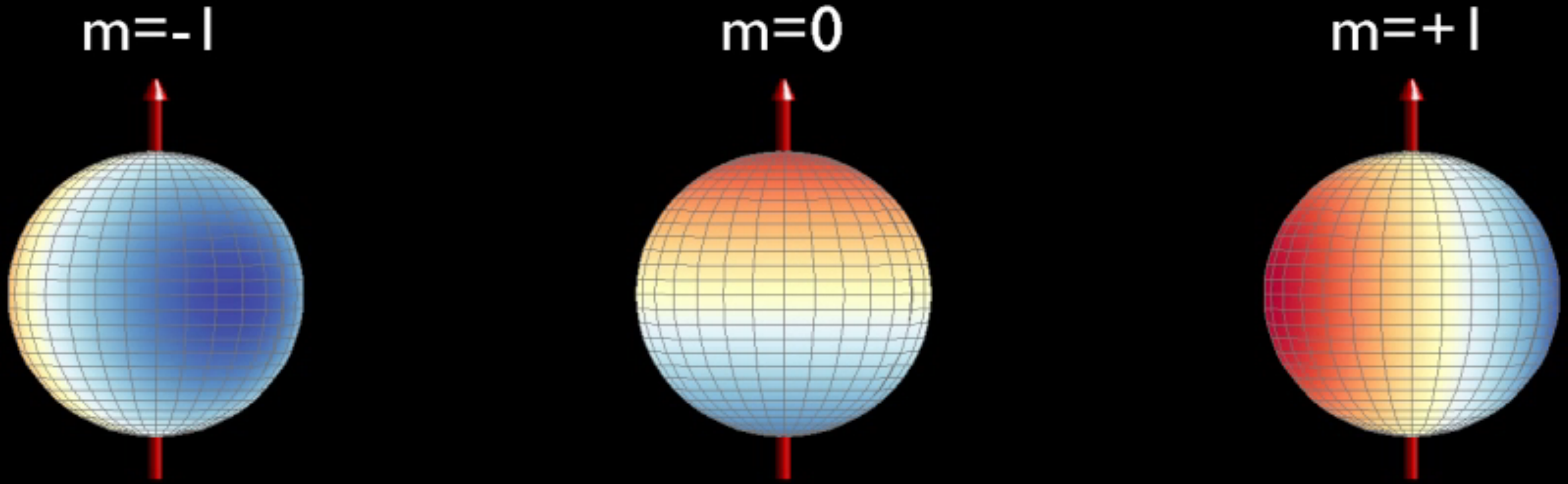
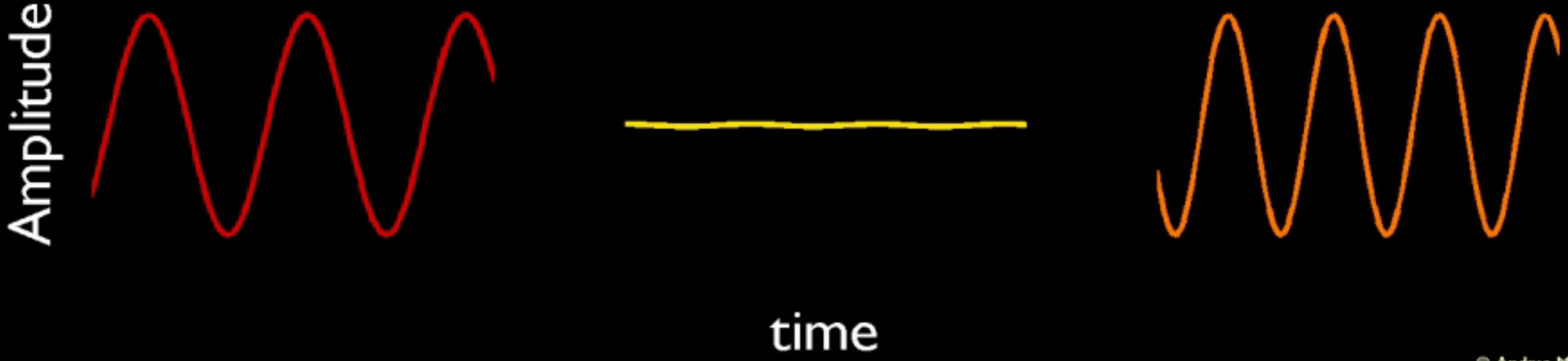
Asteroseismology

