e}_z$
  - Magnetic field varies with position

    Inhomogeneity $\vec{B}(z) \neq const.$

  - Deviations from rotational symmetry - Ellipticity

Real Penning traps II

- In general:
  - Nonharmonic motion – Frequencies depend on oscillation amplitude
  - Coupling of “independent” modes – Frequency in one mode depends on oscillation of other mode

$$\frac{\Delta\omega_z}{\omega_z} = \frac{1}{qV_0}\frac{3C_4}{C_2^2}\left(-\frac{1}{2}\left(\frac{\omega_z}{\omega_+}\right)^2 E_+ + \frac{1}{4}E_z - |E_-|\right)$$

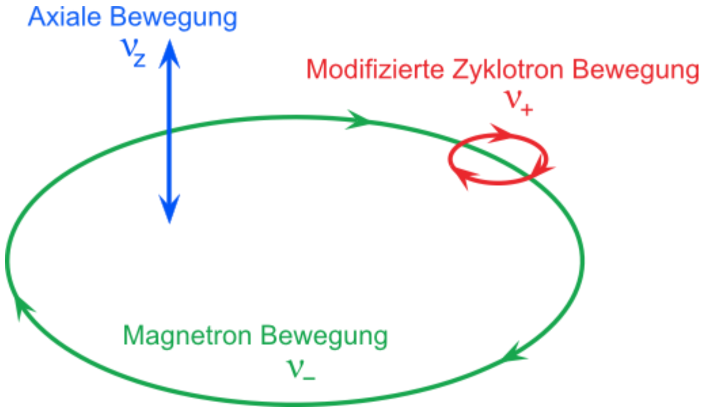
  - Frequencies depend on tilting angle

In detail

- In the end want to measure the true frequncies

$$\frac{\Delta\omega_+}{\omega_+} = \frac{1}{qV_0}\frac{3C_4}{C_2^2}\left(-\left(\frac{\omega_z}{\omega_+}\right)^4 E_+ + \frac{1}{2}\left(\frac{\omega_z}{\omega_+}\right)^2 E_z - \left(\frac{\omega_z}{\omega_+}\right)^2 |E_-|\right)$$

$$\frac{\Delta\omega_-}{\omega_-} = \frac{1}{qV_0}\frac{3C_4}{C_2^2}\left(-\left(\frac{\omega_z}{\omega_+}\right) E_+ + E_z - |E_-|\right)$$



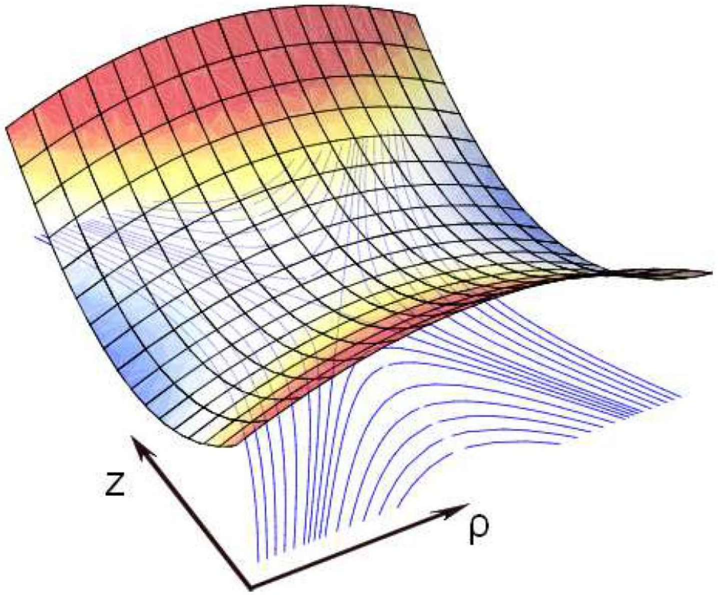
$$\omega_c = \frac{q}{m} B = \omega_+ + \omega_-$$

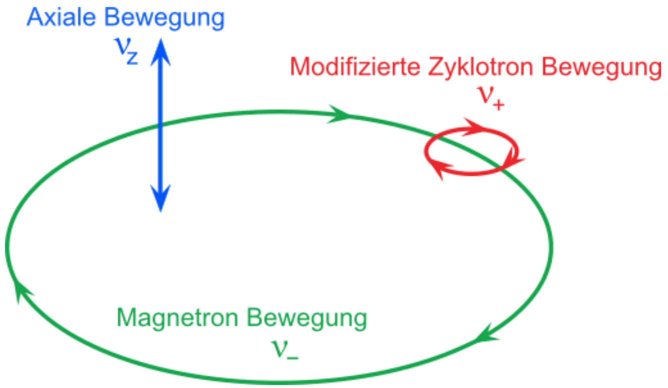
Would determine wrong charge-to-mass ratio

Need to compensate these effects

An invariance theorem

- However two of the above identified shifts cancel
- Tilt changes all frequencies





$$\omega_z = \sqrt{\frac{q}{m_p} 2c_2 V_0}$$

$$\omega_- = \frac{\omega_c}{2} - \sqrt{\left(\frac{\omega_c}{2}\right)^2 - \frac{\omega_z{}^2}{2}}$$

$$\omega_+ = \frac{\omega_c}{2} + \sqrt{\left(\frac{\omega_c}{2}\right)^2 - \frac{\omega_z{}^2}{2}}$$

<u>Invariance-Theorem:</u> $\nu_c^2 = \nu'^2_+ + \nu'^2_z + \nu'^2_-$

$$\nu_c^2 = \nu_+^2 + \nu_z^2 + \nu_-^2$$

- Makes Penning traps powerful tool

L. S. Brown and G. Gabrielse, Phys. Rev. A **25**, 2423 (1982).

Real Penning trap I

- In reality it does not look that nice
    - Errors in construction, e.g. no ideal hyperbolic surfaces,

      Anharmonicity $\Phi(z,\rho)_z = V_0(c_2 z^2 + c_4 z^4 + c_8 z^8 \ldots.)$
    - ~~Misalignment: $\vec{B} \neq B\vec{e}_z$~~
    - Magnetic field varies with position

      Inhomogeneity $\vec{B}(z) \neq const.$
    - ~~Deviations from rotational symmetry - Ellipticity~~

Real Penning trap I

- In reality it does not look that nice
  - Errors in construction, e.g. no ideal hyperbolic surfaces,

    Anharmonicity $\Phi(z,\rho)_z = V_0(c_2 z^2 \boxed{+ c_4 z^4 + c_8 z^8 ....)}$

  - ~~Misalignment:~~ ~~$\vec{B} \neq B\vec{e}_z$~~

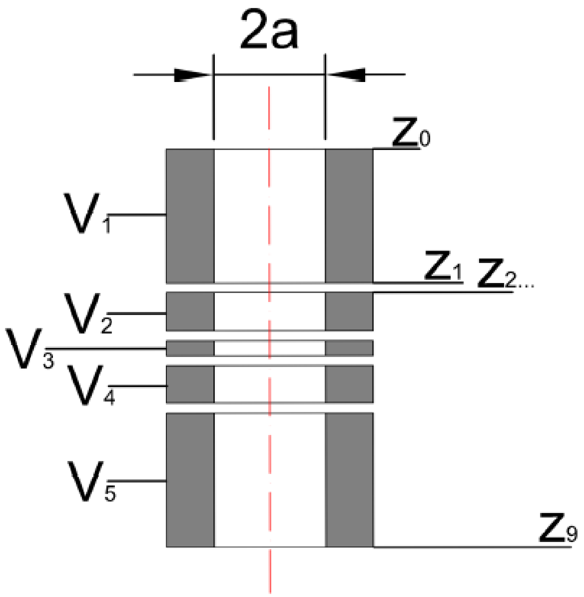
    How to remove these terms?

  - Magnetic field varies with position

    Inhomogeneity $\vec{B}(z) \neq const.$

  - ~~Deviations from rotational symmetry - Ellipticity~~

Cylindrical traps design

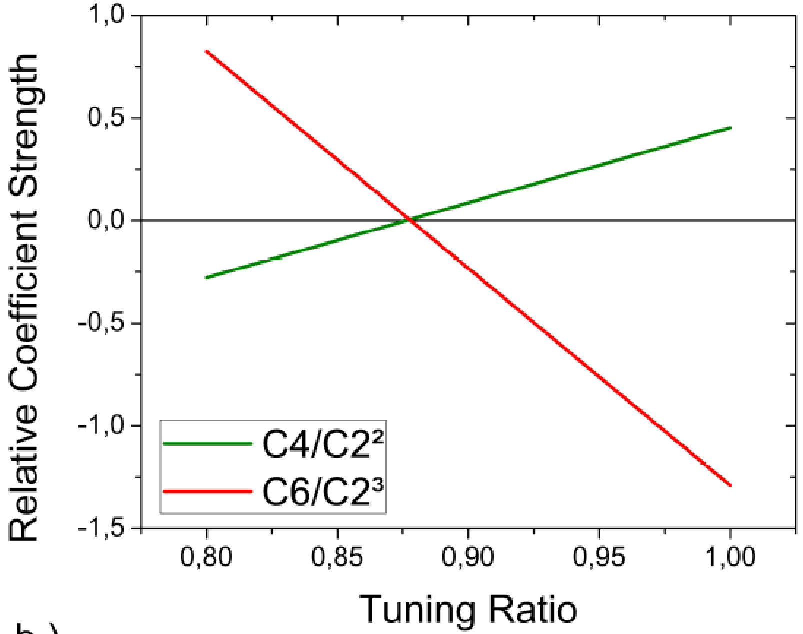


Degrees of freedom for defined radius:

1.) Length of ring electrode

2.) Length of correction electrode

3.) Ratio of Compensation to Ring electrode voltage – called Tuning Ratio



b.)