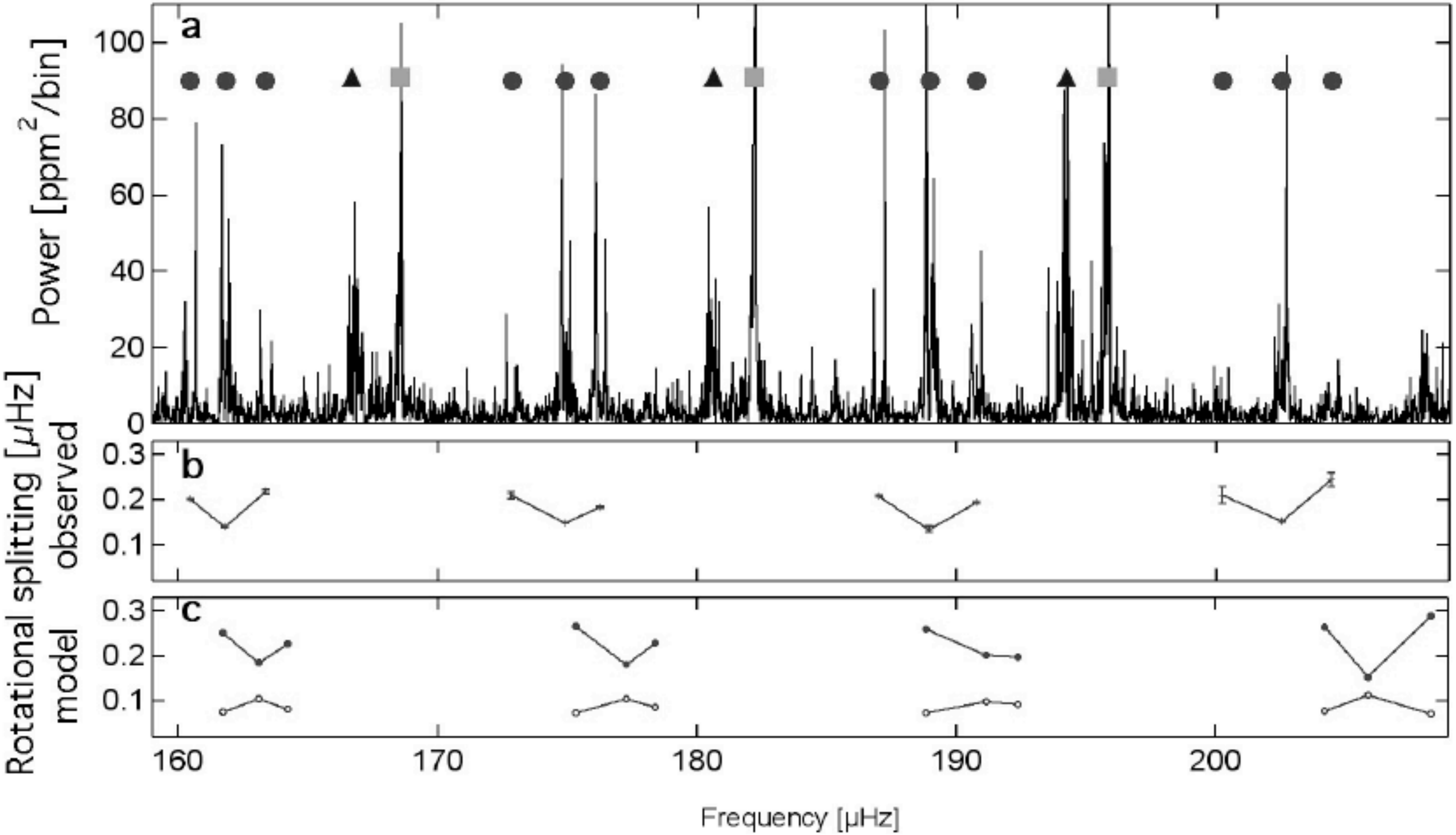
Solar-like oscillators and synergies



Inclination = 90°

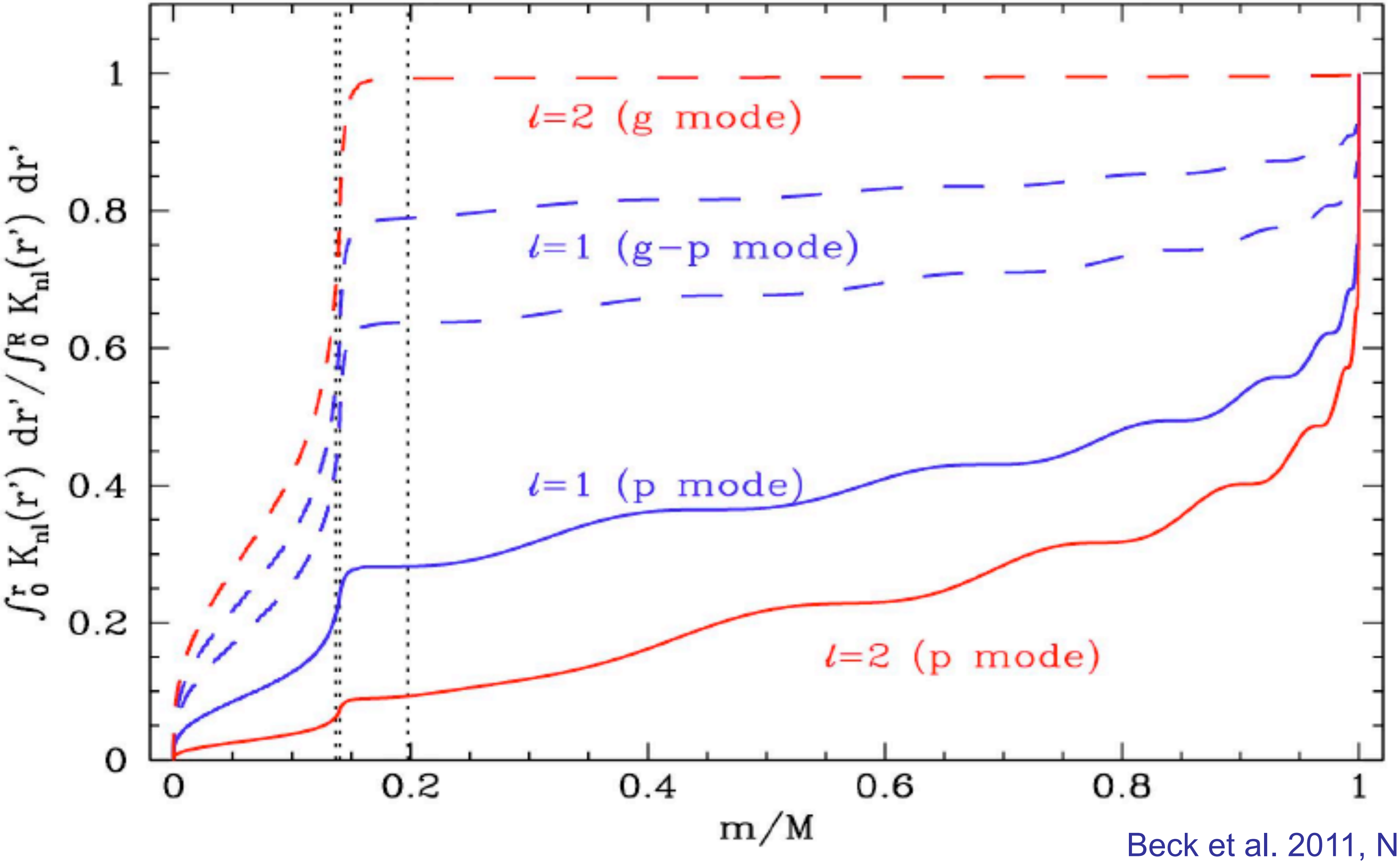


Rotation in red giants



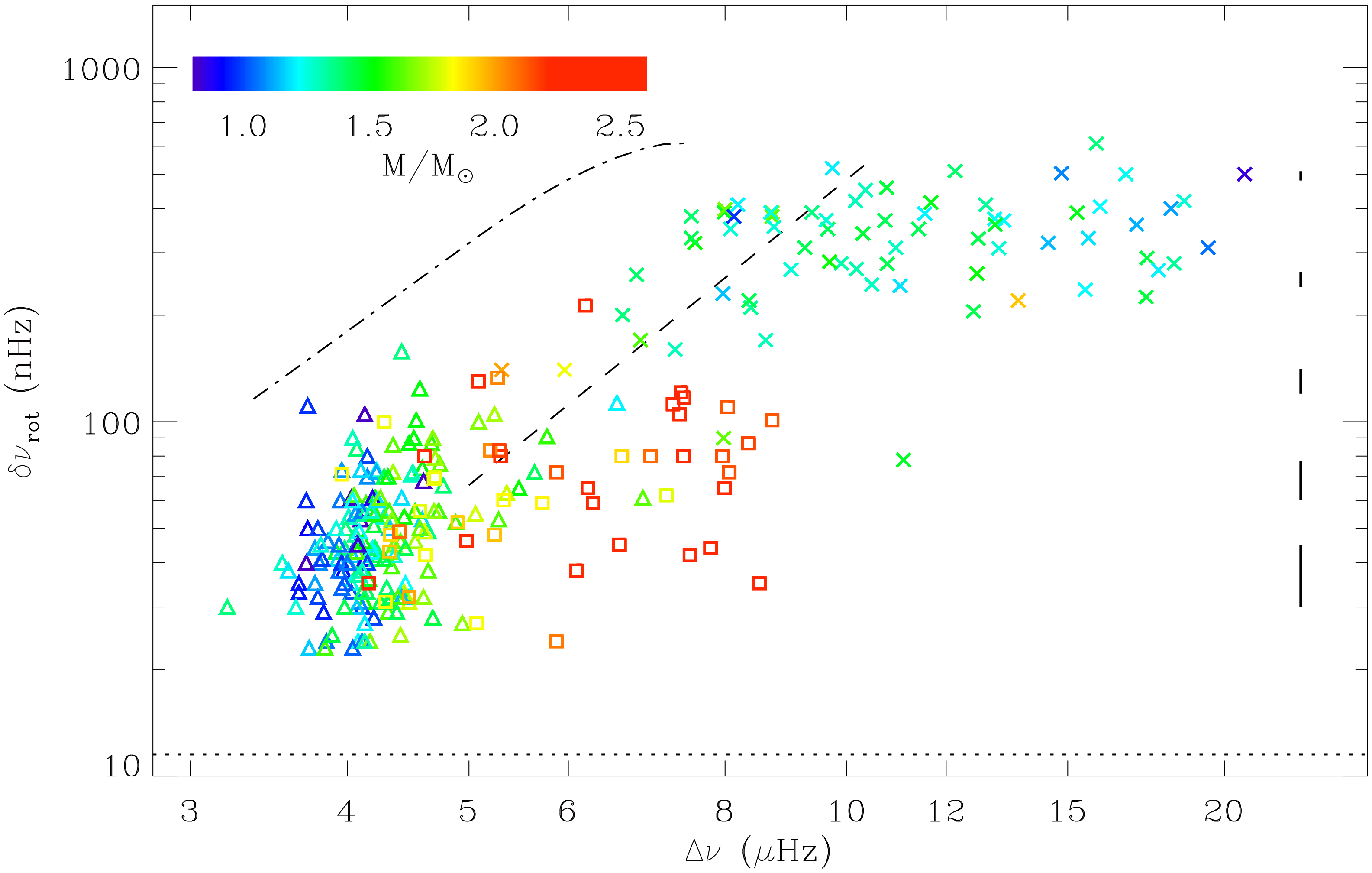
Beck et al. 2011, Nature

Rotation in red giants



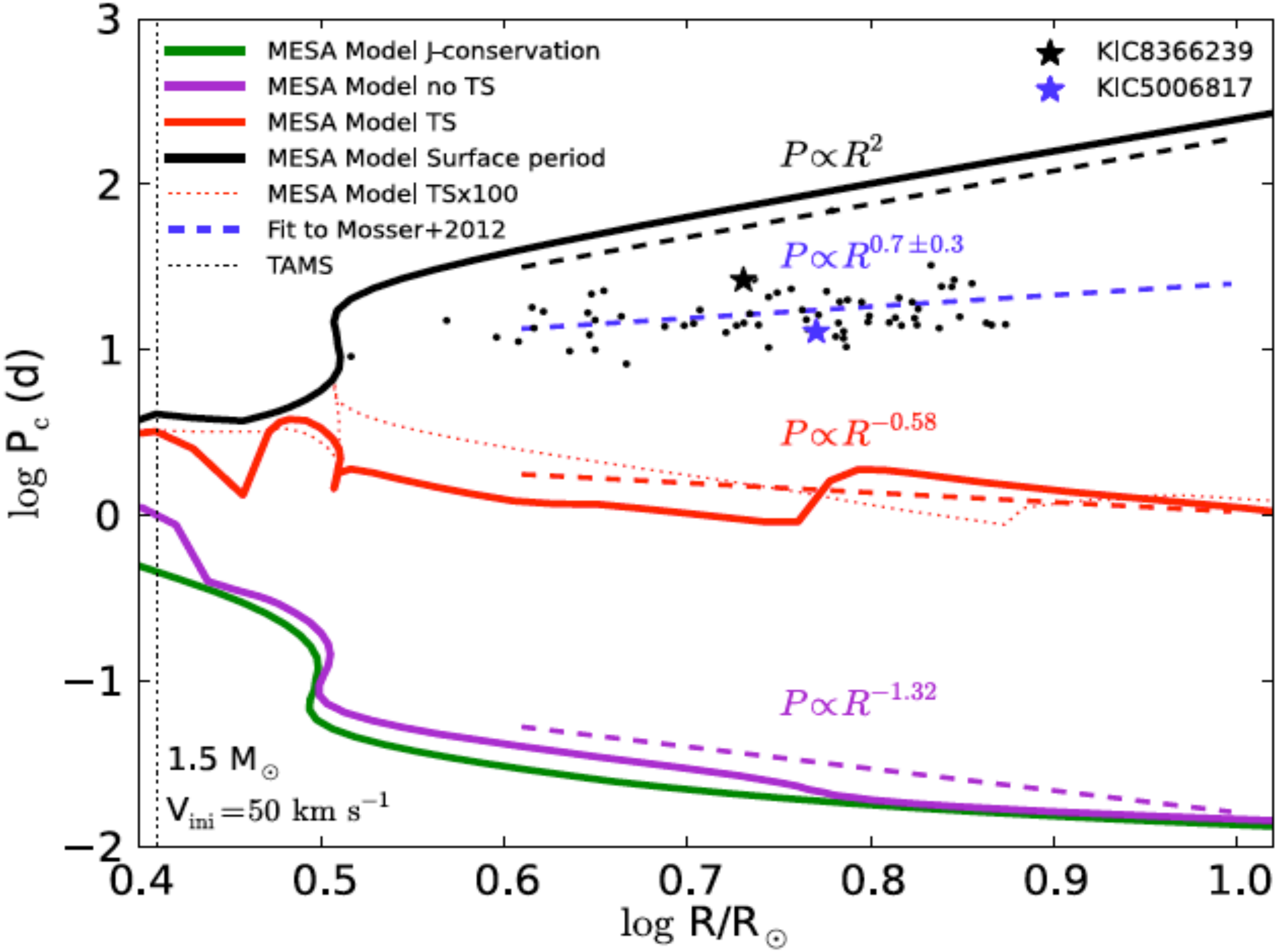
Beck et al. 2011, Nature

Observed core rotation



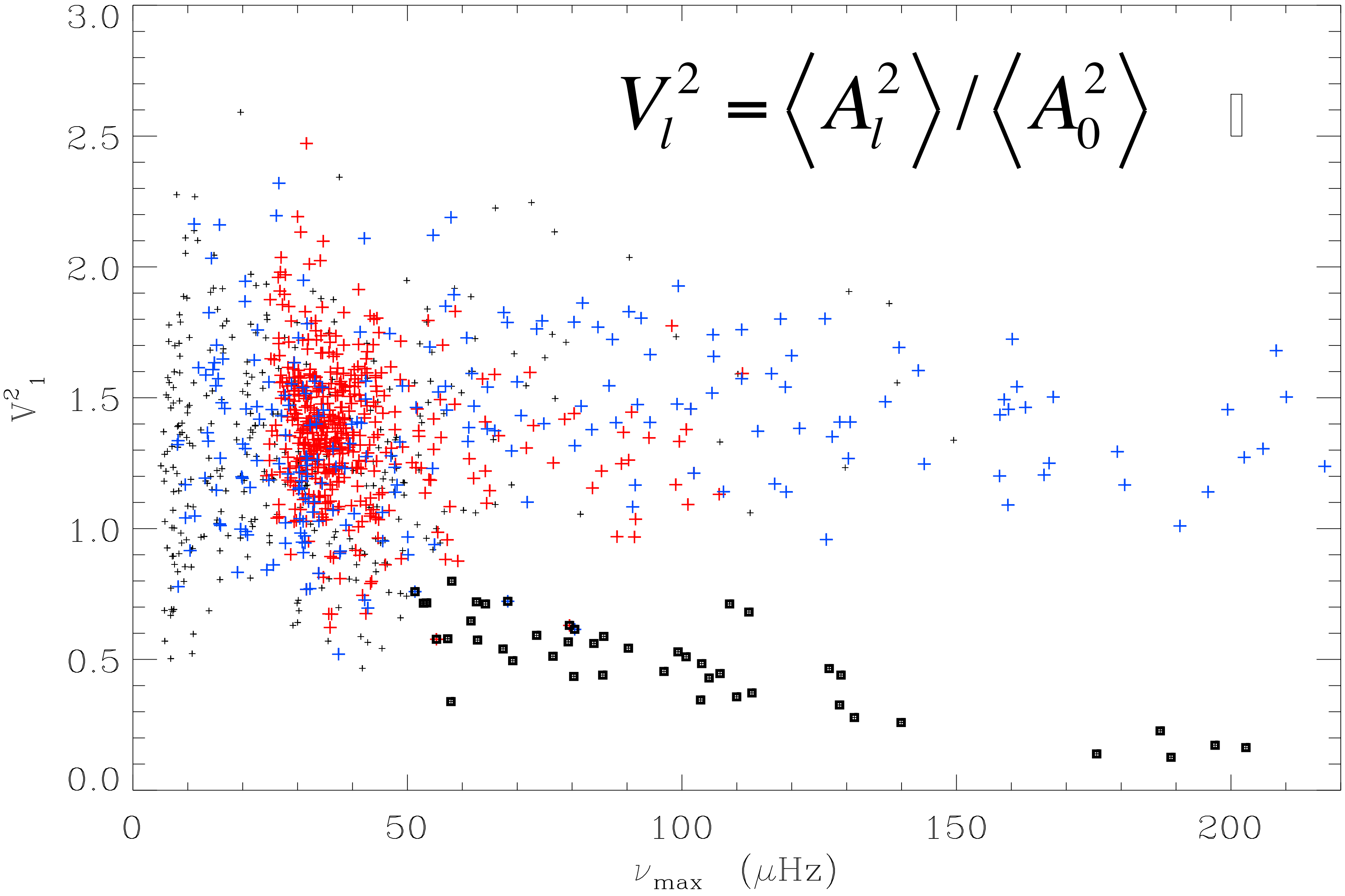
Mosser et al. 2012

Predicted core rotation



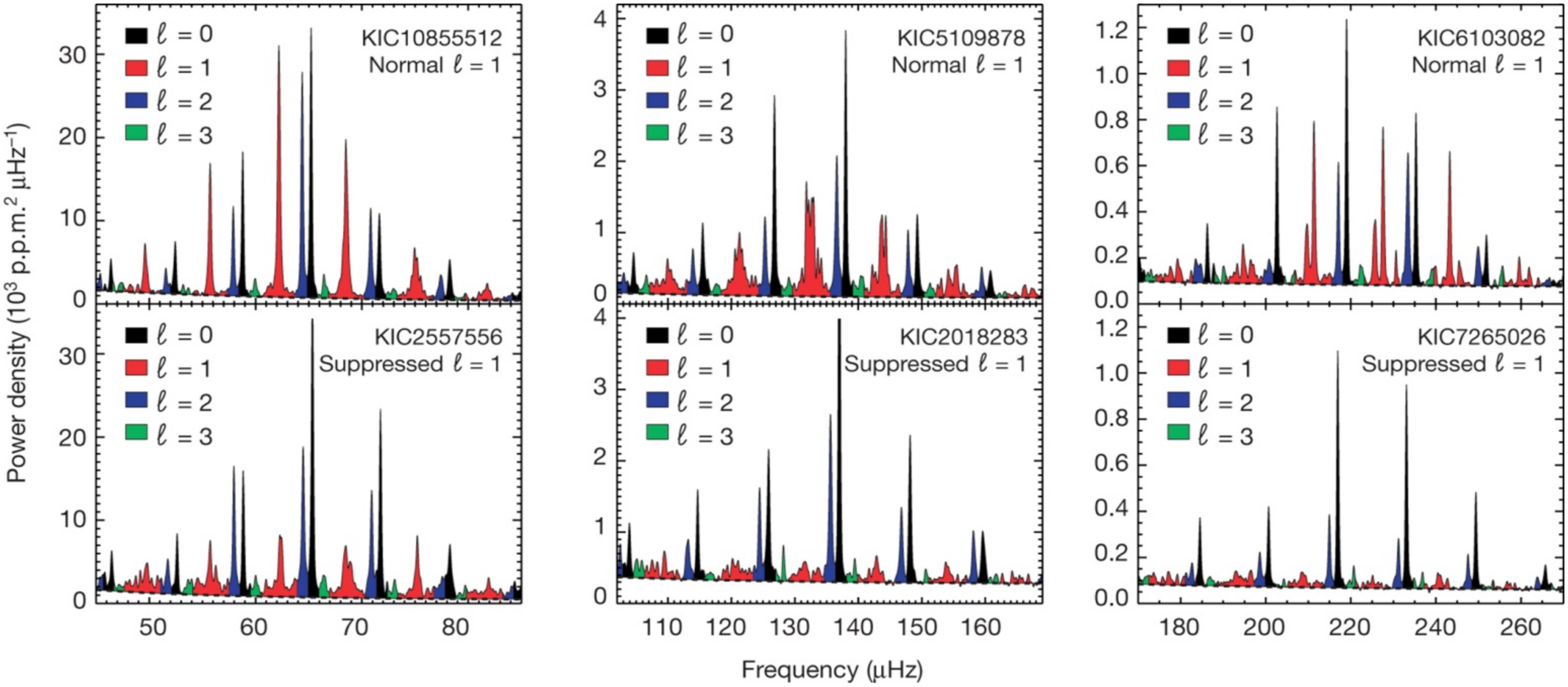
Cantiello et al. 2014

Suppressed dipole modes



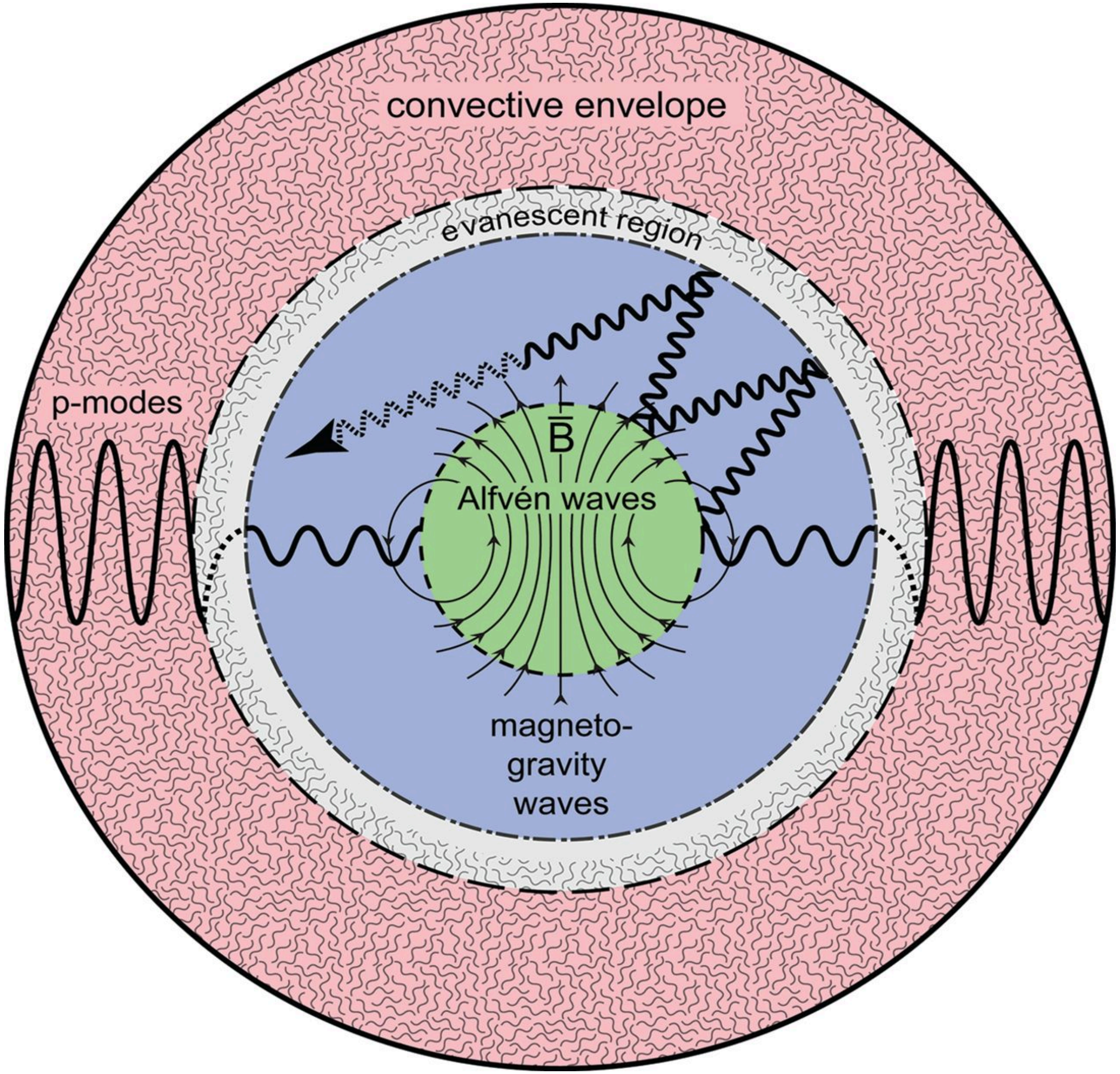
Mosser et al. 2011

Suppressed dipole modes



Stello et al. 2016

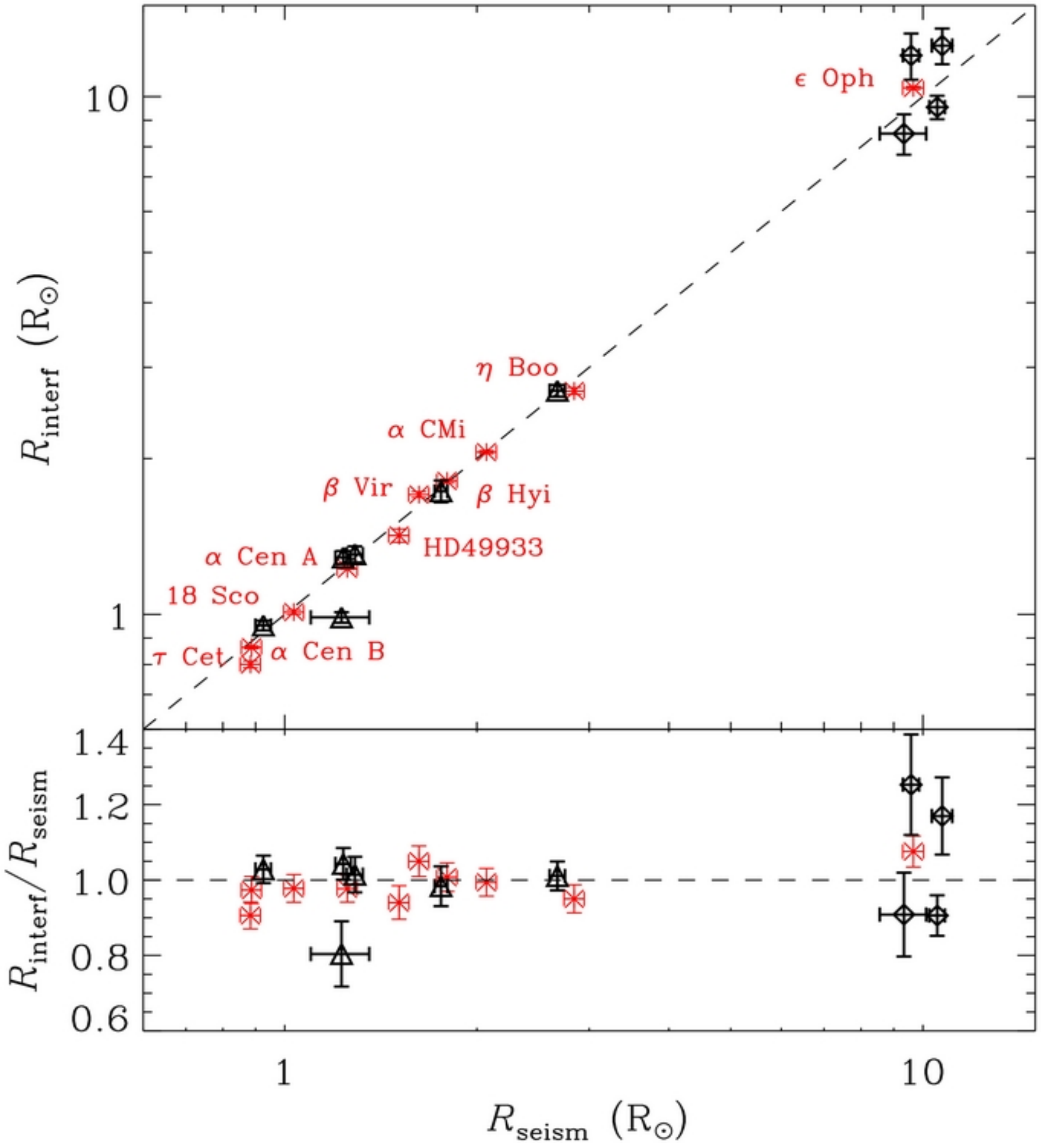
Suppressed modes: magnetic greenhouse



Fuller et al. 2015

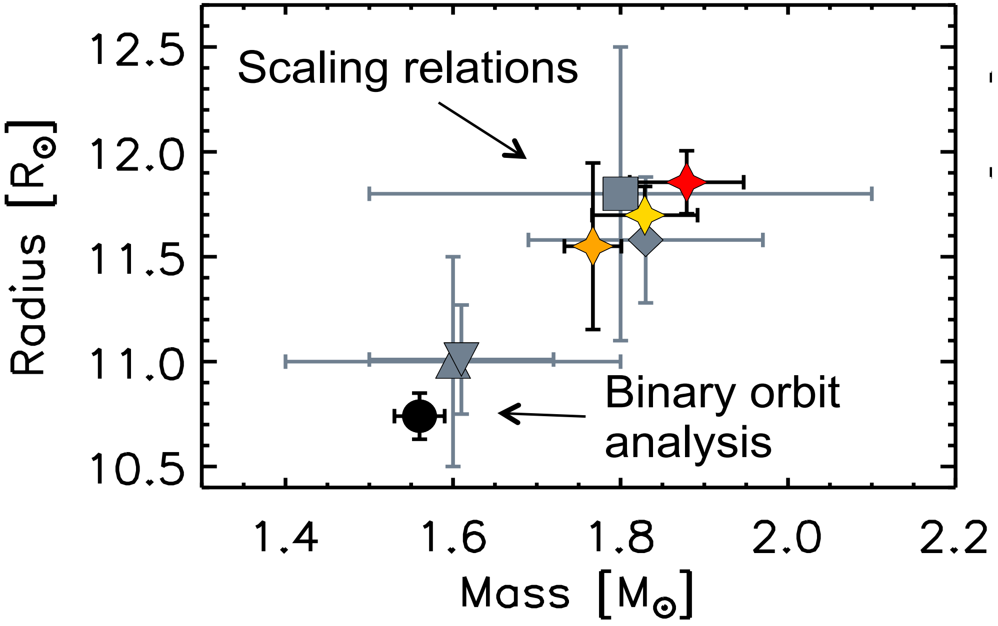
Testing asteroseismology

Testing asteroseismic radii



Huber et al. 2012

Testing scaling relations

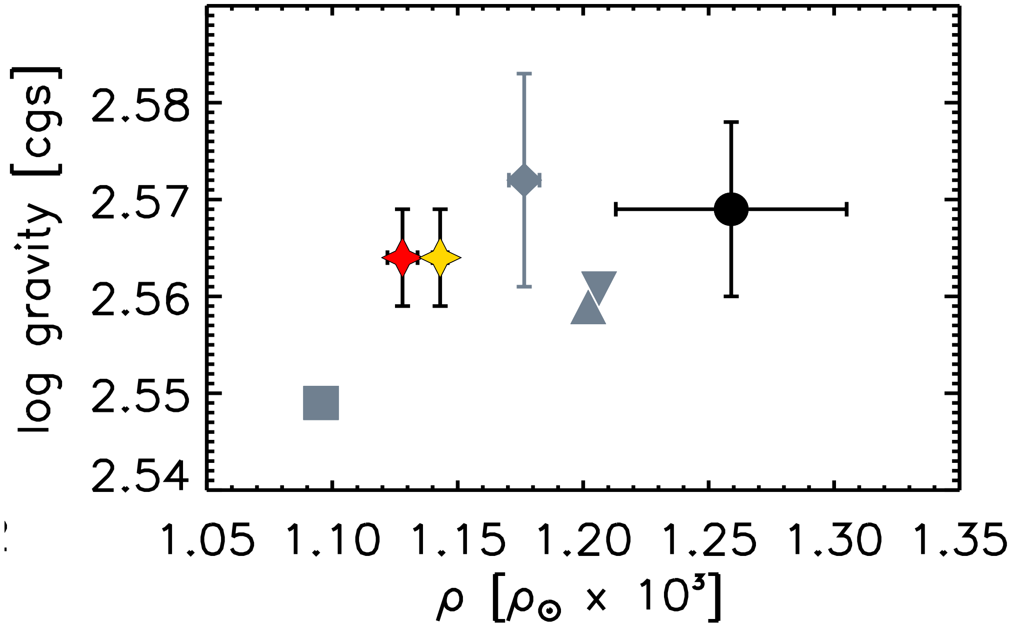


orbital solution

scaling relation + $\Delta\nu_\odot$

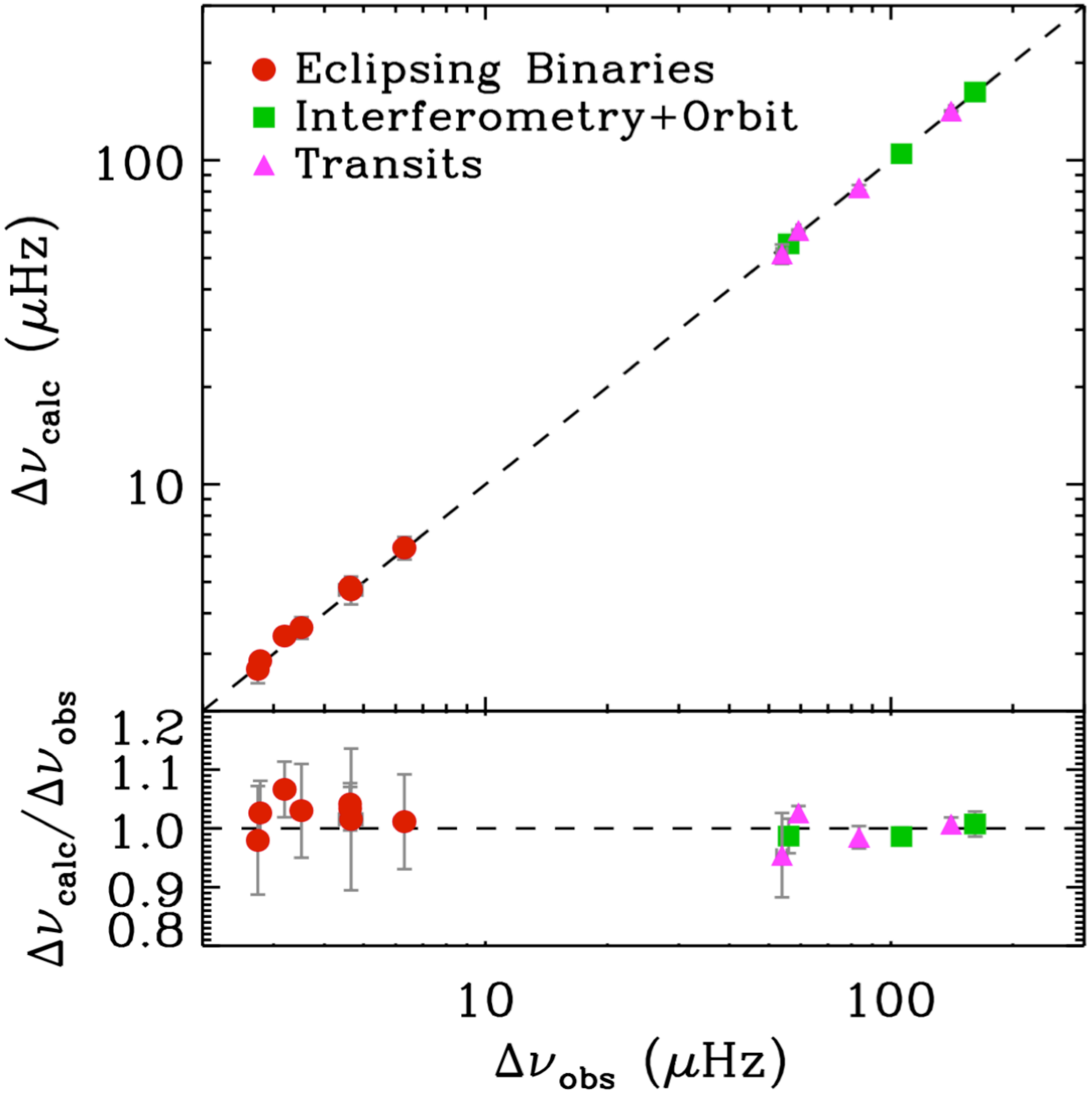
scaling relation + $\Delta\nu_{ref}$

grid-based modeling



Themeßl et al. in prep.

Testing asteroseismic densities

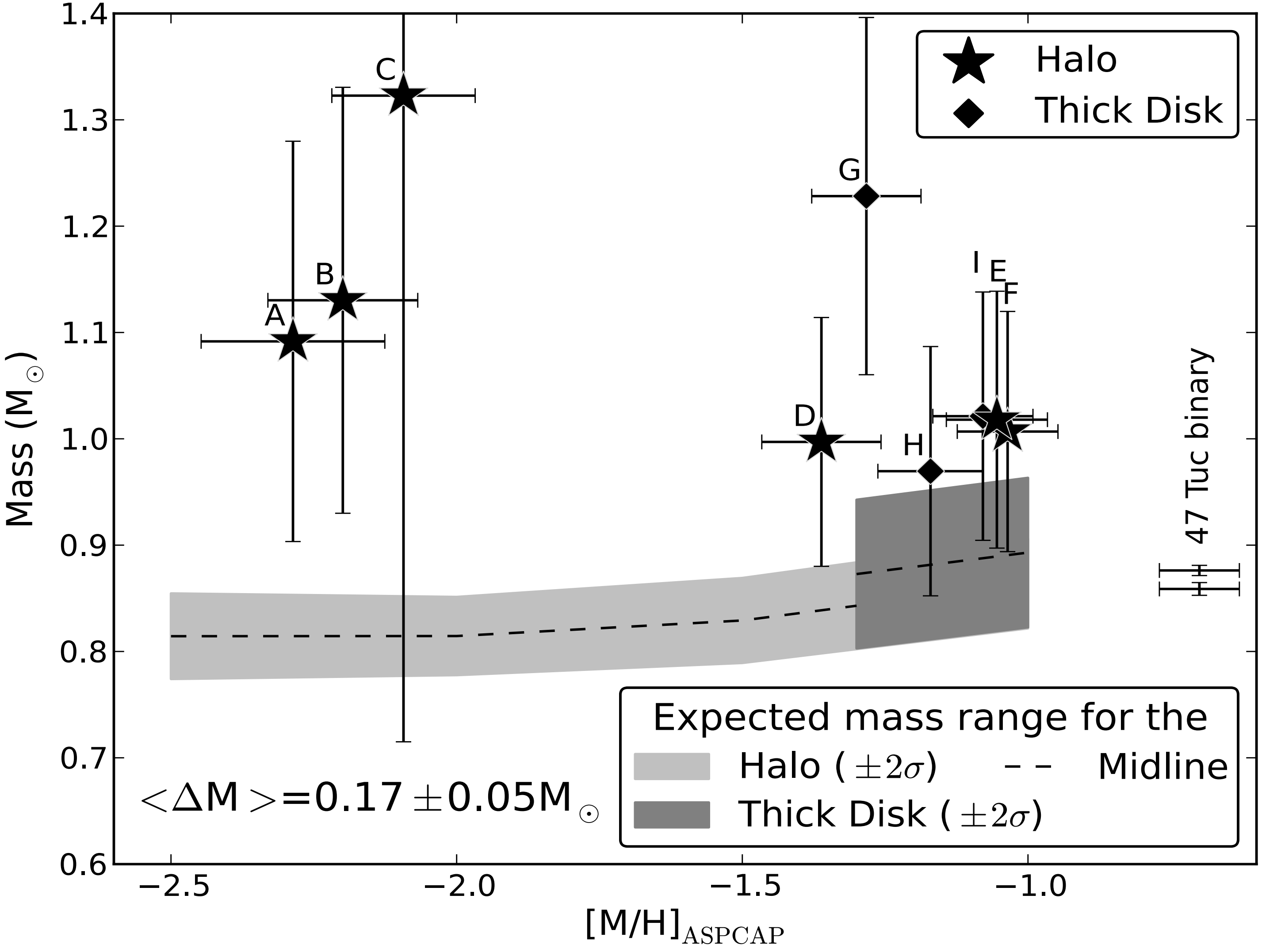


$$\rho_{\star,\rm transit} = \frac{3\pi}{GP^2}\left(\frac{a}{R_\star}\right)^3$$

Circular orbit!