Degrees of freedom three adjustable parameters

1.) C4 = 0 2.) C6 = 0 3.) Orthogonality

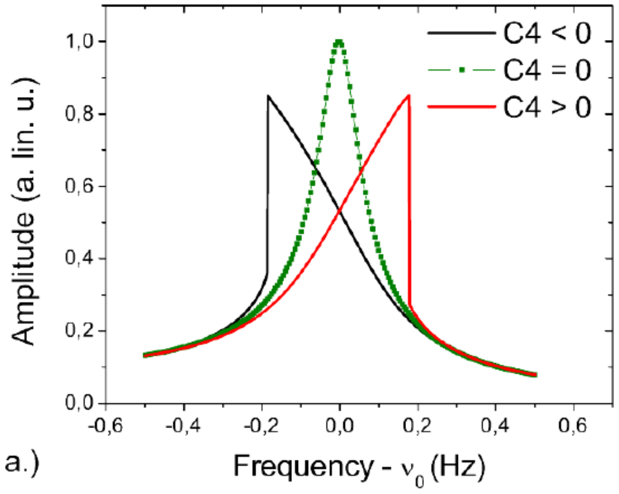
Cylindrical traps are at least as good as hyperbolic traps.
Easier to understand / Simpler to optimize

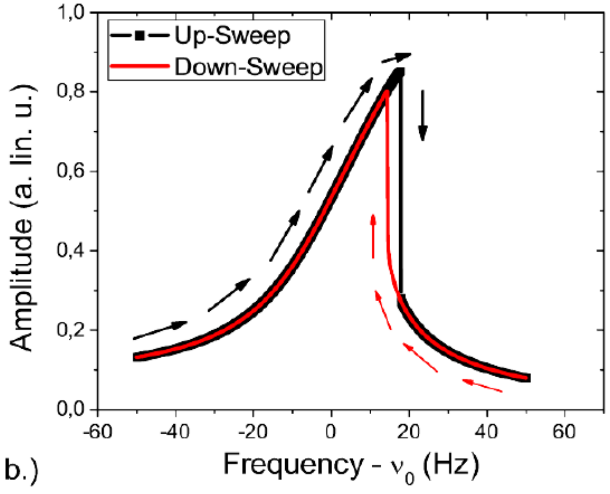
Nonlinear dynamics in a Penning trap

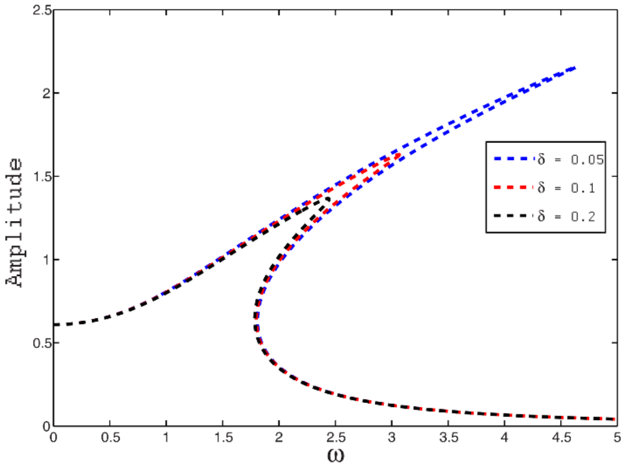
- The Duffing Oscillator

$$\Phi(z,\rho)_z = V_0(c_2 z^2 + c_4 z^4) \Longrightarrow m\ddot{z} + \gamma\,\dot{z} + qV_0 c_2 z + qV_0 c_4 z^3 = A\cos(\Omega t)$$

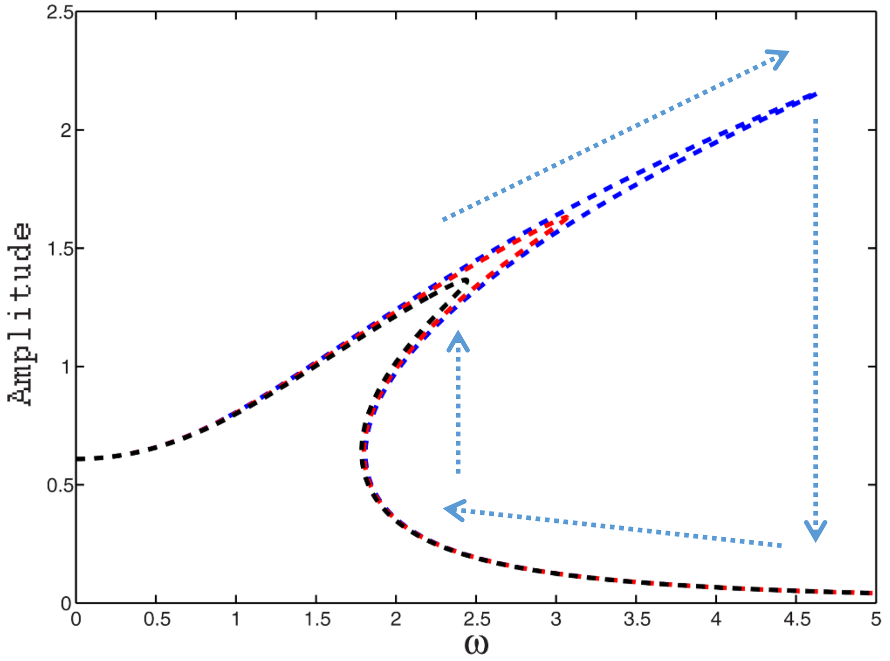
- No closed form for analytical solution – method of harmonic balance
  - See e.g. Brennan, M.J. Journal of Sound and Vibration. **318** (4–5): 1250–1261