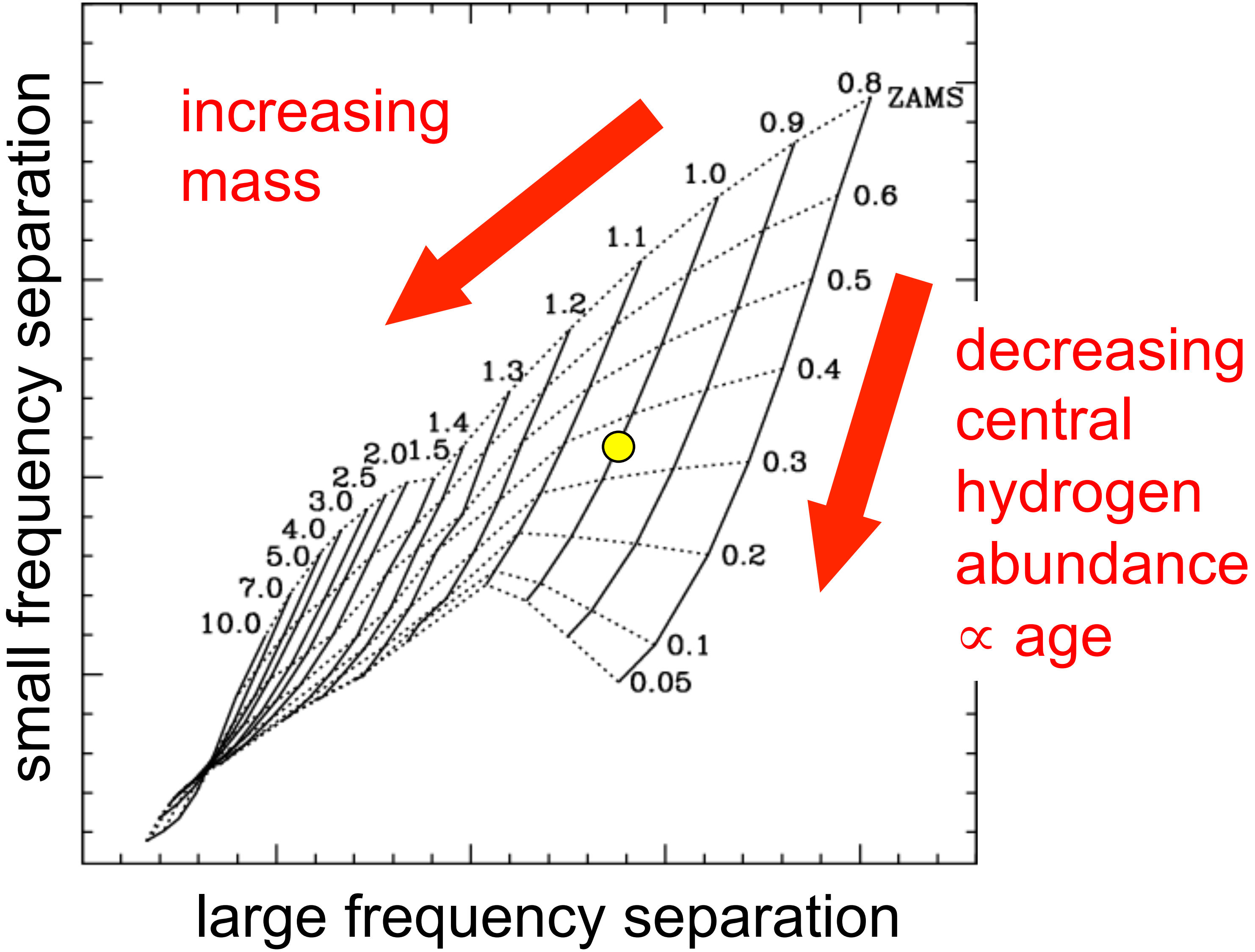
Huber et al. 2015

Halo stars



Epstein et al. 2014

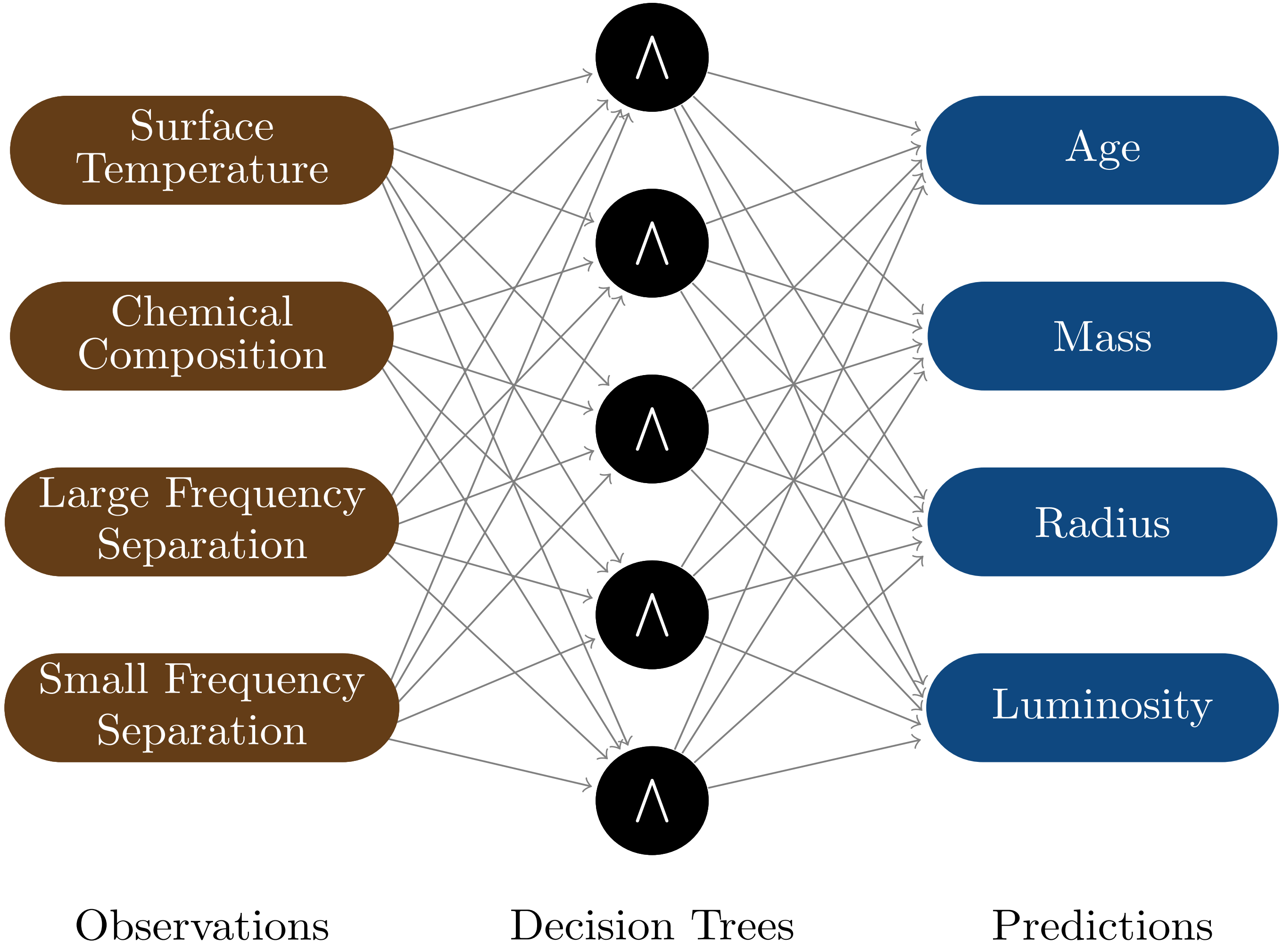
C-D Diagram



increasing mass

decreasing central hydrogen abundance $\propto$ age

Stellar parameters with Machine Learning



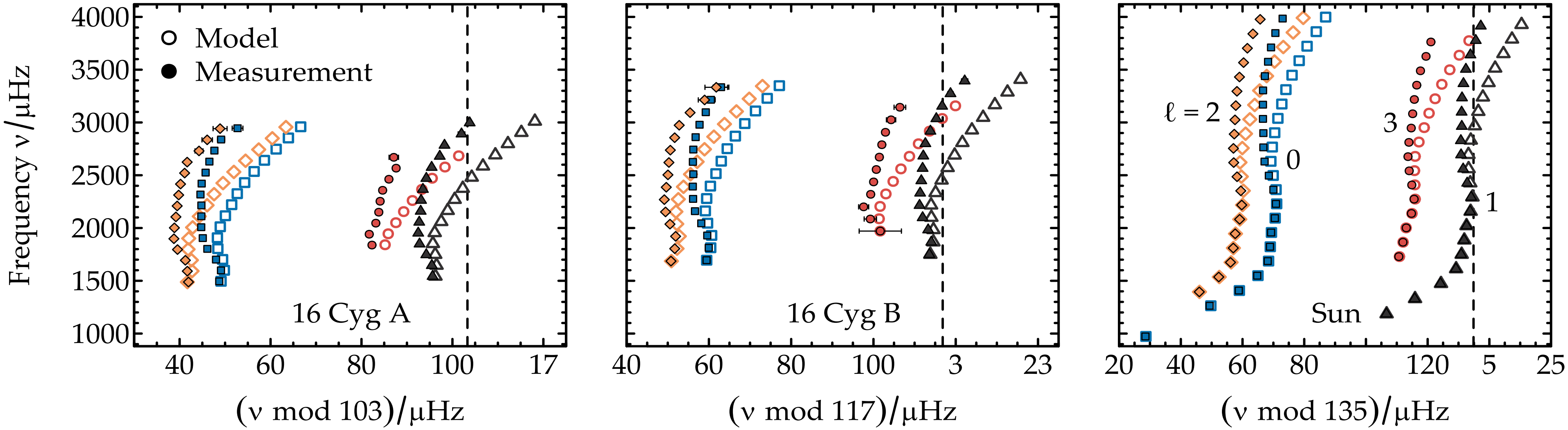
Bellinger et al. 2016

What is the intrinsic accuracy of age determinations?

Parameters		$\mu(\epsilon)$ [Gyr]
$\langle r_{02} \rangle$	$\nu_{\max}$	0.642
$\langle r_{02} \rangle$	log g	0.683
$\langle r_{13} \rangle$	$\nu_{\max}$	0.711
$\langle r_{02} \rangle$	$\langle \Delta\nu_0 \rangle$	0.694
$\langle \Delta\nu_0 \rangle$	$\langle \delta\nu_{02} \rangle$	0.701
$\langle r_{02} \rangle$	$\langle \delta\nu_{02} \rangle$	0.701
PC_2	PC_8	0.767
PC_2	PC_4	0.762
log g	$\langle \Delta\nu_0 \rangle$	1.29
log g	$T_{\rm eff}$	1.53

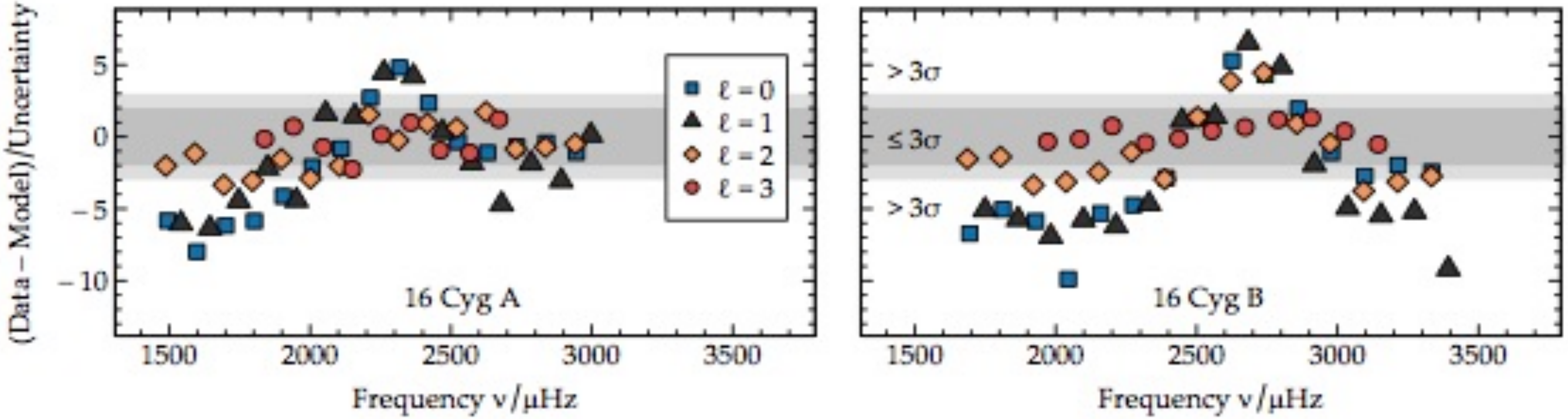
Angelou et al. (2017)

Stellar inversion



Bellinger et al. submitted

Stellar inversion

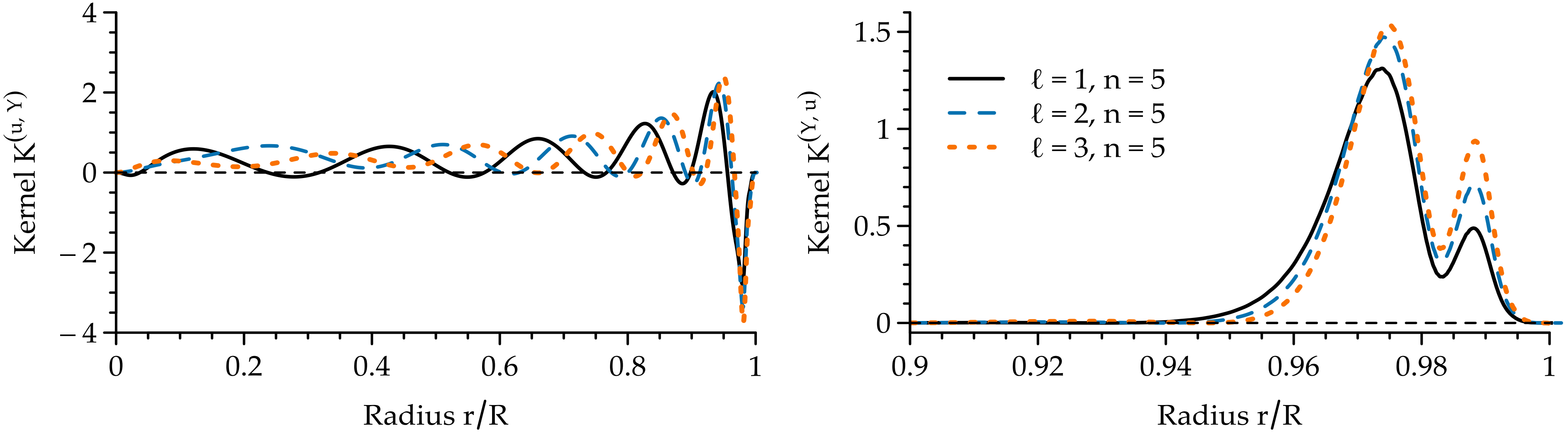


Bellinger et al. submitted

Stellar inversions

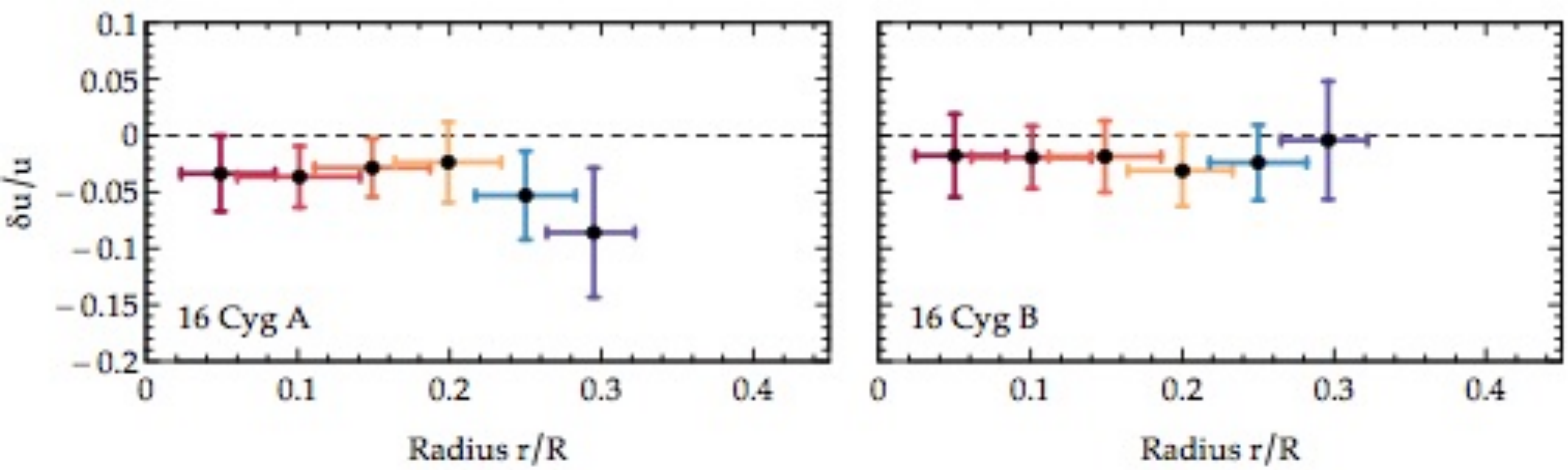
The problem of deducing small differences in structure between a star and a sufficiently close model by comparison of their mode frequencies.

Stellar inversion



Bellinger et al. submitted

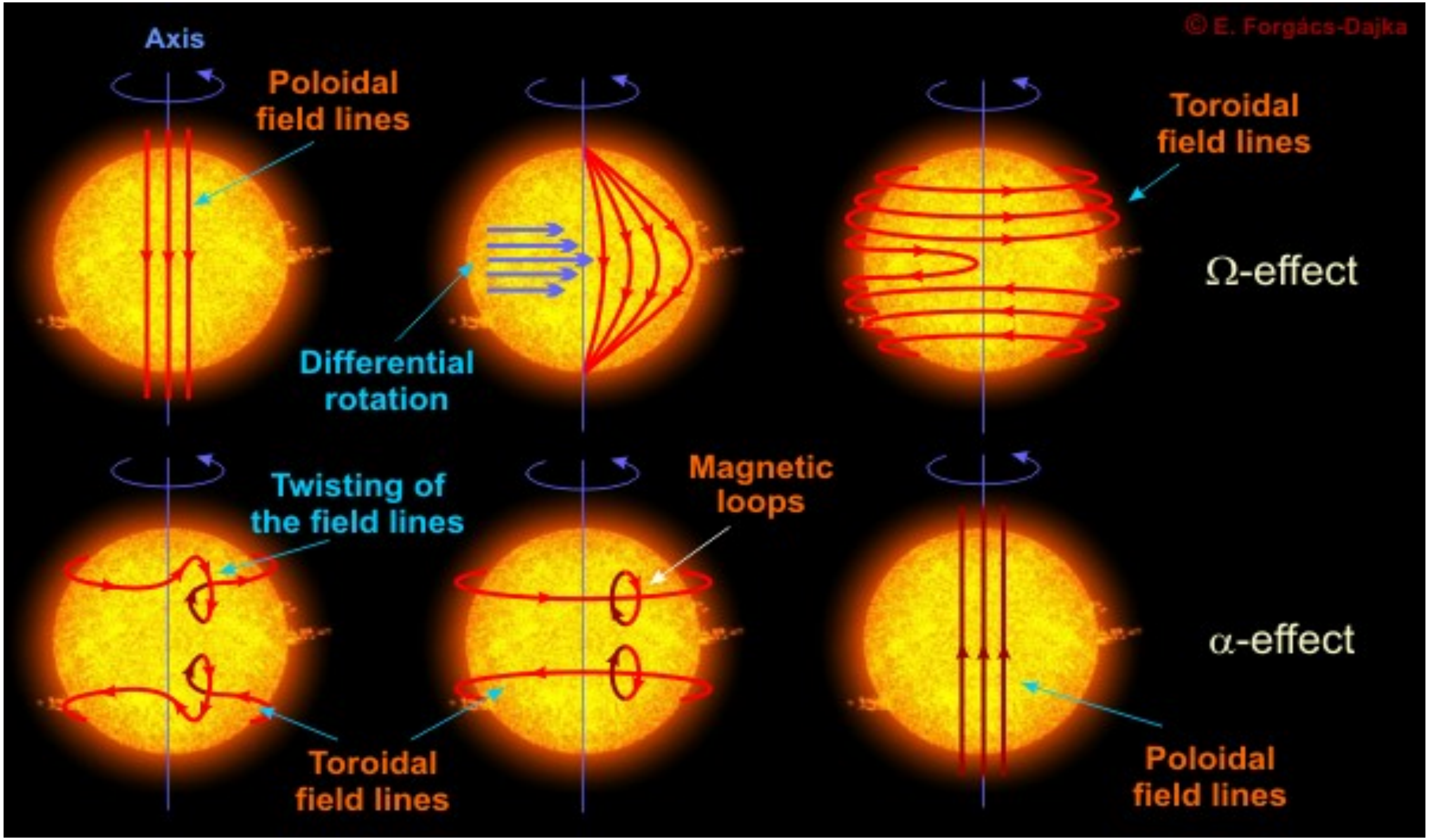
Stellar inversion



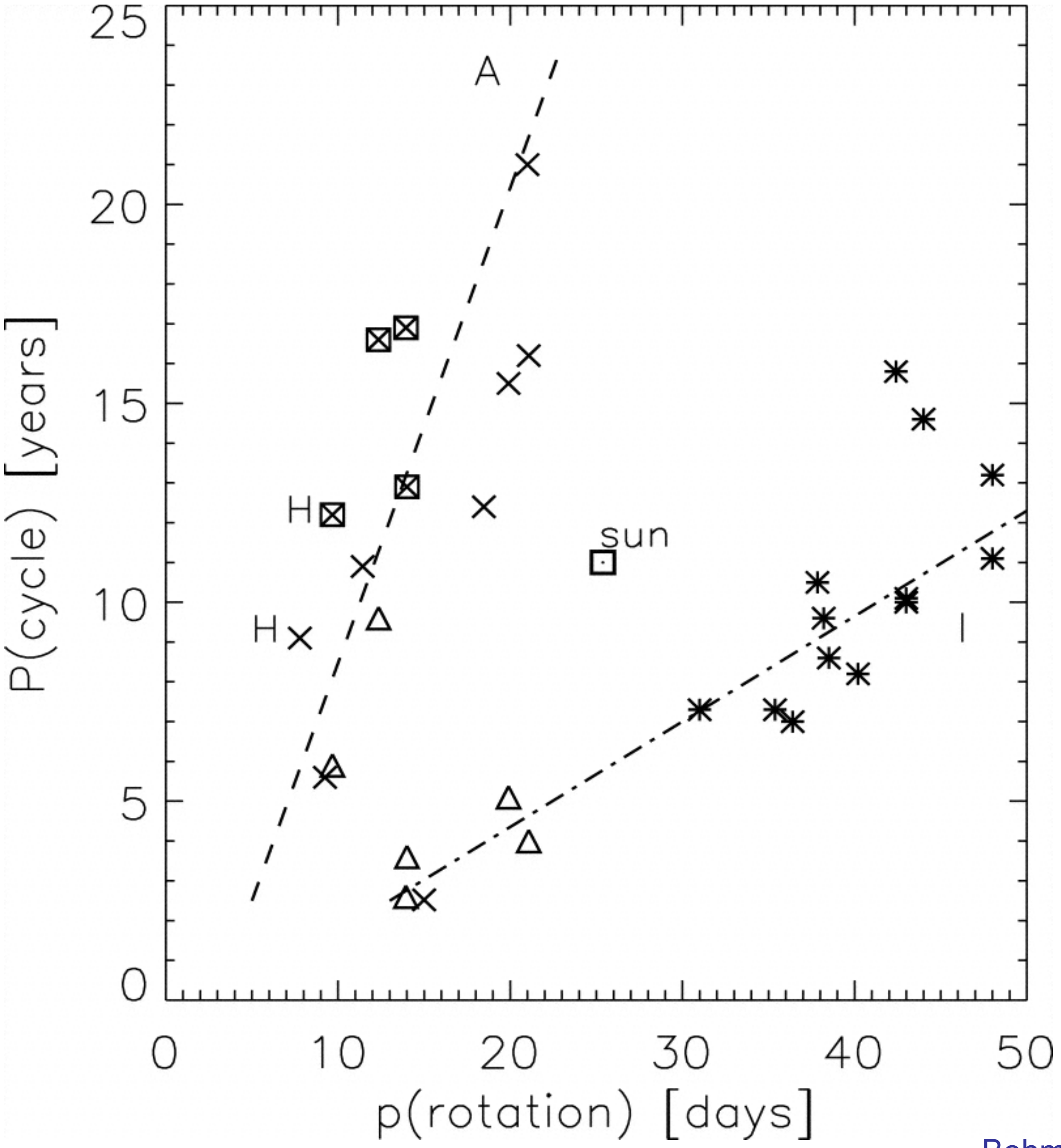
Bellinger et al. submitted

SYNERGIES

Hale cycle



Created by E. Forgacs-Dajka

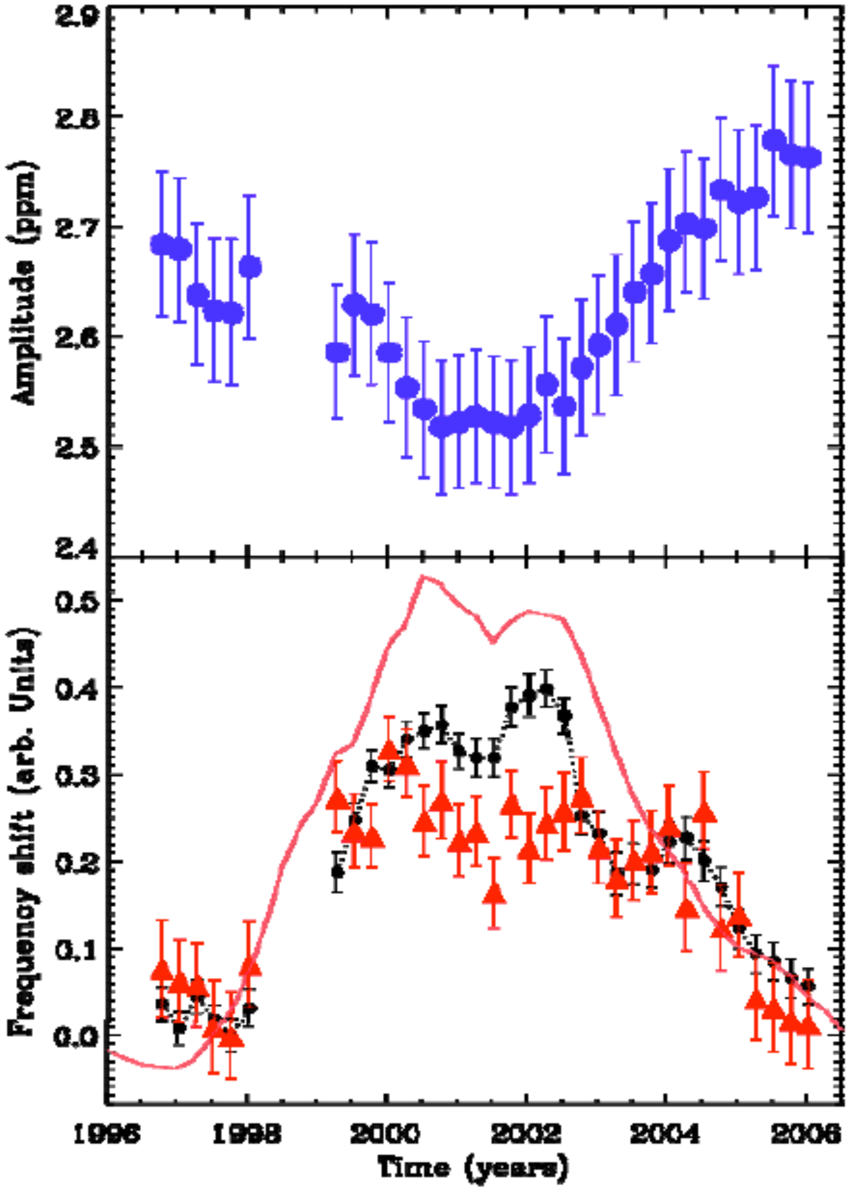


Bohm-Vitense 2007, ApJ 657, 486

Effects on stellar properties

- Perturbations induced by the magnetic fields in the outer parts of the star influence the oscillation cavity and thus the frequencies with high upper-turning points.
- Magnetic structures are strong absorbers of p-mode oscillations by diminishing the turbulent velocities in a convective unstable layer affecting the driving of the modes.

Sun



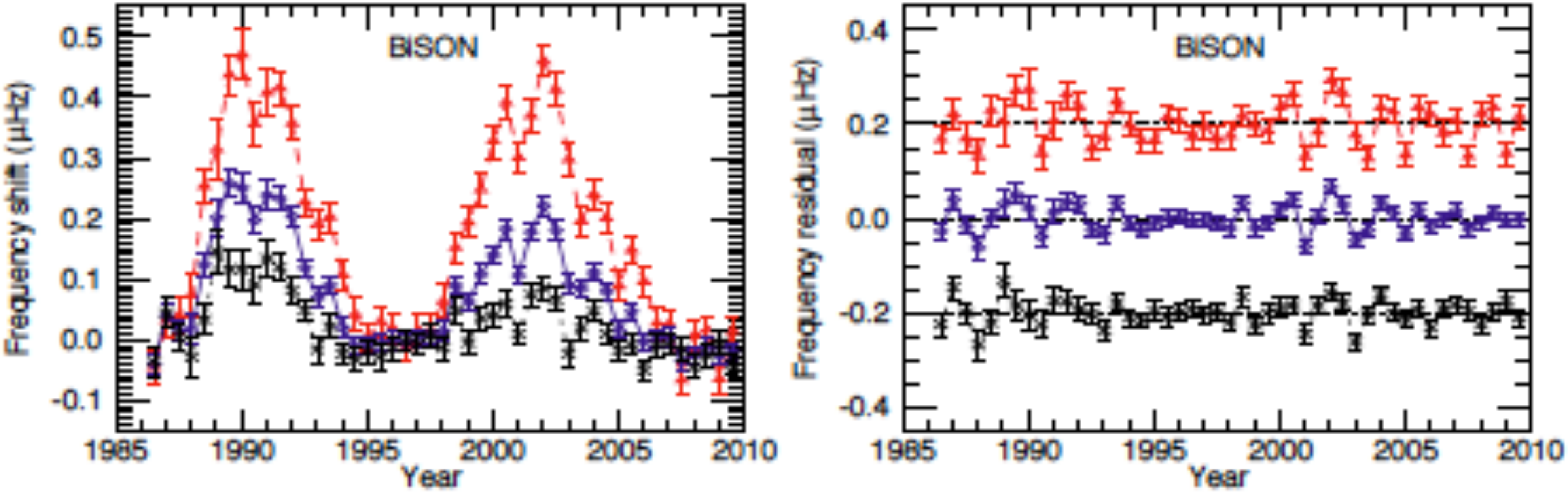
▲ Measured freq shifts (cross-correlation)

● Measured freq shifts (ind. Freq)

— International Sunspot number

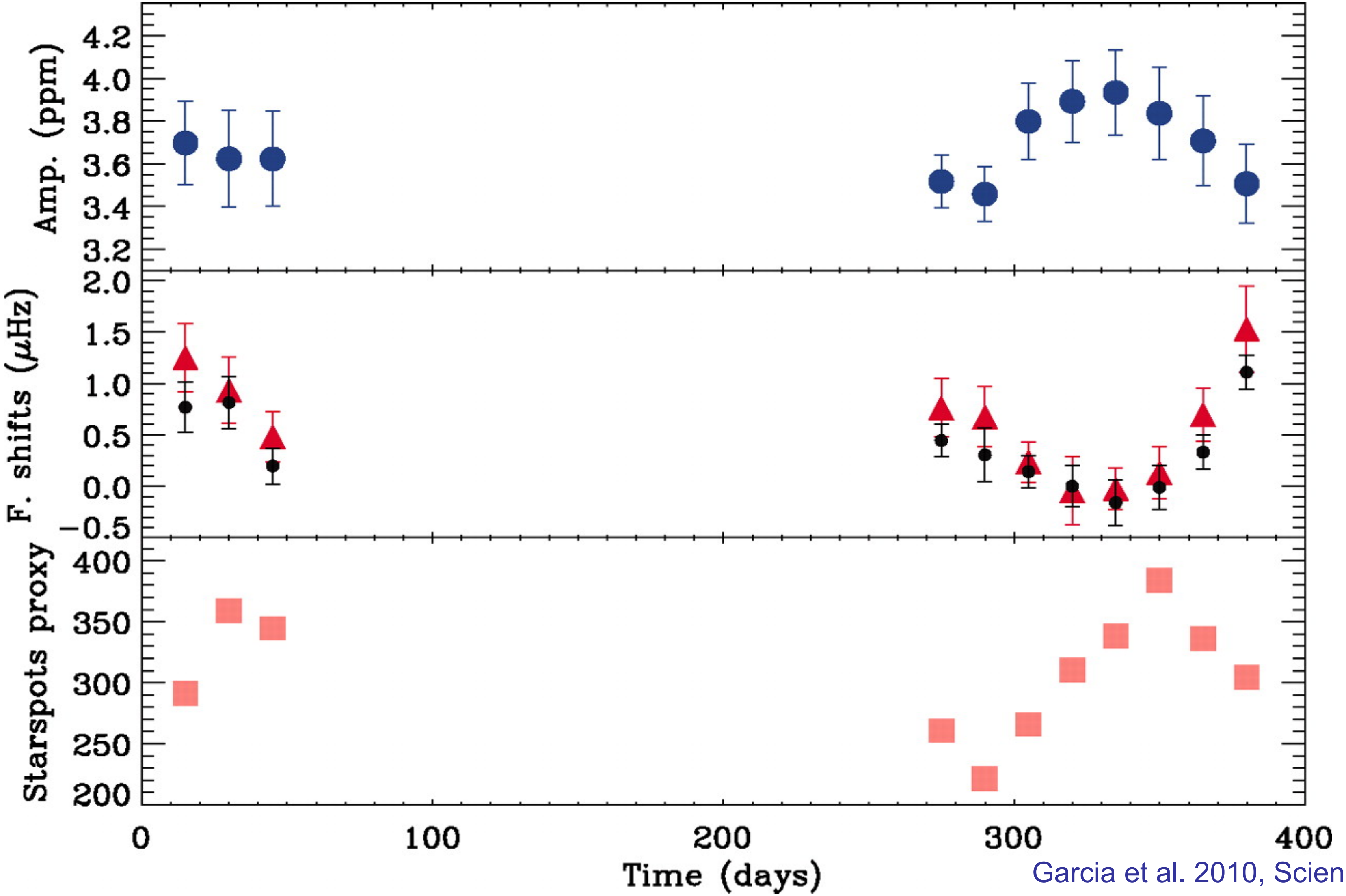
Garcia et al. 2010, Science 329, 1032

Sun: second dynamo...?