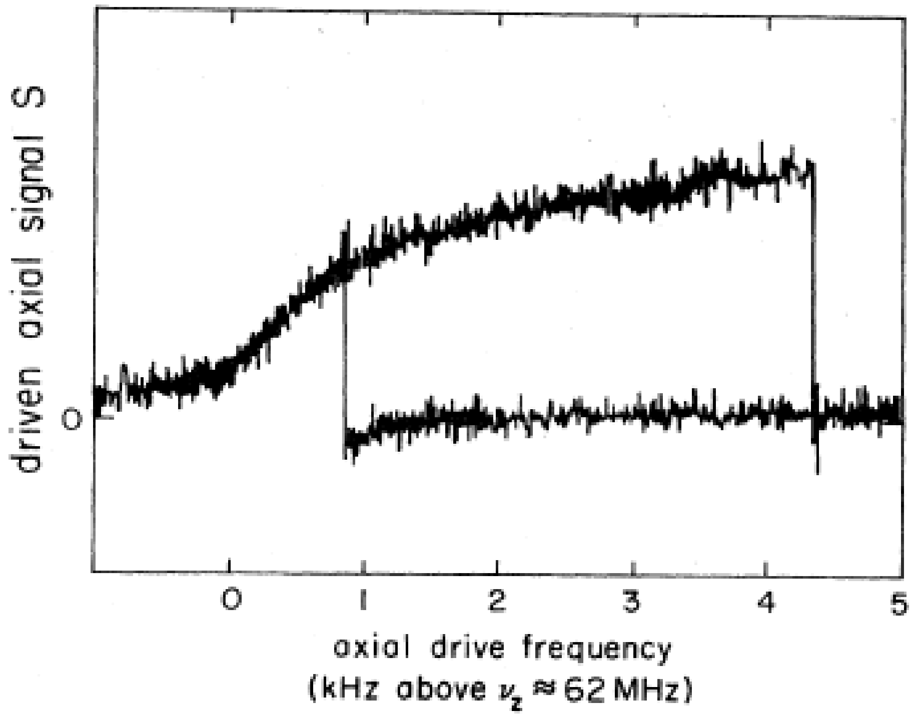






Nonlinear dynamics for an electron





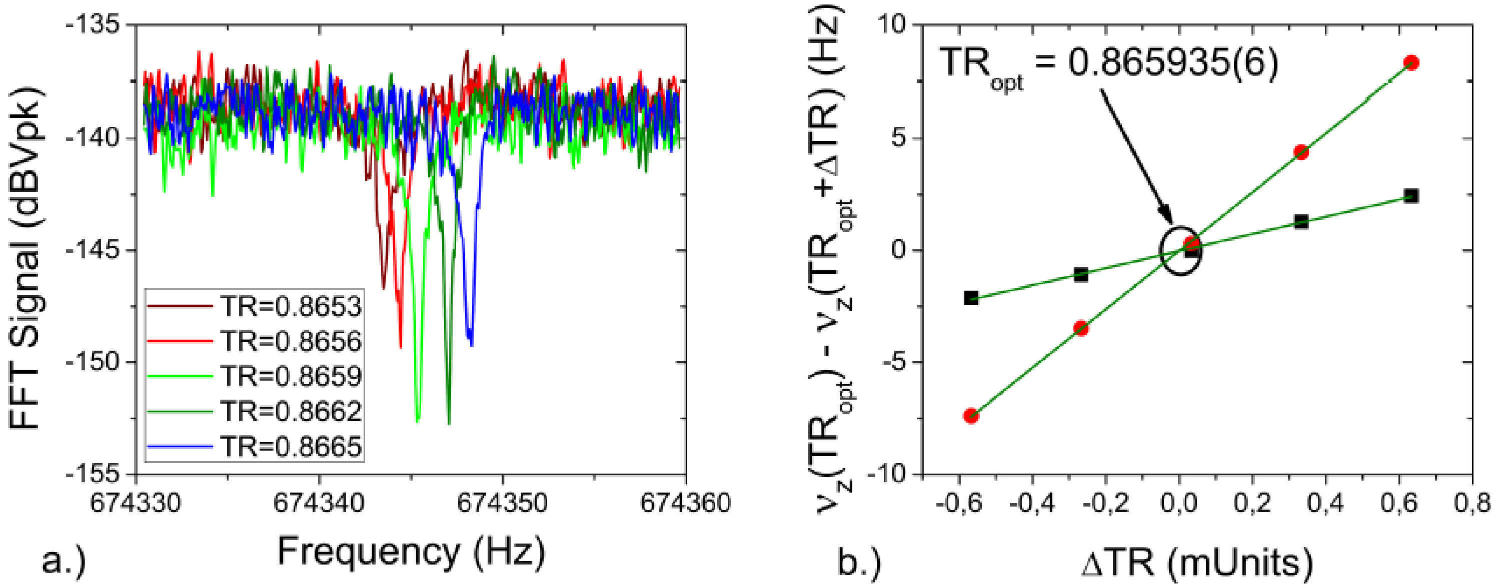
L. S. Brown and G. Gabrielse, Phys. Rev. A **25**, 2423 (1982).

Can be used to optimize for harmonic trap

Tuning of the trap

- The ideal tuning ratio is not known – need to search for it

$$\frac{\Delta\omega_z}{\omega_z} = \frac{1}{qV_0}\frac{3C_4}{C_2^2}\left(-\frac{1}{2}\left(\frac{\omega_z}{\omega_+}\right)^2 E_+ + \frac{1}{4}E_z - |E_-|\right)$$



- Voltages optimized to 10^{-5}
- Residual frequency shift mHz only

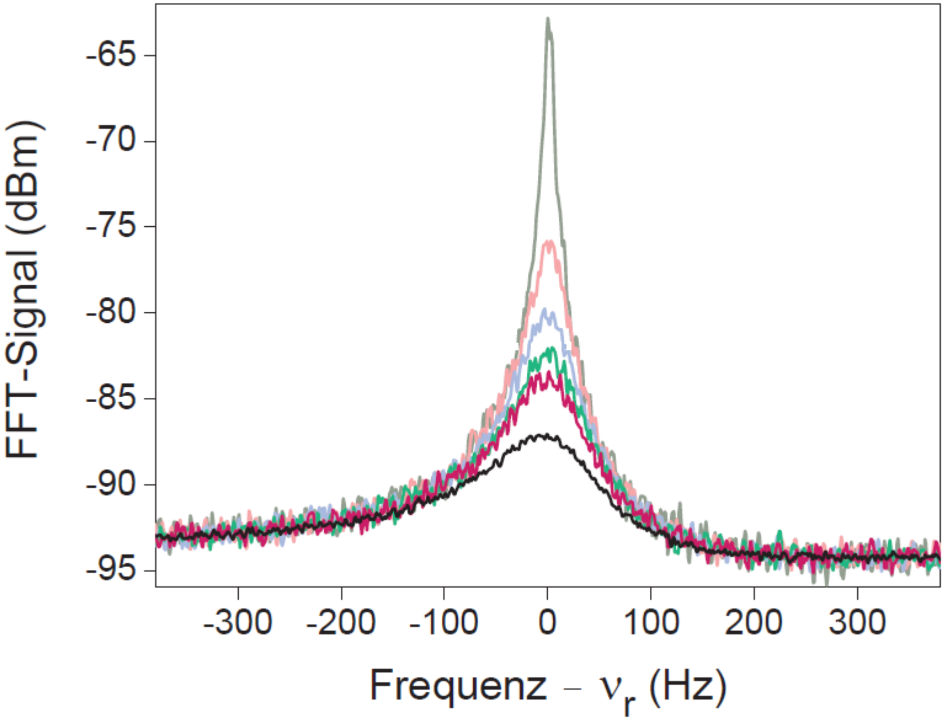
How did we change the Energy (Temperature)

- Ion in permanent interaction with detector
    - Ion has same temperature as detector

- Apply additional white noise to one of the electrodes to increase temperature of system

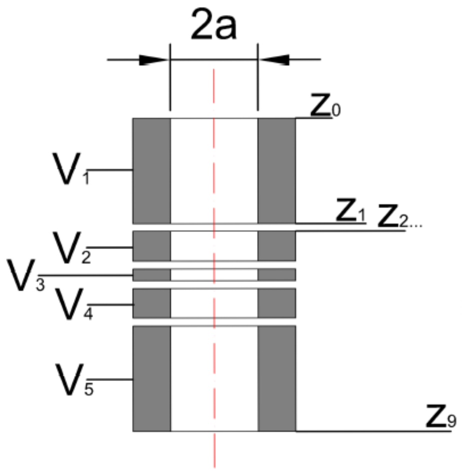
$$u_{tot}^2 = 4k_B T_{eff} R = 4k_B T_0 R + u_w^2$$

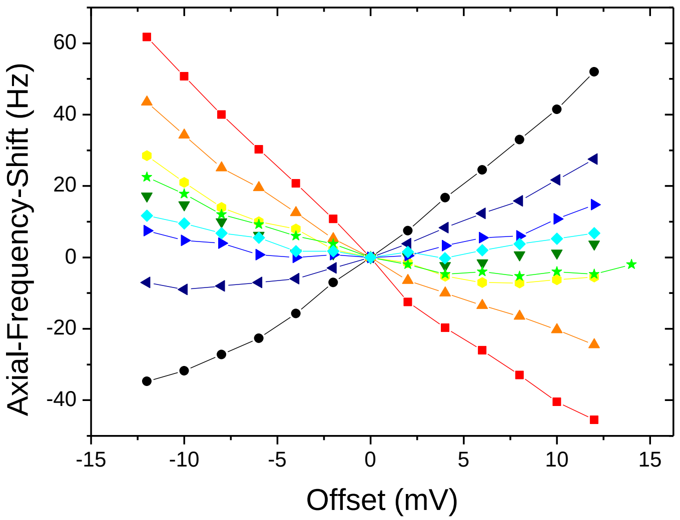
$$\Delta T = T_{eff} - T_0 = \frac{u_w^2}{4k_B R}$$



One step further

- Offset potentials on electrodes due to Seebeck effect
    - High energy electrons at hot end diffuse to cold end
    - Low energy electrons at cold end diffuse to end
    - Diffusion proportional to temperature – net current
    - In equilibrium additional voltage compensates for current
- Offset potentials in the order of 10mV – needed to compensate for 10µV





Are there other Shifts which can not be compensated?

- Relativistic shift

Relativity tells us that the effectiv mass depends on the velocity

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}}$$

Really important – we were taking about meV (4K) experiments

- Example

Electron in a 5T magnetic field at 4K:

Frequency = 160GHz , Motional Amplitude = 10nm ⇒ Velocity = 15000m/s

$$\Rightarrow \quad m = \left(1+1.2*10^{-9}\right) m_0$$

Important if you plan to measure frequencies with 10^{-9} precision or better

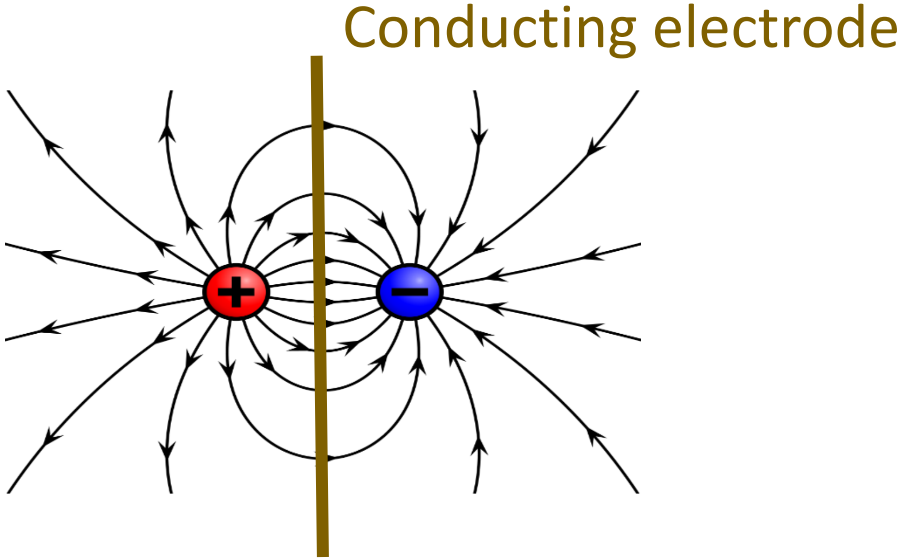
Are there other Shifts which can not be compensated?