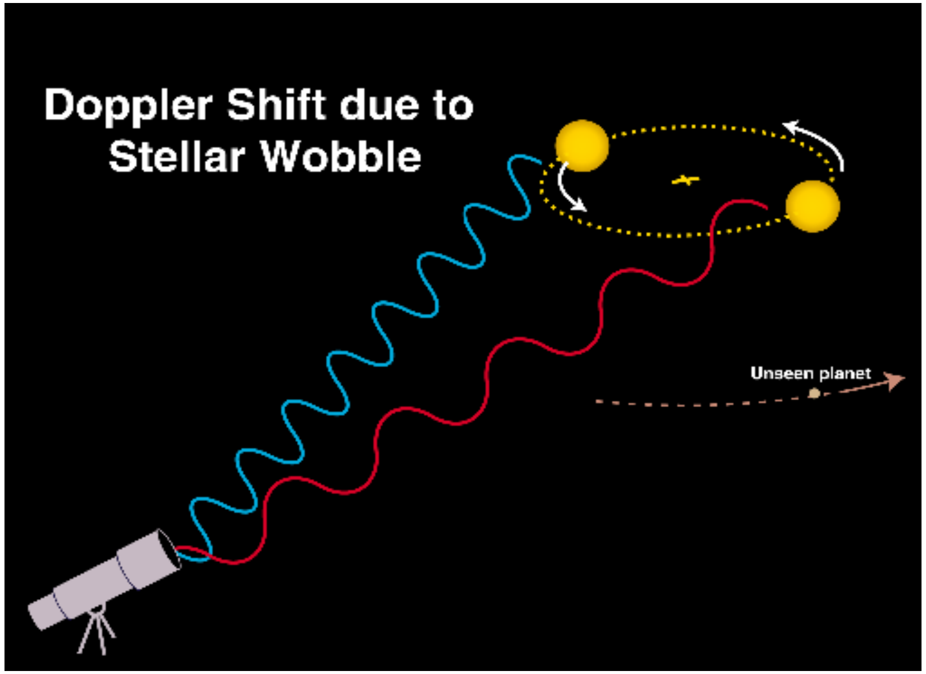
Broomhall et al. 2012, MNRAS 420, 1405

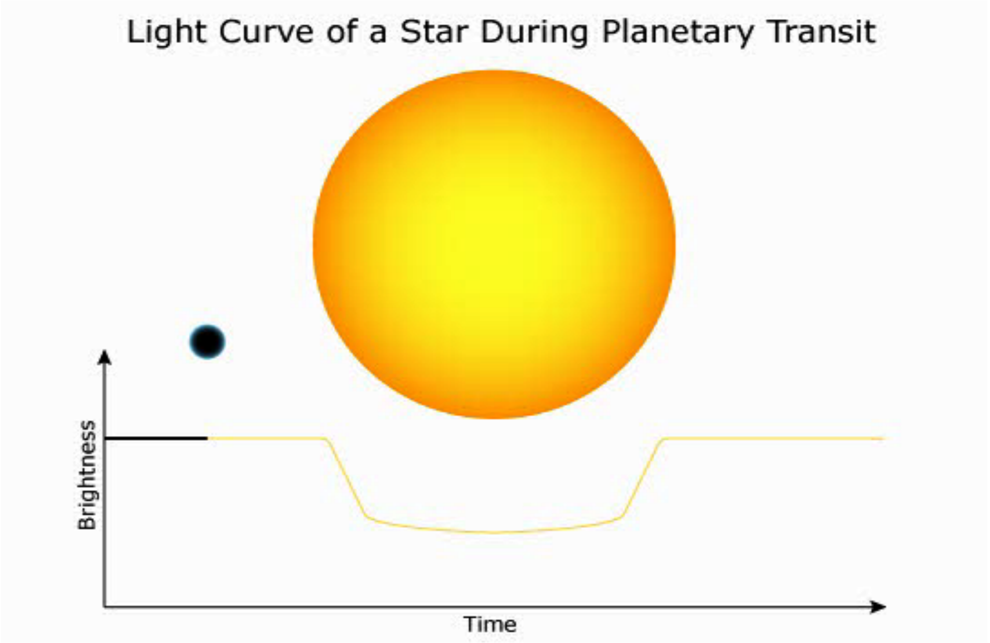
HD 49933



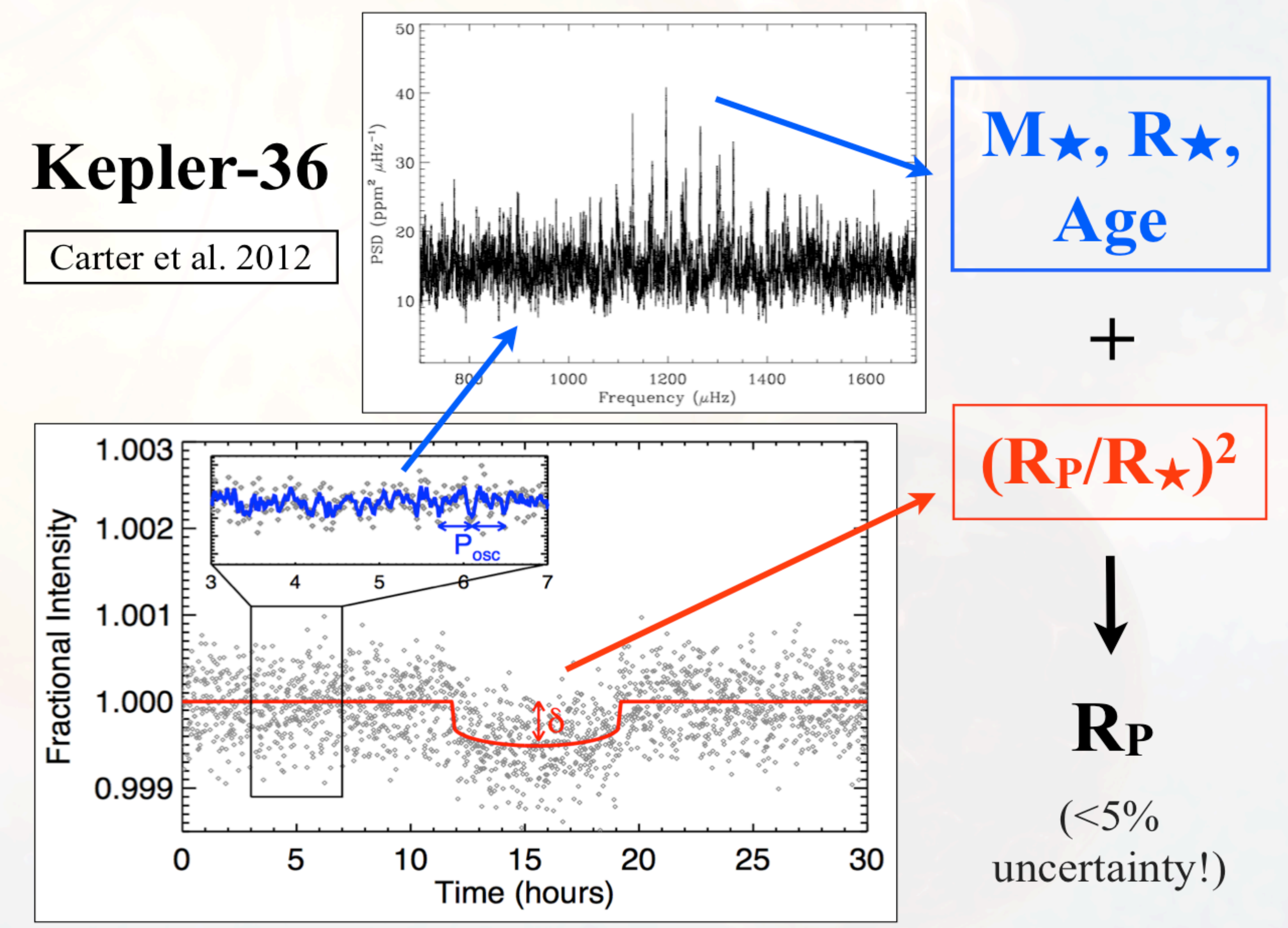
Garcia et al. 2010, Science 329, 1032

Exoplanets


Doppler Shift due to Stellar Wobble
Unseen planet


Light Curve of a Star During Planetary Transit
Brightness
Time

Planet radius



Kepler-36

Carter et al. 2012

$M_\star$, $R_\star$, Age

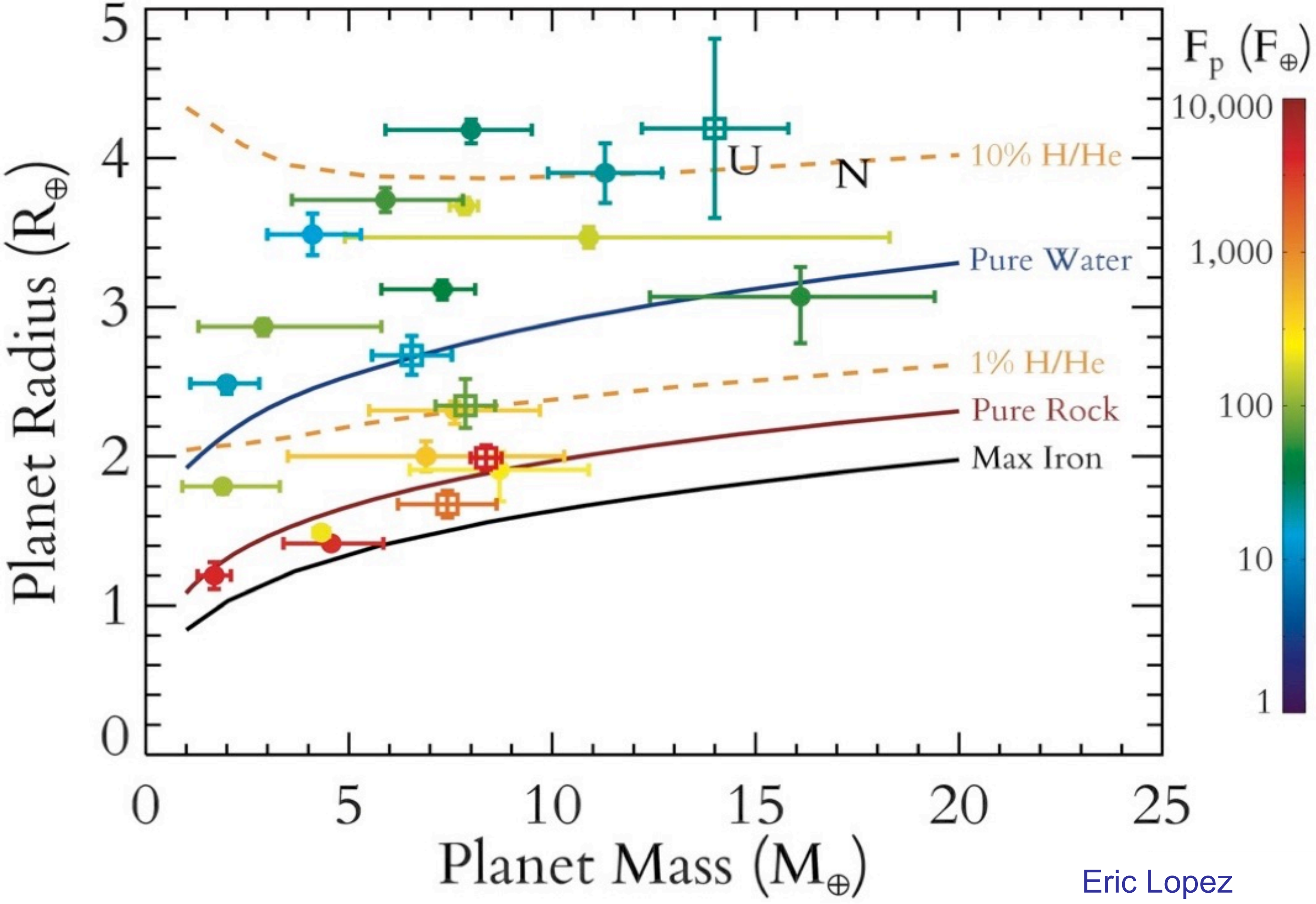
+

$(R_P/R_\star)^2$

R_P

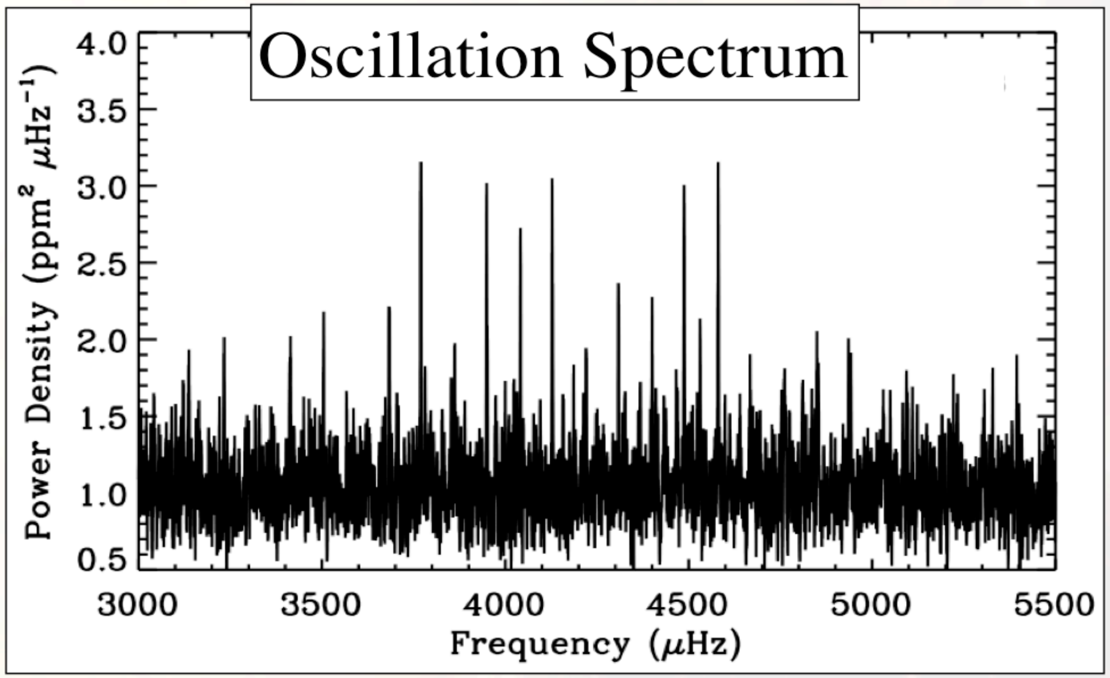
(<5% uncertainty!)

Planet composition



Eric Lopez

Kepler 444



T_{eff} ~ 5000K, [Fe/H] ~ -0.6, high proper motion!

Asteroseismology + Spectroscopy:

$R = 0.75 +/- 0.01\ R_\odot$

$M = 0.76 +/- 0.04\ M_\odot$

Age = 11.2 +/- 0.9 Gyr

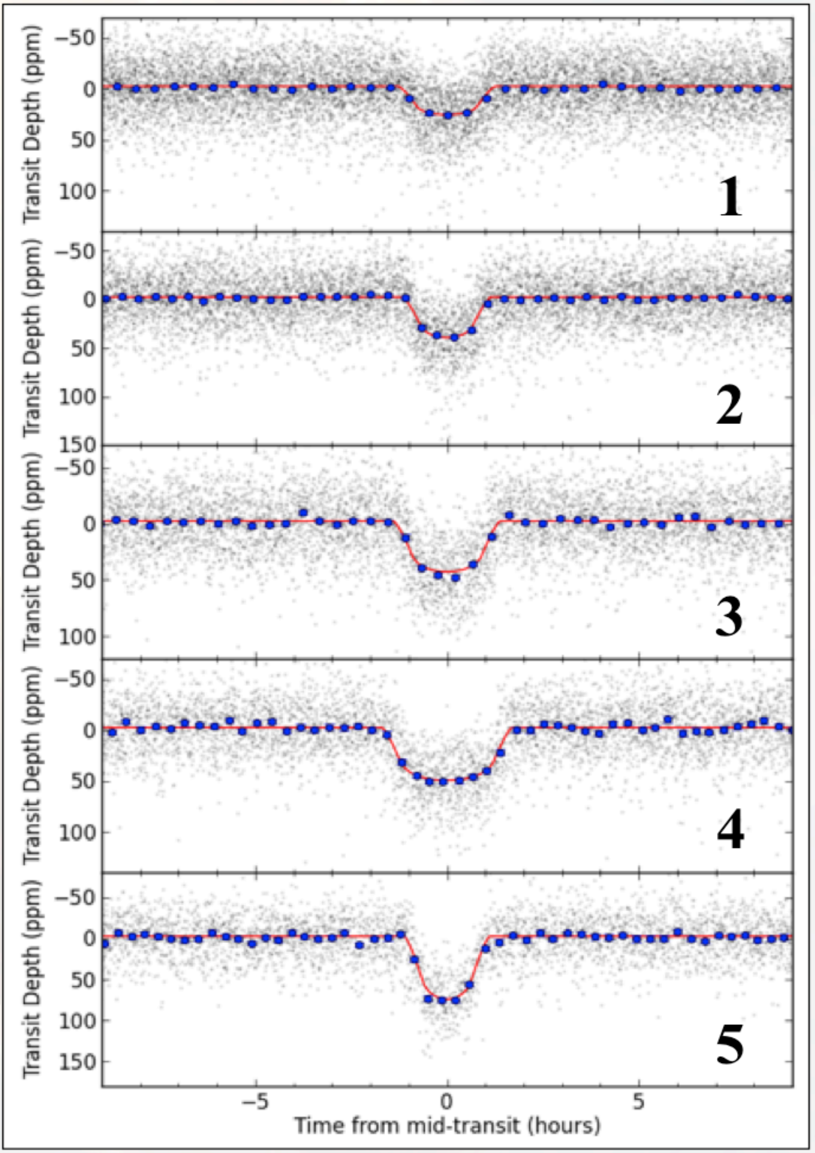
Campante et al. 2015

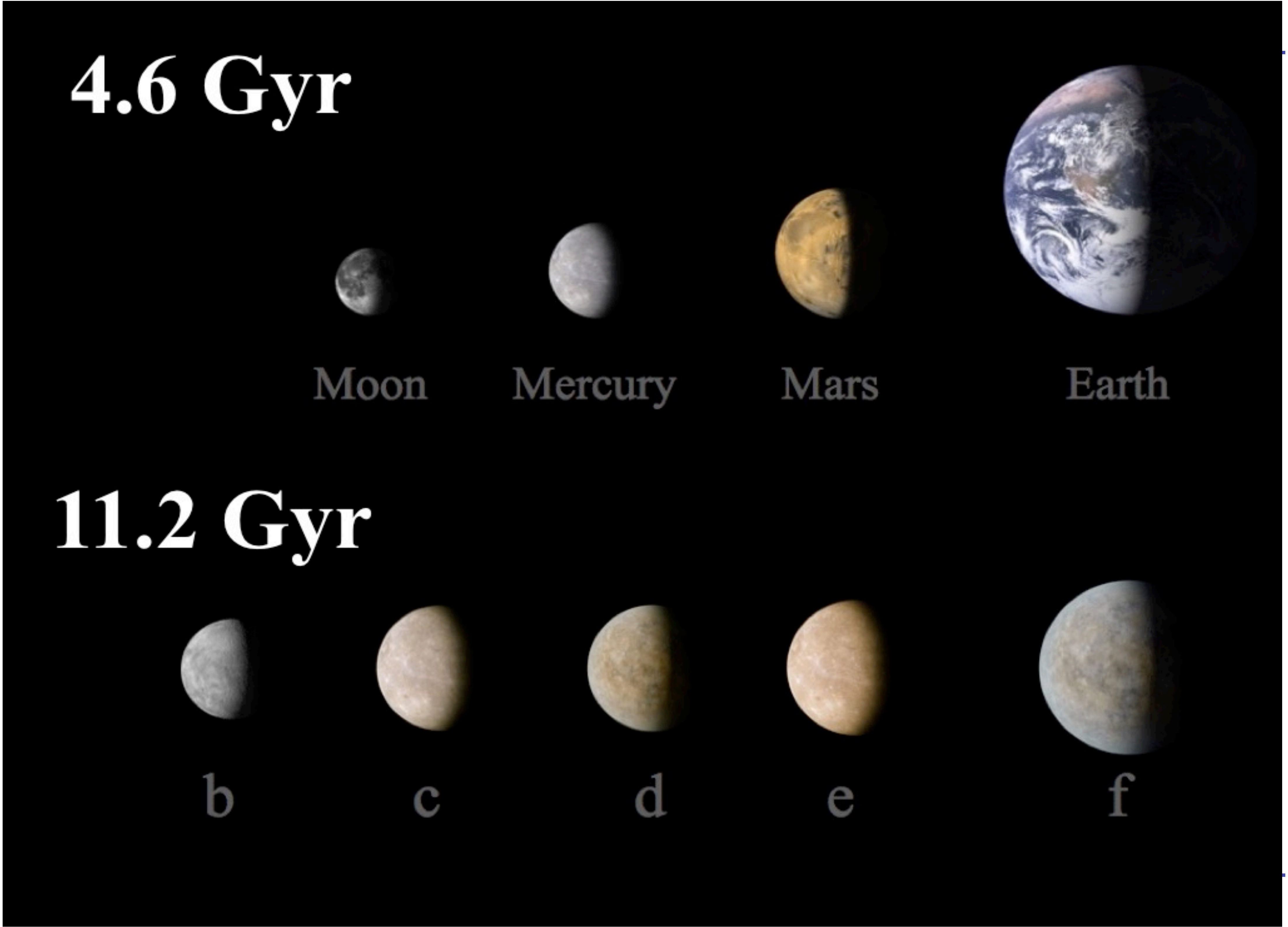
Kepler 444

5 transiting planets: all smaller than Earth, all with periods < 10 days!

$R_{01} = 0.395+/-0.015\ R_{\oplus}$
$R_{02} = 0.499+/-0.025\ R_{\oplus}$
$R_{03} = 0.511+/-0.018\ R_{\oplus}$
$R_{04} = 0.535+/-0.017\ R_{\oplus}$
$R_{05} = 0.725+/-0.061\ R_{\oplus}$