- Image charge:

Image charge produces an additional potential term

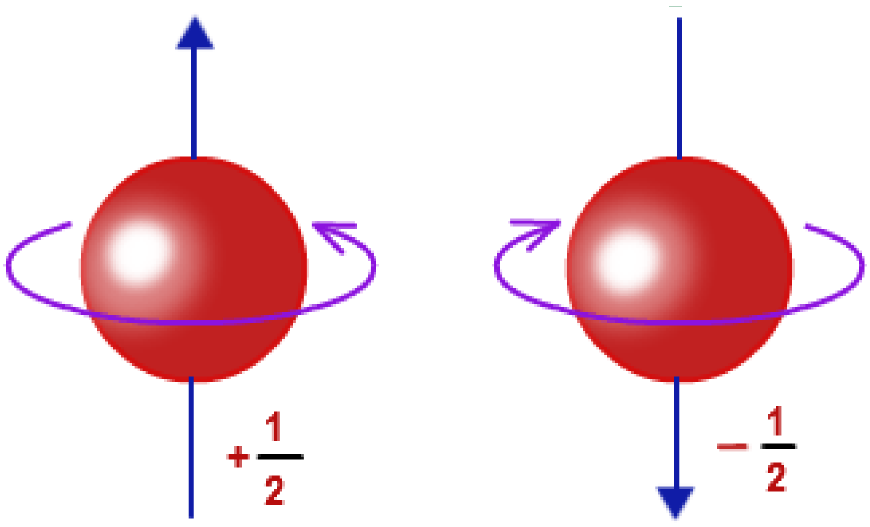
$$\Phi_{ideal} \rightarrow \Phi_{ideal} + \Phi_{image} \qquad \Phi_{image}(\rho) = \frac{q}{8\pi\varepsilon_0 r_0}\rho^2$$

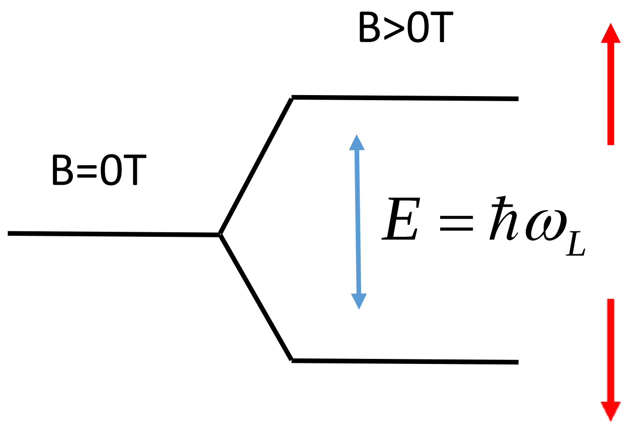
$$\Delta\omega_+ \approx \frac{q^2}{4\pi\varepsilon_0 m r_0^3 \omega_c} \qquad {}^{12}\mathrm{C}^{5+}\text{:}\ \frac{\Delta\omega_+}{\omega_+} \approx 300\,ppt$$



Imprortant for ions with large charge state – increase distance to electrodes, hence larger trap

There is one additional degree





$E = \hbar\omega_L$

In a magnetic field a spin caring particle precesses with the Larmorfrequency

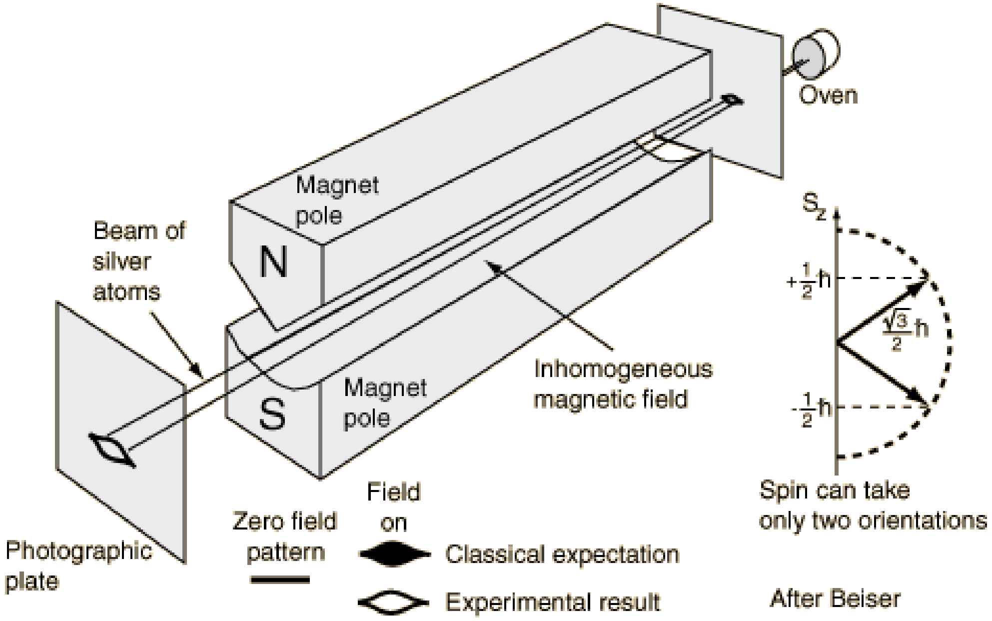
Unfortunately this is not accompanied by a movement of charge

How to measure?

Back to the roots

Idea:

Need to make particle movement depend on spin orientation – has been done before



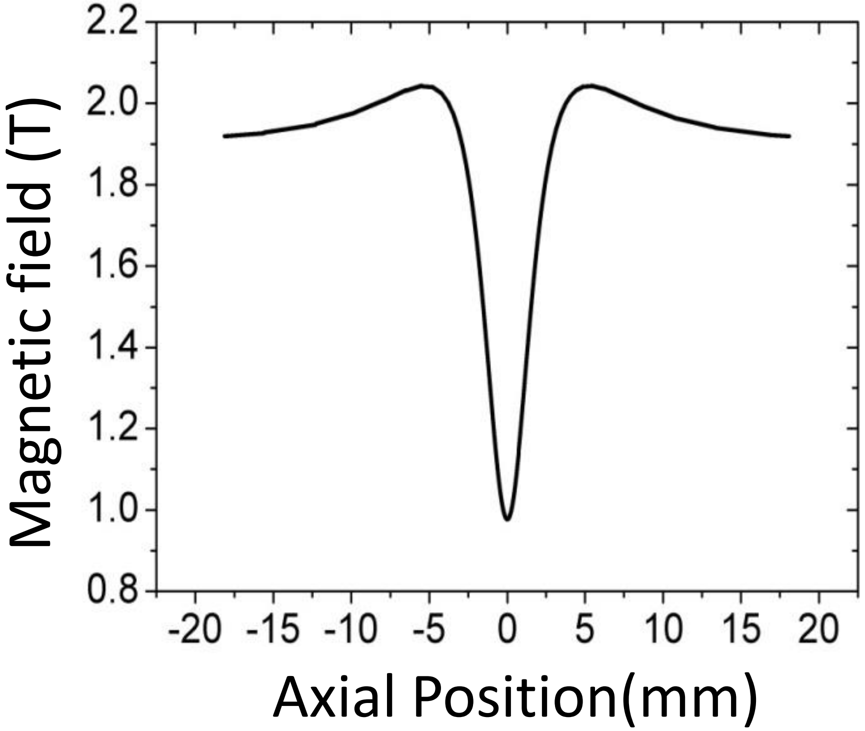
Beam of Silver Atoms in inhomogeneous Magnetic field

$$\vec{F} = \nabla\left(\vec{\mu}\cdot\vec{B}\right) = \begin{pmatrix} 0 \\ 0 \\ \mu_z \cdot \frac{\partial B}{\partial z} \end{pmatrix}$$

The classical Stern-Gerlach effect – quantization of spin

Applied to Penning traps

Introduce magnetic inhomogeneity, so-called magnetic bottle



$$B_z = B_0 + B_2\left(z^2 - \frac{\rho^2}{2}\right)$$

Additional spin-dependent force acting on particle

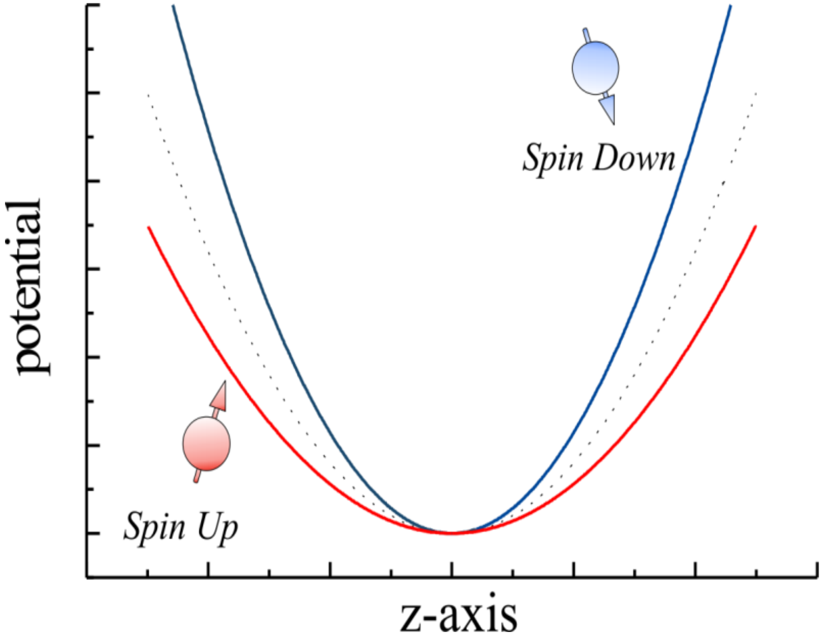
$$F_z = \pm\frac{\partial}{\partial z}\Phi_z = \frac{\partial}{\partial z}\pm\mu_p B_z$$

Continuous Stern-Gerlach effect

$$B_z = B_0 + B_2\left(z^2 - \frac{\rho^2}{2}\right)$$

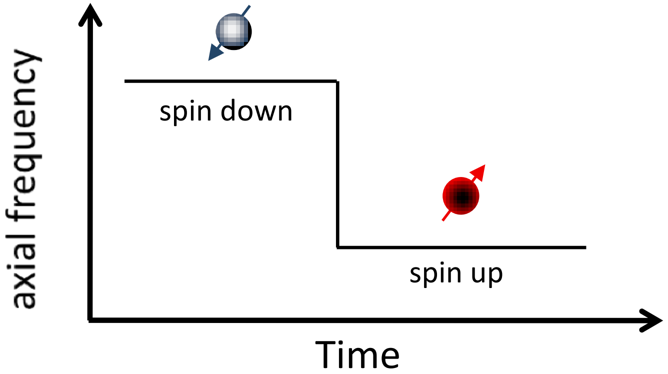
$$m\ddot{z} = q\frac{\partial}{\partial z}\Phi_{el} \pm \frac{\partial}{\partial z}\Phi_{mag}$$

$$= \frac{q}{m}V_0 z \pm \mu B_2 z = \left(\frac{q}{m}V_0 \pm \mu B_2\right)z$$

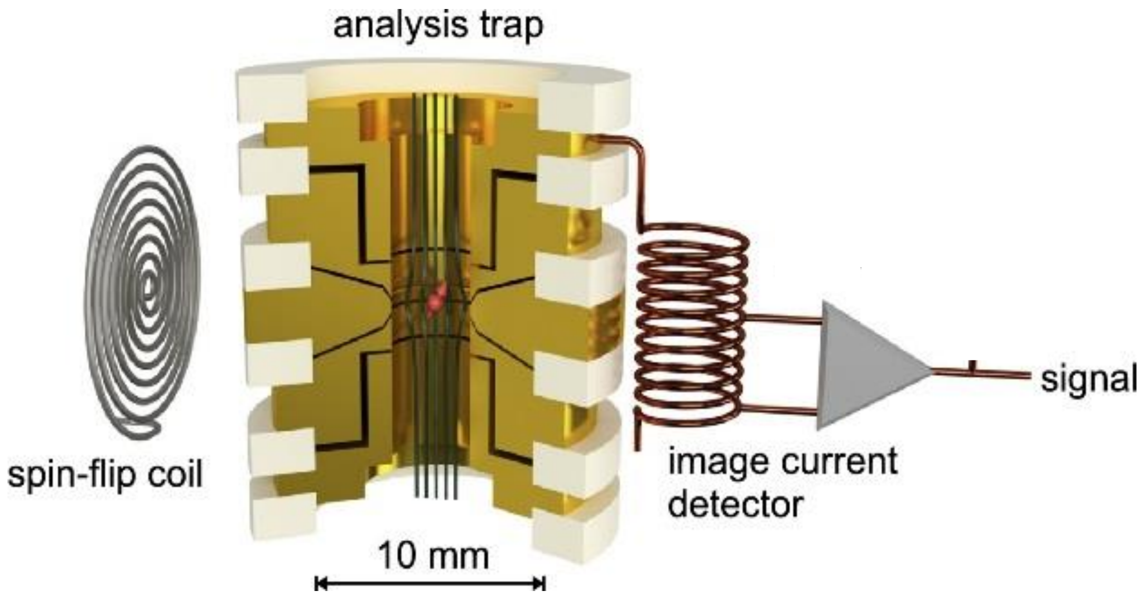


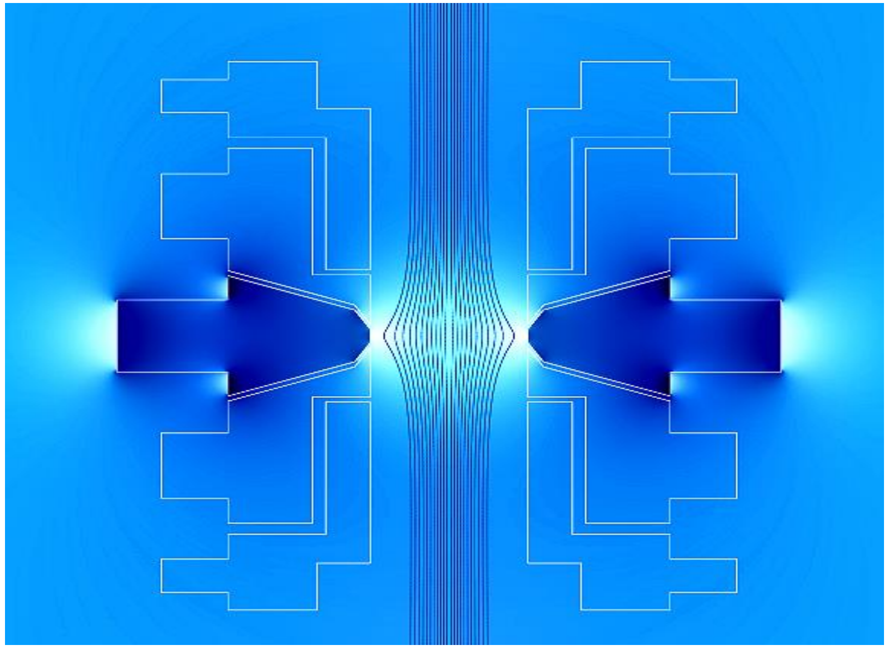
Frequency depends on Spin State

$$\nu_z \propto \frac{\mu_p}{m}B_2$$



An Example


analysis trap
spin-flip coil
image current detector
signal
10 mm



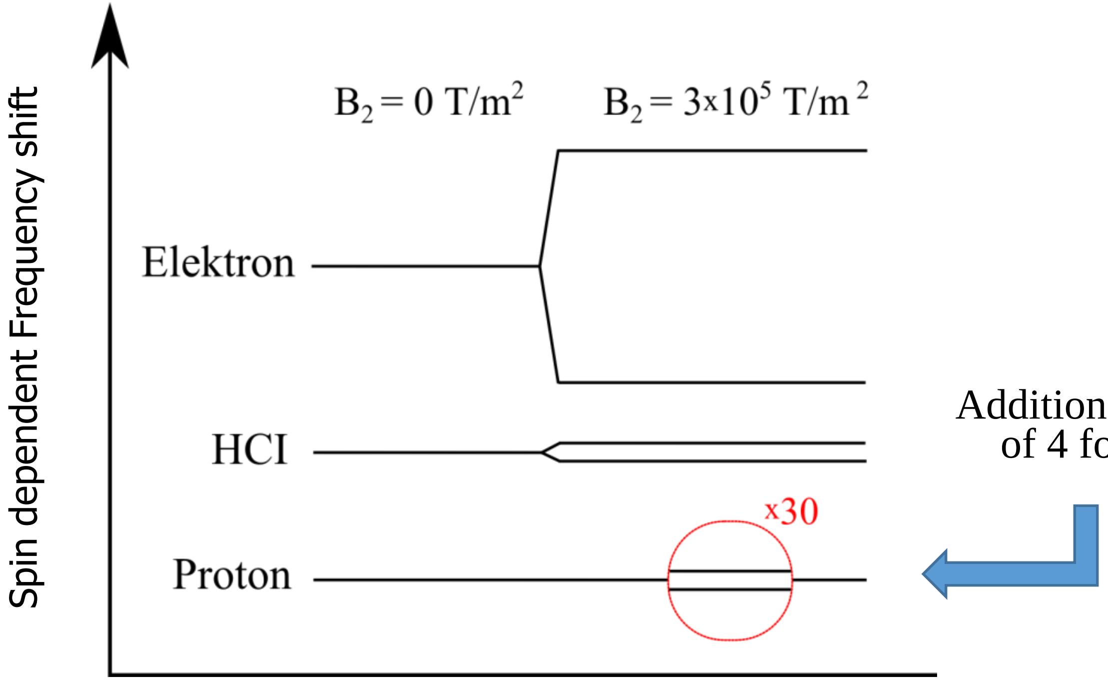
Comparison of different systems

Size of frequency jump depends on size of magnetic moment

$$\Delta\nu_z \propto \frac{\mu_p}{m} B_2$$

$$\mu_\mathrm{N} = \frac{e\hbar}{2m_\mathrm{p}}$$

$$\mu_\mathrm{B} = \frac{e\hbar}{2m_\mathrm{e}}$$