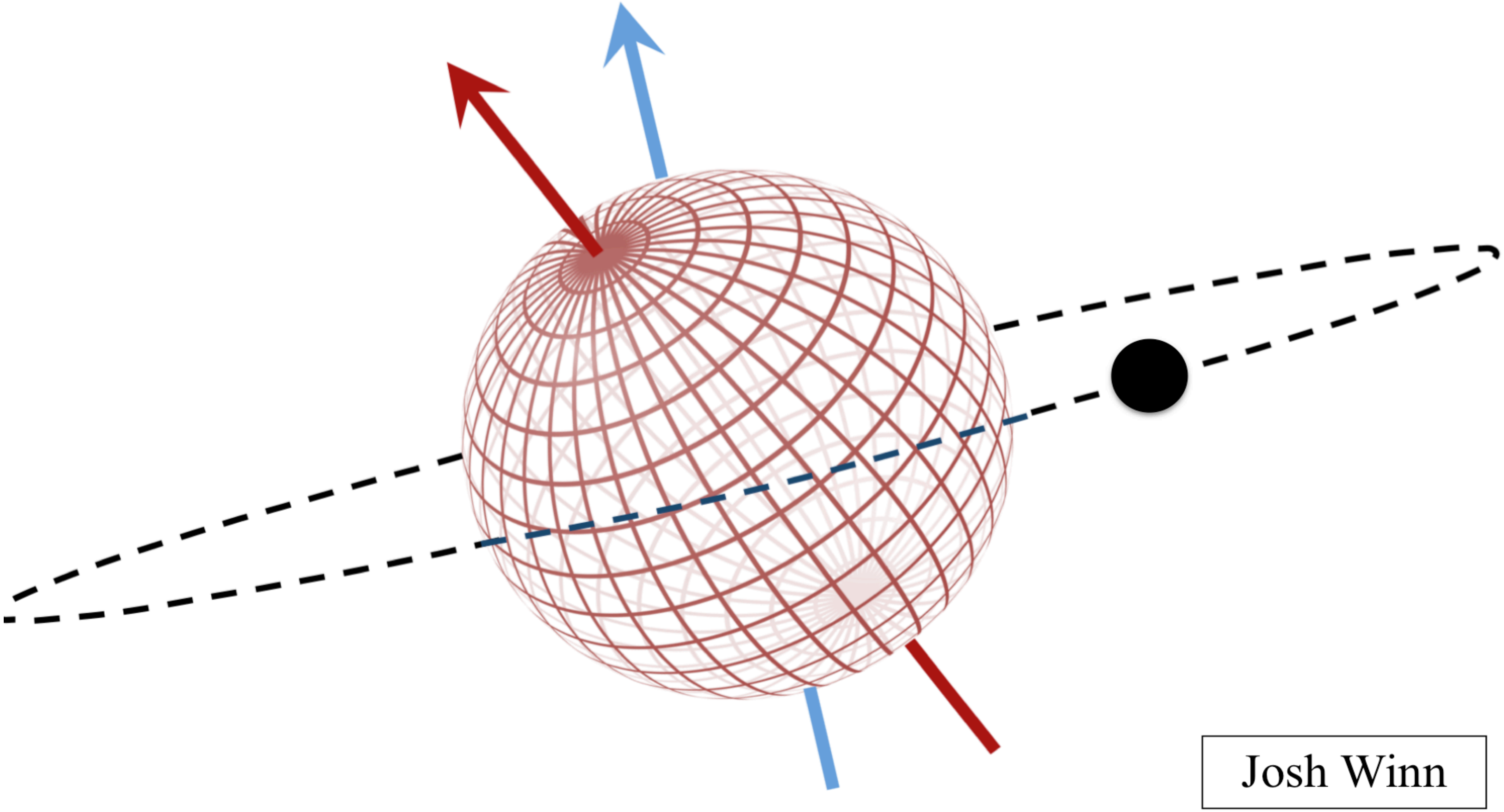
Campante et al. 2015



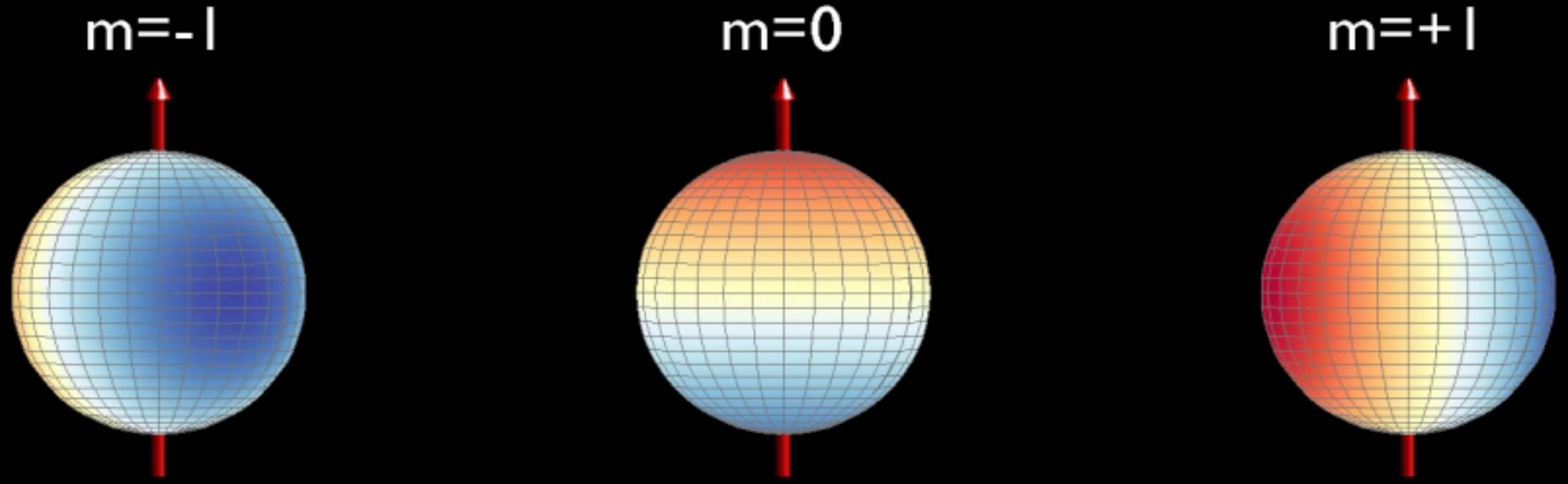

4.6 Gyr
Moon
Mercury
Mars
Earth
11.2 Gyr
b
c
d
e
f

Obliquity

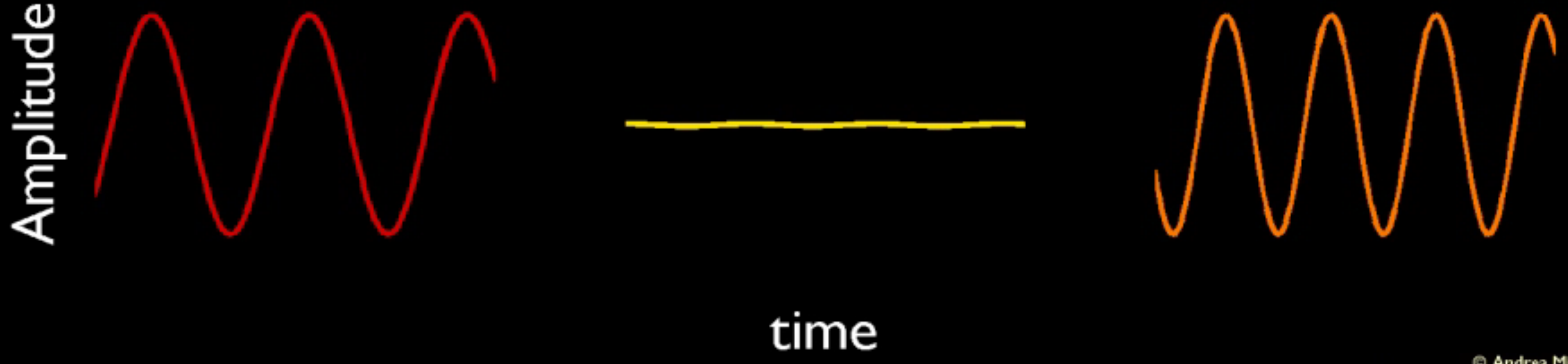
Angle between the **spin** and **orbital** vectors



Josh Winn

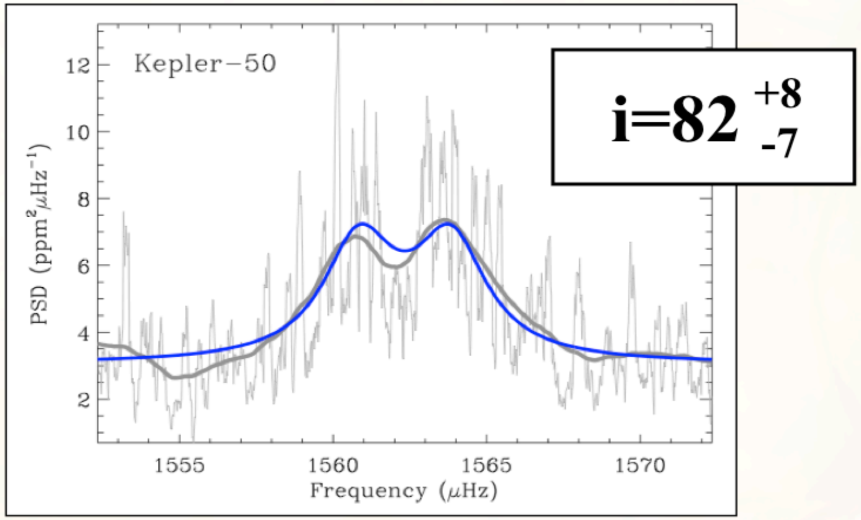


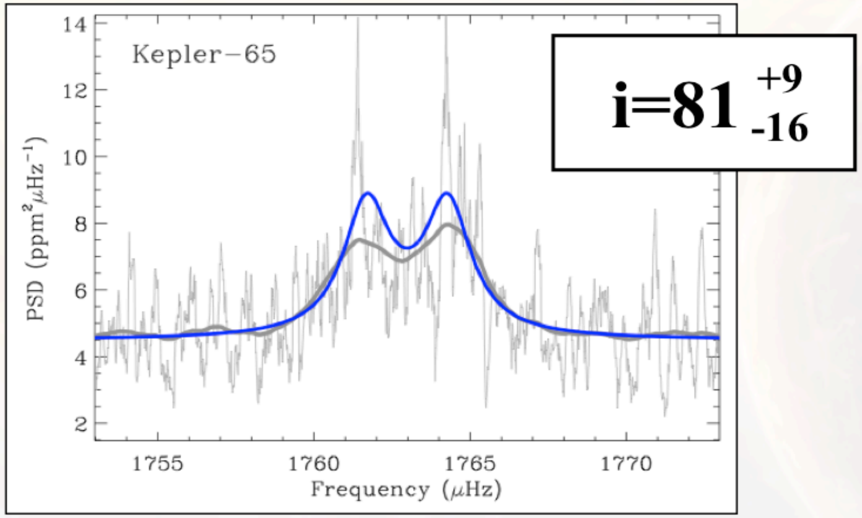
Inclination = 90°



© Andrea Miglio
University of Birmingham, UK

Obliquity





Kepler-50

$R_b = 1.7\,R_\oplus$ $P_b = 7.8$d
$R_c = 2.2\,R_\oplus$ $P_c = 9.4$d

aligned

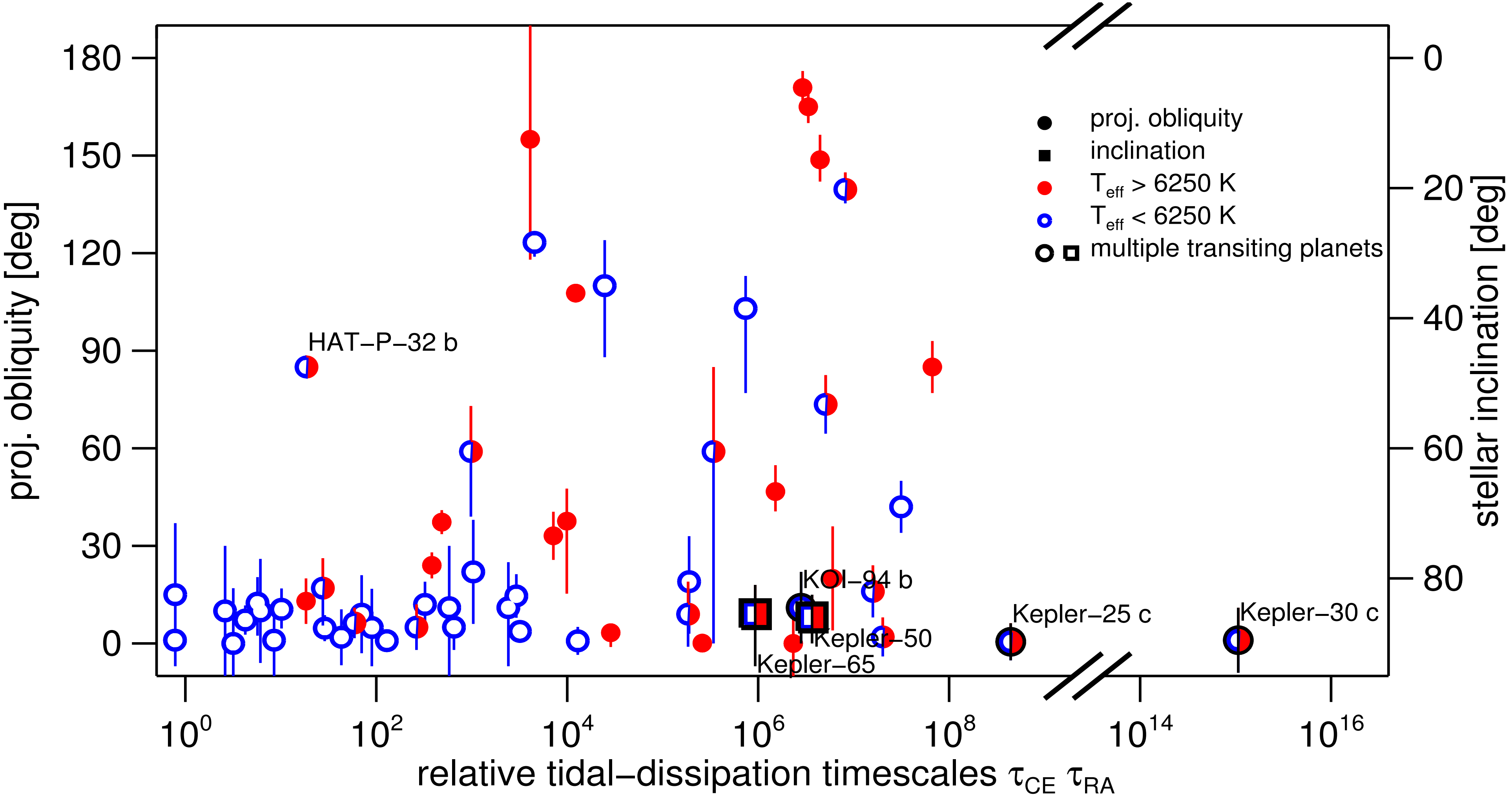
Kepler-65

$R_b = 1.4\,R_\oplus$ $P_b = 2.2$d
$R_c = 2.6\,R_\oplus$ $P_c = 5.9$d
$R_d = 1.5\,R_\oplus$ $P_d = 8.1$d

aligned

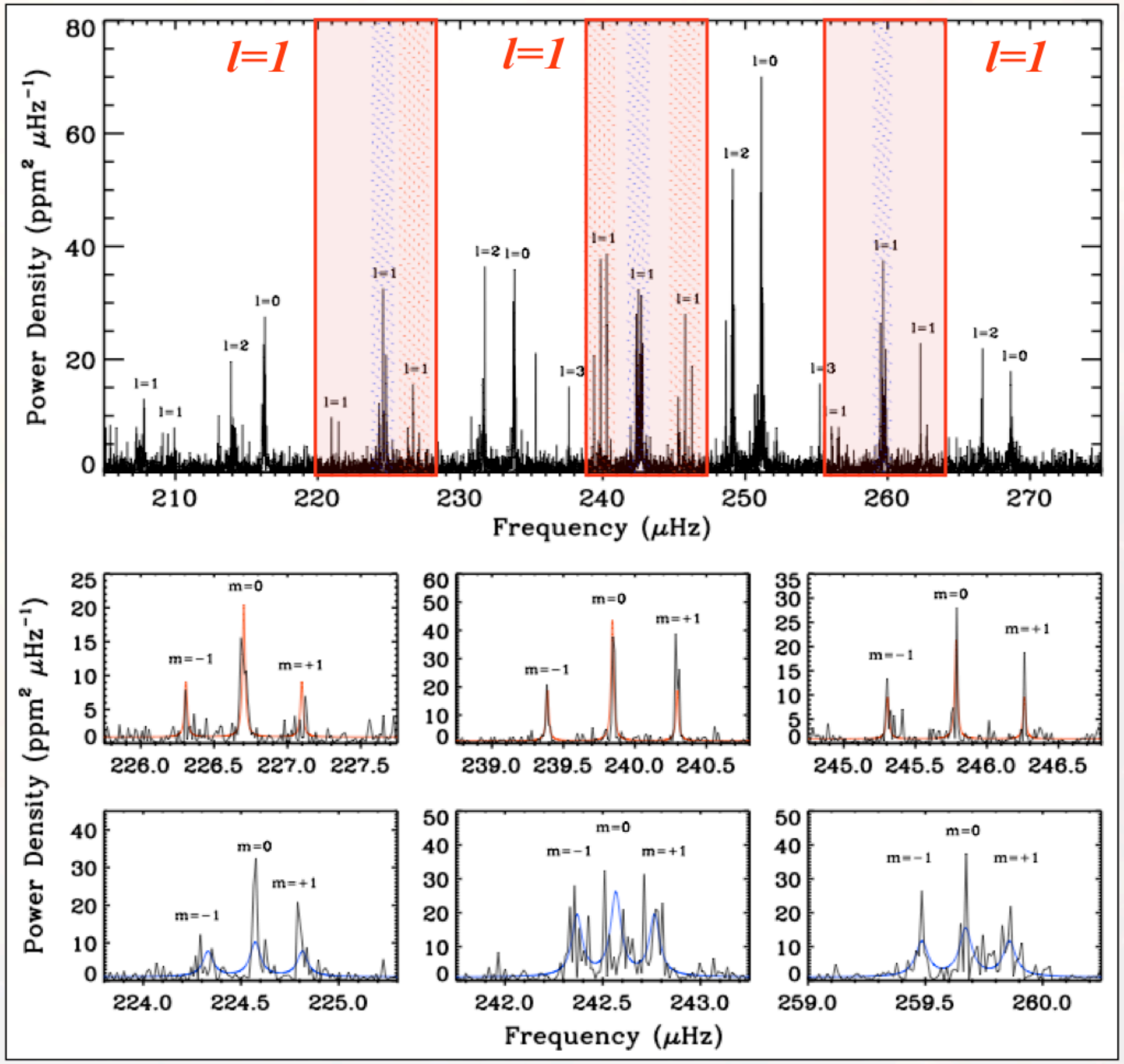
Chaplin et al. (2013)