Additional factor of 4 for ^{3}He

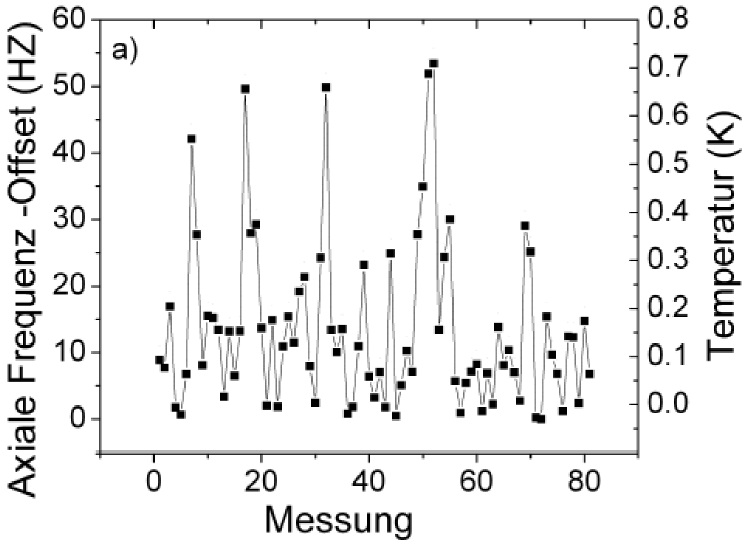
A Single Particle Thermometer

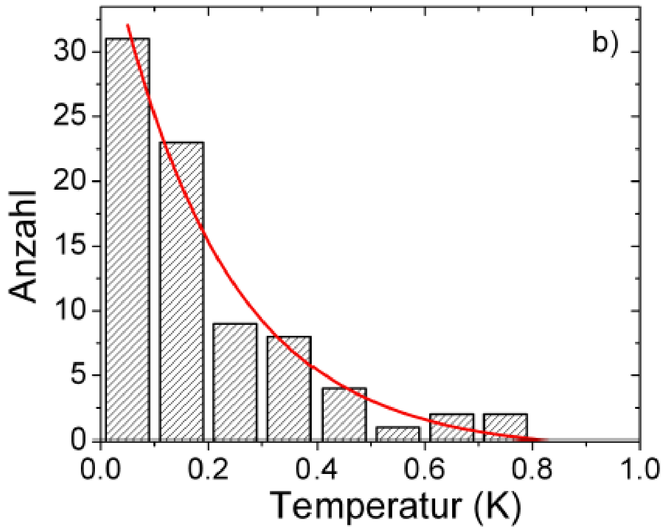
Sequence:

1. Measure axial frequency
2. Sideband coupling of magnetron mode to thermal bath (axial resonator)
3. Measure axial frequency

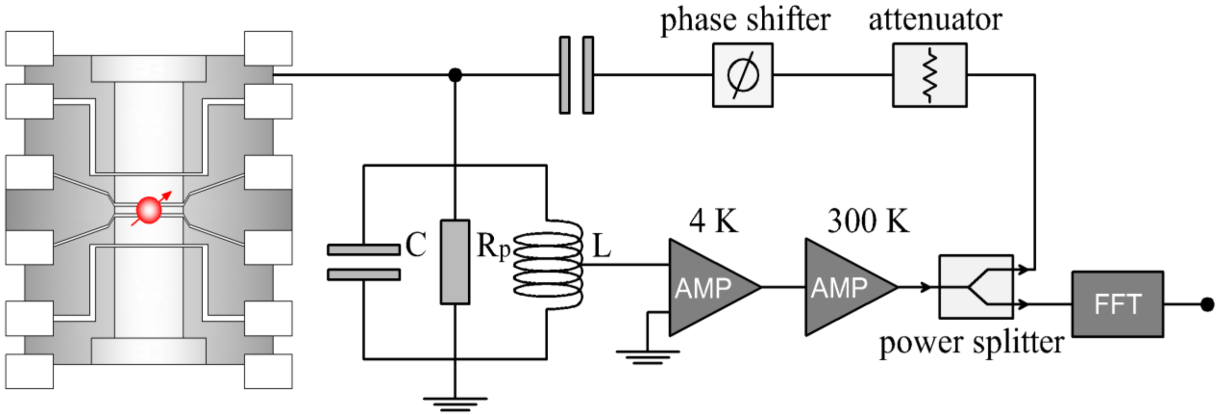
$$\Delta \nu_z \propto \frac{\mu_p}{m} B_2$$

Test of ergodic hypothesis: Array Average = Time Average





Aktive electronic feedback

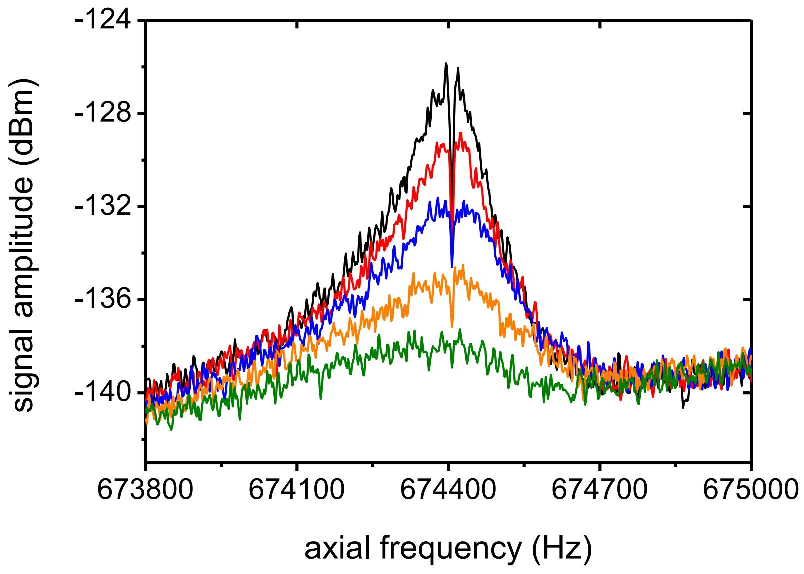


$$R_{FB} = R_p\left(1 \pm G_{FB}\right)$$
$$T_{FB} = R_0\left(1 \pm G_{FB}\right)$$

- Particle signal is fed back to the trap
- Particle temperature becomes adjustable:

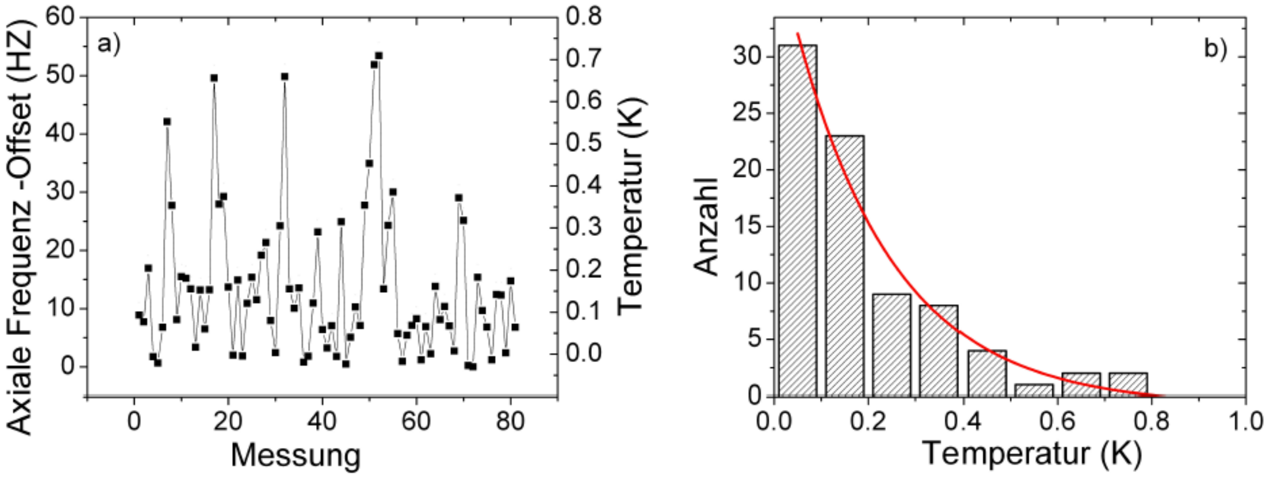
Negative feedback: lower temperature, smaller oscillation amplitude.

Positive feedback: higher temperature, higher signal-to-noise ratio.

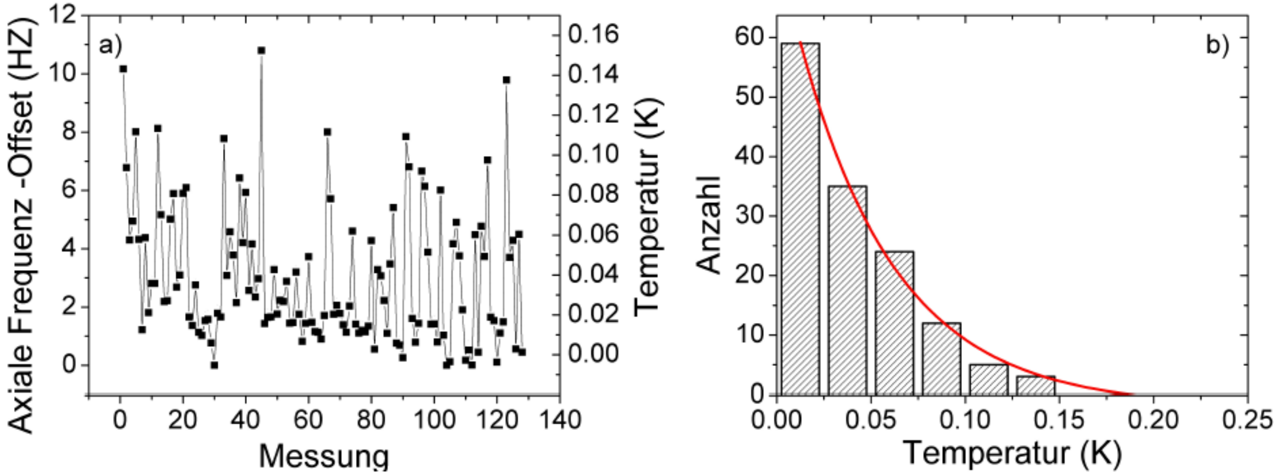


A Single Particle Thermometer II

No feedback



Negative feedback



Overview

Different ions demand different setups

- Electron/Positron: Low mass – large relativistic shift
  - Precise control over Cyclotron quantum state

$$\boxed{\frac{\Delta m}{m} = \frac{E}{mc^2}}$$

- Highly charged ions: Large charge state – large image charge shift
  - Larger traps to increase ion-trap distance

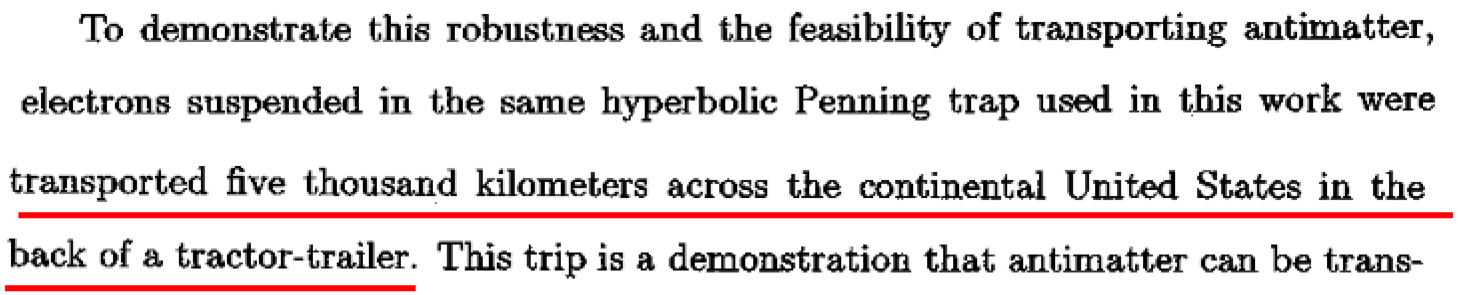
$$\boxed{\Delta\omega_+ \approx \frac{q^2}{4\pi\varepsilon_0 m r_0^3 \omega_c}}$$

- Proton/Aniproton: Small magnetic moment – small frequency jump
  - Large inhomogeneous magnetic field

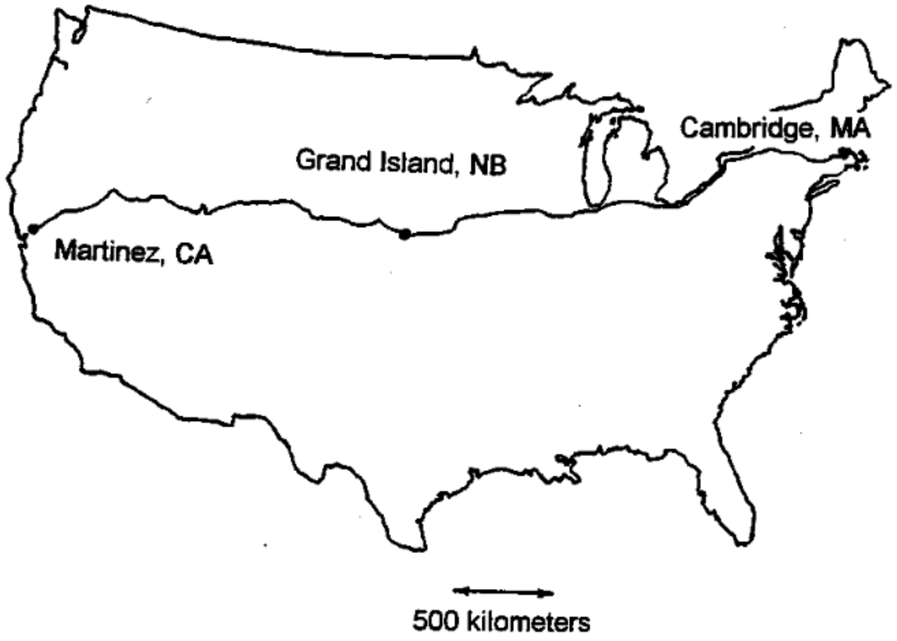
$$\boxed{\Delta\nu_z \propto \frac{\mu_p}{m} B_2}$$

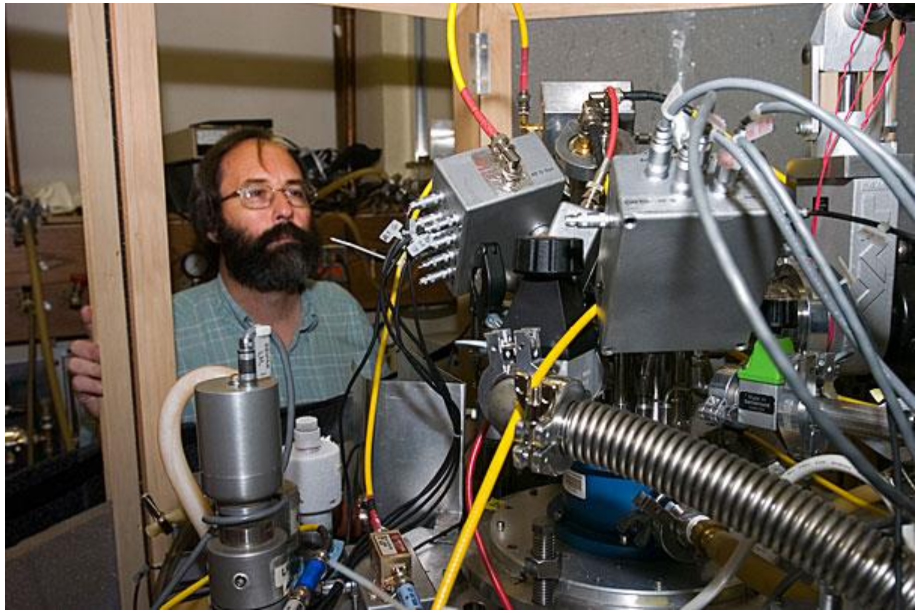
More to come Tomorrow

The capabilities of Penning traps



To demonstrate this robustness and the feasibility of transporting antimatter, electrons suspended in the same hyperbolic Penning trap used in this work were transported five thousand kilometers across the continental United States in the back of a tractor-trailer. This trip is a demonstration that antimatter can be trans-





Summary

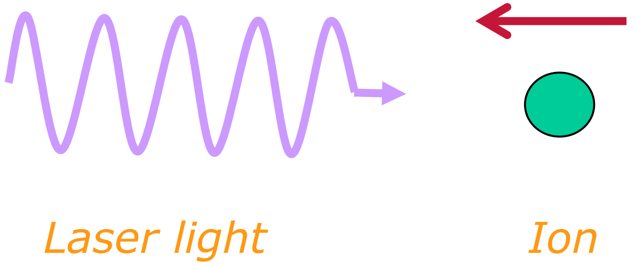
- Real Penning traps introduce systematic shifts of frequencies
- Frequency shifts scale with Energy and size of the correction (e.g C4, B2)
- However possibility to non-destructively detect ion and tune energy as well as correction
- Can compensate effects to allow accurate measurement – can do systematic measurements

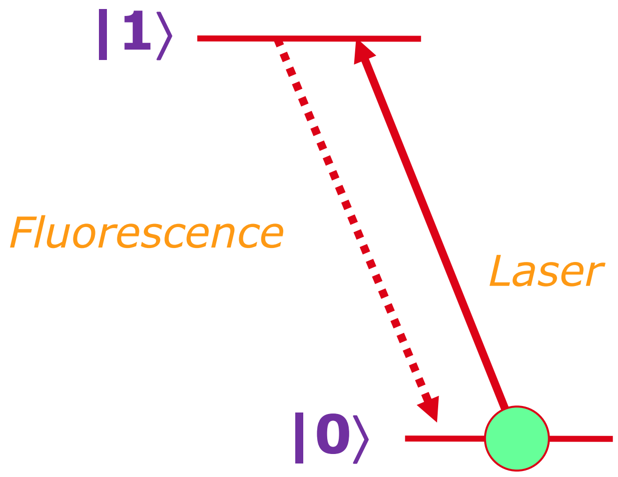
What we have is what Dehmelt called a Geonium atom – an artificial atom where we can precisely control the potentials seen by the ion

Ersatz Folien I

Basic Idea

- We use radiation pressure for laser cooling of ions
  - Same principle as for Doppler cooling of atoms
  - Laser excites the ion to an excited state, slows it down slightly
  - An ion with v ~ 500 m s^{-1} can be stopped in 1 ms (10^5 cycles)
- Force must be turned off when ion moving away from laser
  - Otherwise the process will reverse and it will speed up again
  - So set laser frequency just below ion resonance
- Minimum temperature (Doppler limit) is ~100muK





First Lasercooling in a trap

- First laser cooling demonstration (Wineland et al 1978) used the “Bolometric technique” – detect electrical noise across electrodes due to ion motion

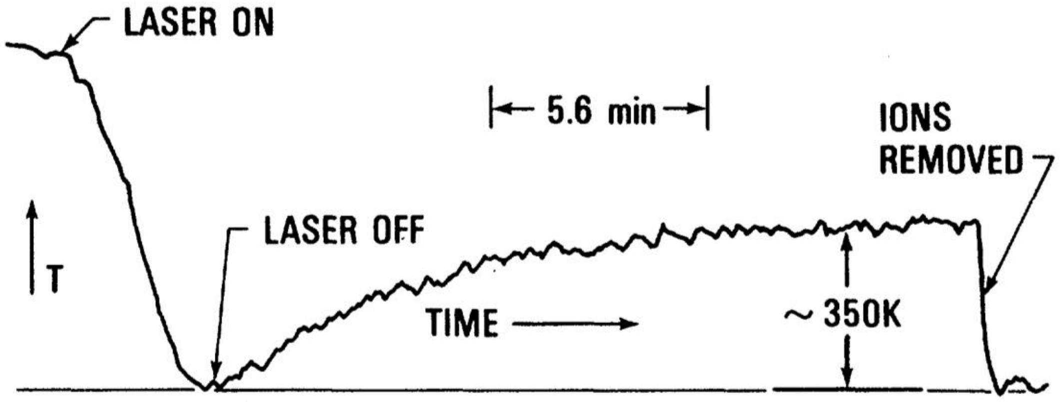
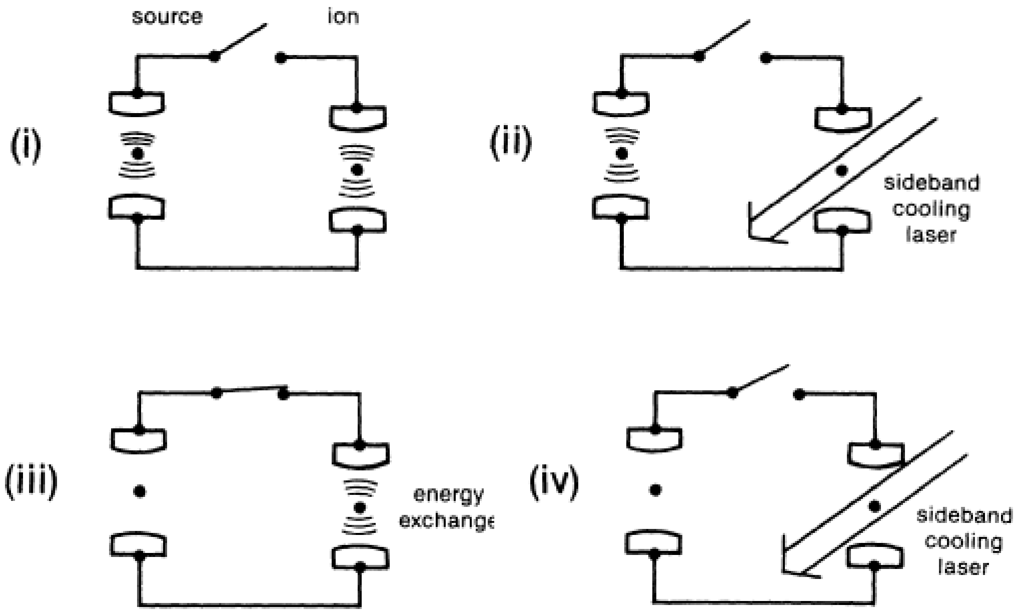


FIG. 2. Ion temperature vs time when laser cooling is applied for fixed $\nu_L - \nu_0$. The ions were initially heated above equilibrium temperature with the laser. Laser cooling was then applied on the $-\frac{1}{2} \leftrightarrow -\frac{3}{2}$ transition for a fixed time until a temperature approaching 0 K (<40 K) was achieved. After the laser is turned off, the ions rethermalize to the ambient temperature.

Sympathetic laser cooling

- Sympathetic cooling – no addressable internal states for proton/antiproton
- Proposal by D.J. Wineland in the 1990ies



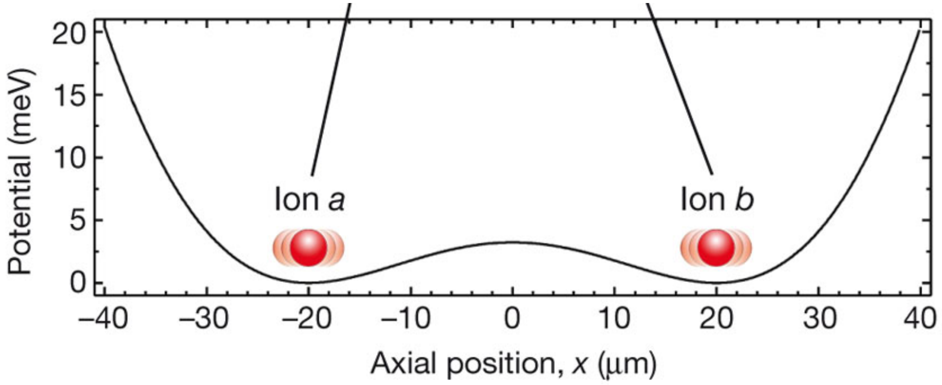
Two Options…

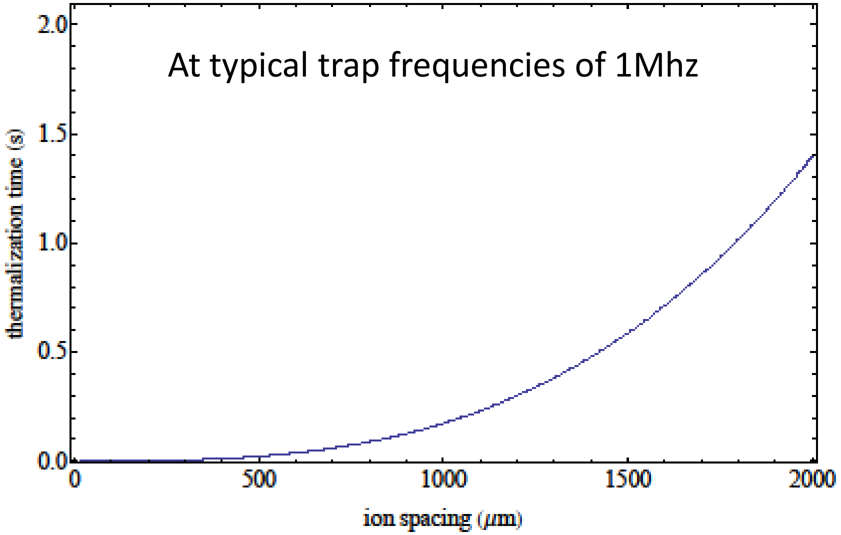
Heinzen and Wineland Phys. Rev. A 42, 2977 (1990)

Option I – Direct coulomb coupling

- Direct Coulomb coupling with ions at close proximity

$$\Omega_{\text{ex}} \equiv \frac{q_a q_b}{4\pi\varepsilon_0 s_0^3 \sqrt{m_a m_b}\sqrt{\omega_{0a}\omega_{0b}}}$$





- Coupling times in the order of seconds
- Increases for additional Be ions

- Demands development of „miniature“ Penning trap which allows for small ion separations at equal oscillation frequencies
- Beeing implemented at BASE HANNOVER

Option II – Common Endcap coupling

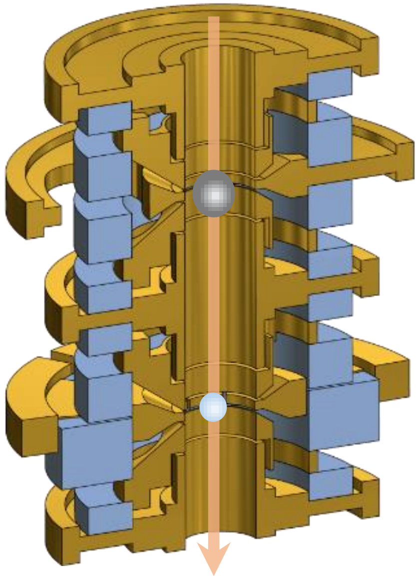
- Interaction via image currents induced in trap electrodes (proposal by D. J. Wineland)

$$\tau = 2\,\pi\,\omega\,C_T\,\frac{\sqrt{m_p m_{Be}}}{q^2}\,D_{eff}\,\frac{1}{\sqrt{N}}$$

- Allows usage of established trap designs
- Better control over static trapping fields

- However coupling times in the order of 30sec

- Beeing implemented at BASE Mainz



Based on existing design - optimized for low trap capacitance

Setup– Common Endcap coupling