Obliquity



Albrecht et al. 2013

Obliquity: Kepler 56



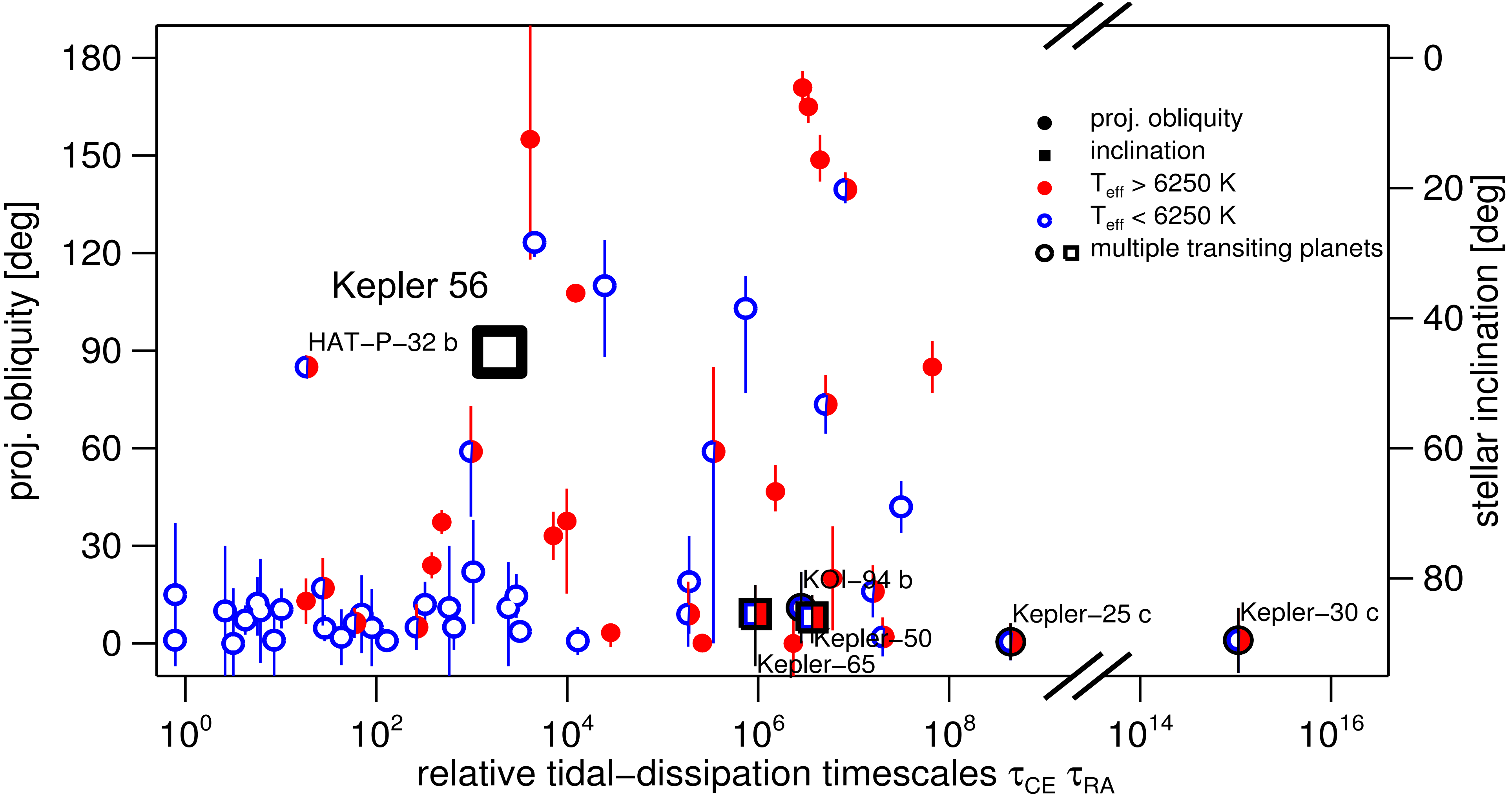
~50 individual frequencies detected

mixed *l=1* modes are split into triplets by rotation

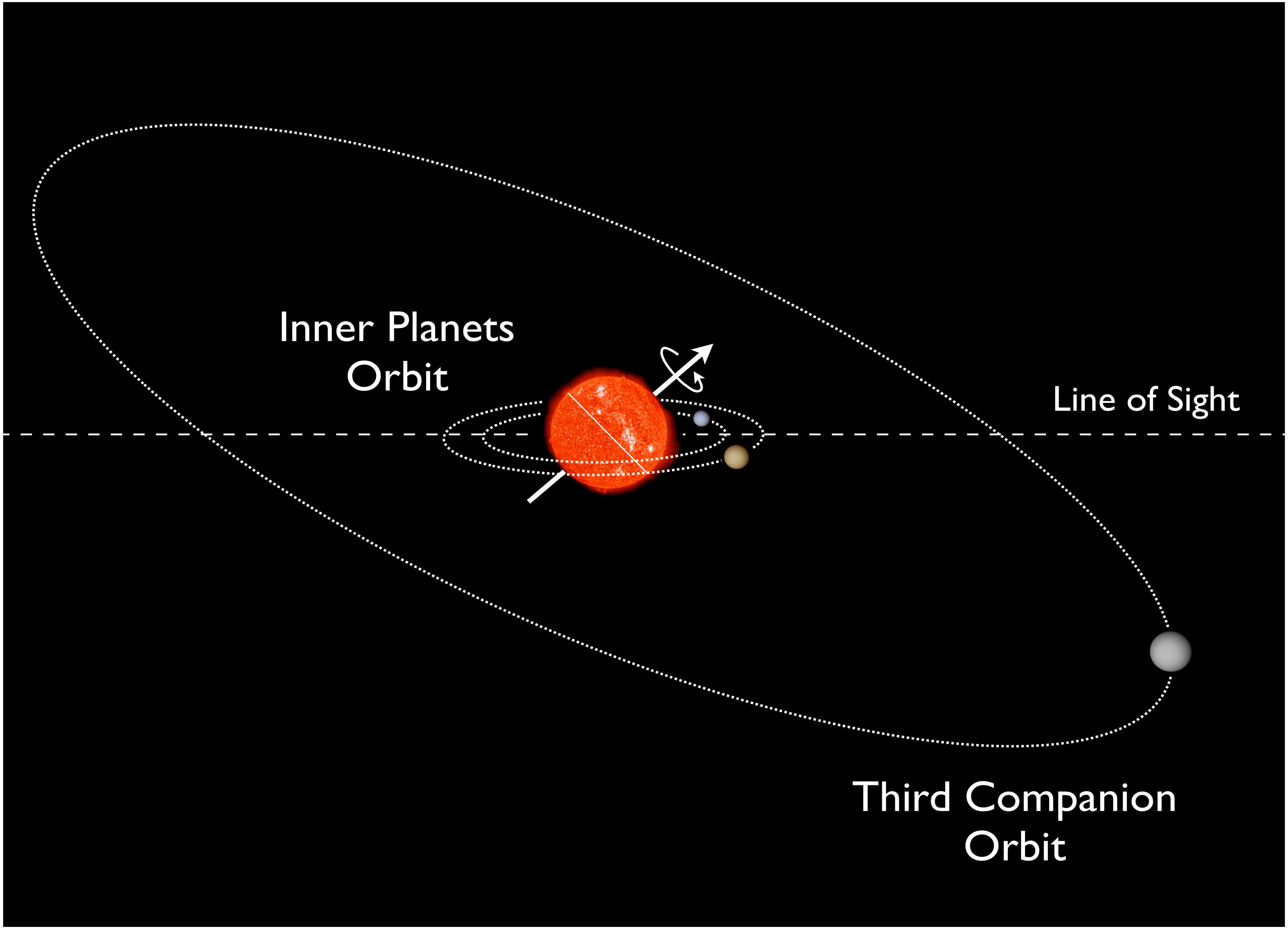
i ~ 45°!

Huber et al. (2013)

Obliquity



Albrecht et al. 2013


Inner Planets
Orbit
Line of Sight
Third Companion
Orbit

Galactic Archeology

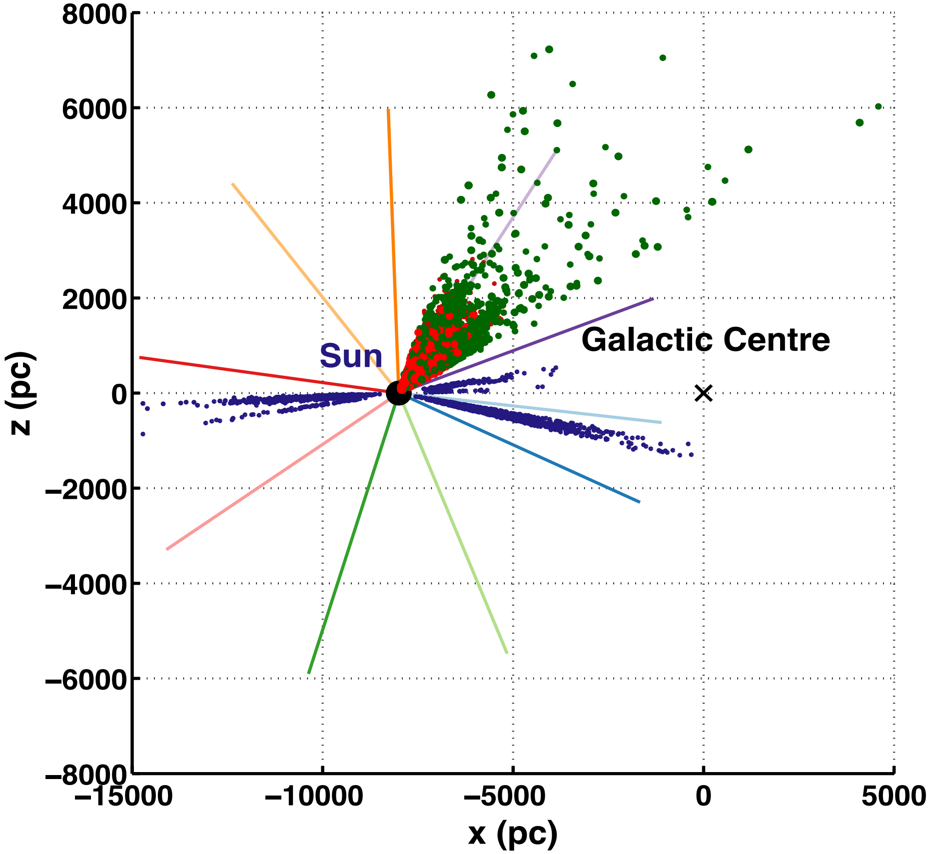
"the study of the formation and evolution of the Milky Way by reconstructing its past from its current constituents"

important parameters:

- position
- distance
- velocity
- chemical composition
- age / evolutionary phase

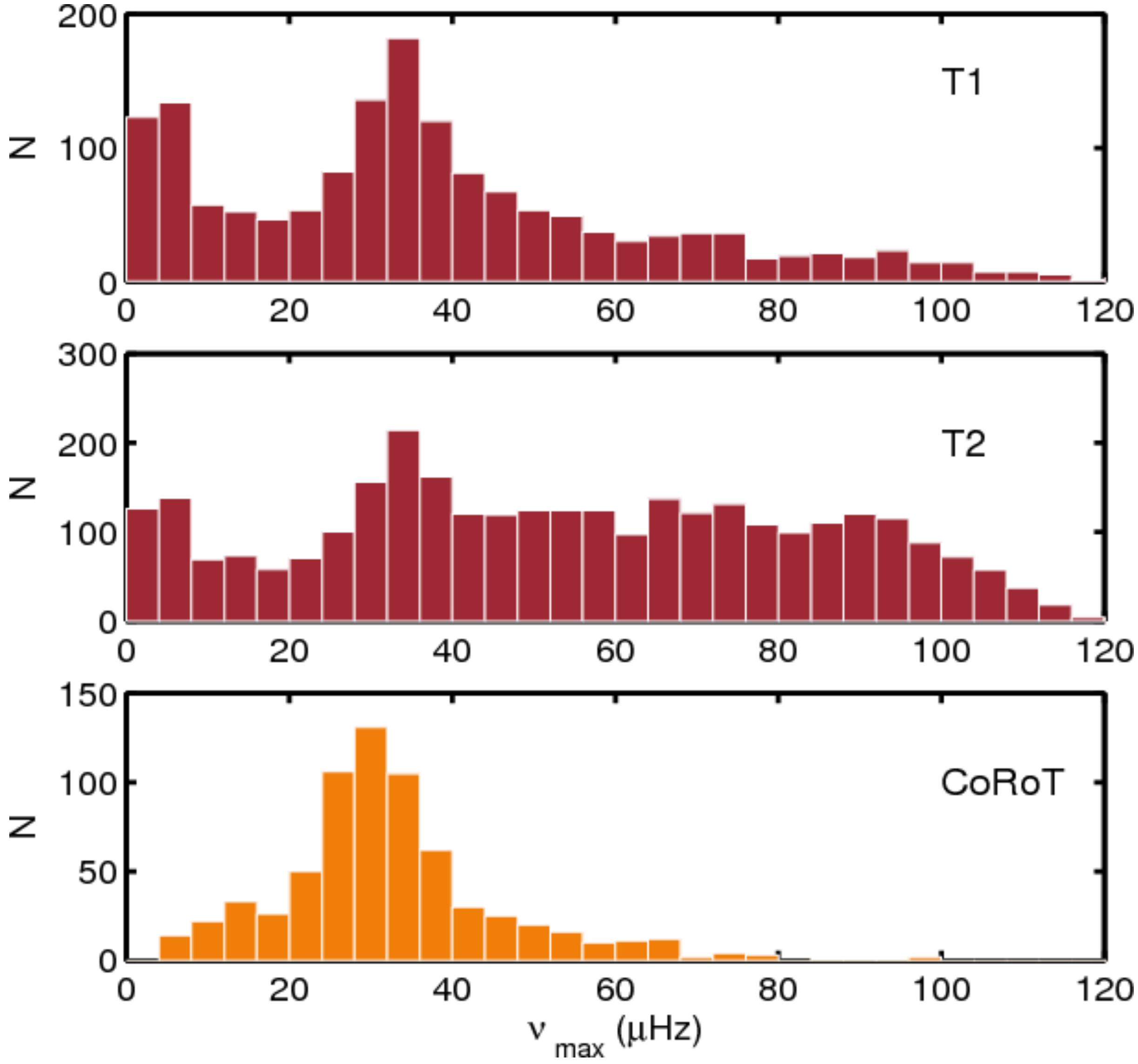
red giants

- many
- intrinsically luminous
- present in all parts of MW
- "direct" probes of M and R through scaling relations



Mathur et al. 2016

Ensemble / population studies



Simulated population (Miglio et al. 2009)

T1: no recent star burst

T2: recent star burst

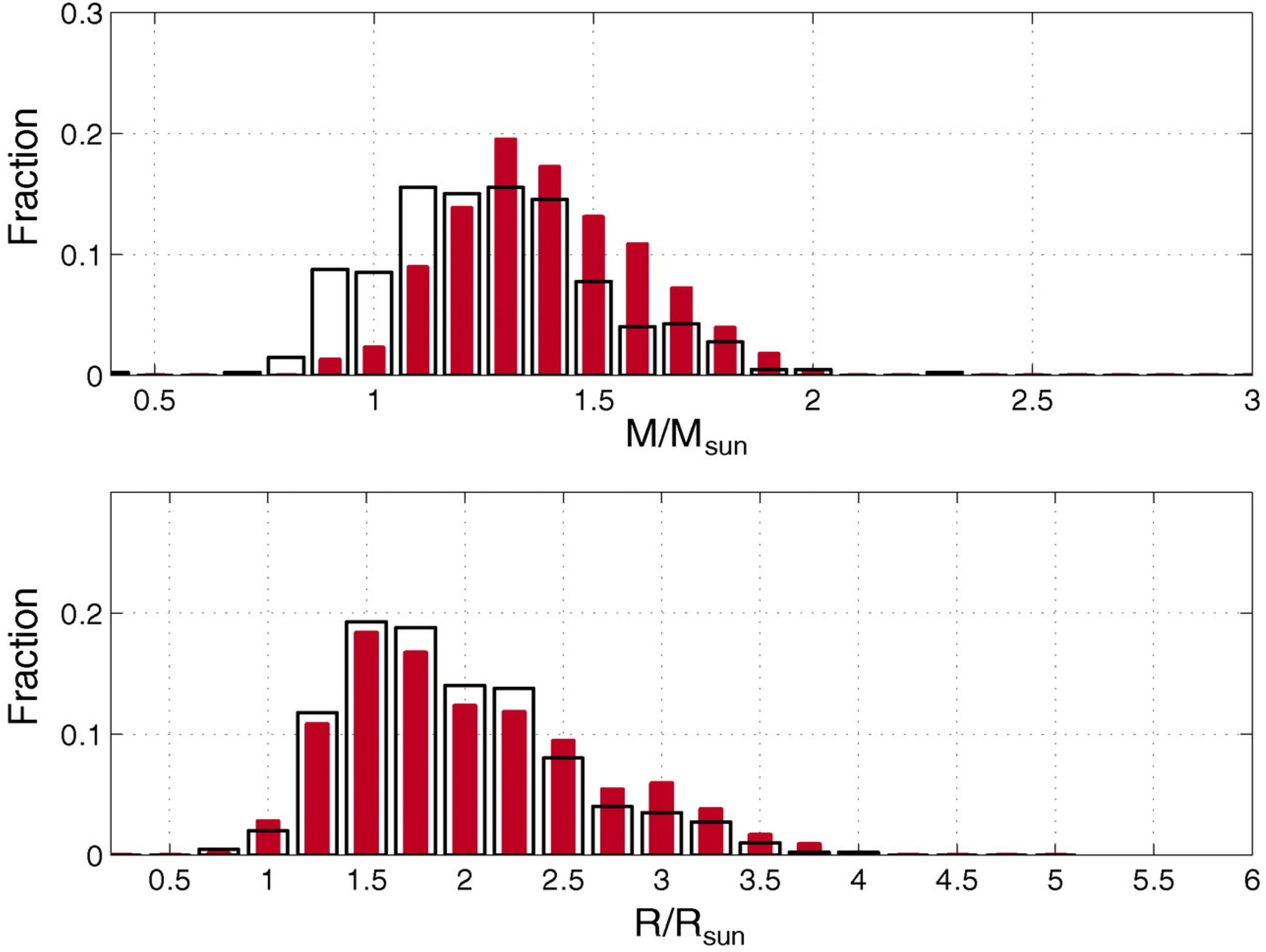
Observations (Hekker et al. 2009)

Note of caution: selection effects / observational biases

Selection effects: which fraction of stars are chosen to be observed out of the total number of stars available

Observational biases: to what parameter space the observations are limited due to limitations of instrumentation / observing strategy

Ensemble / population studies

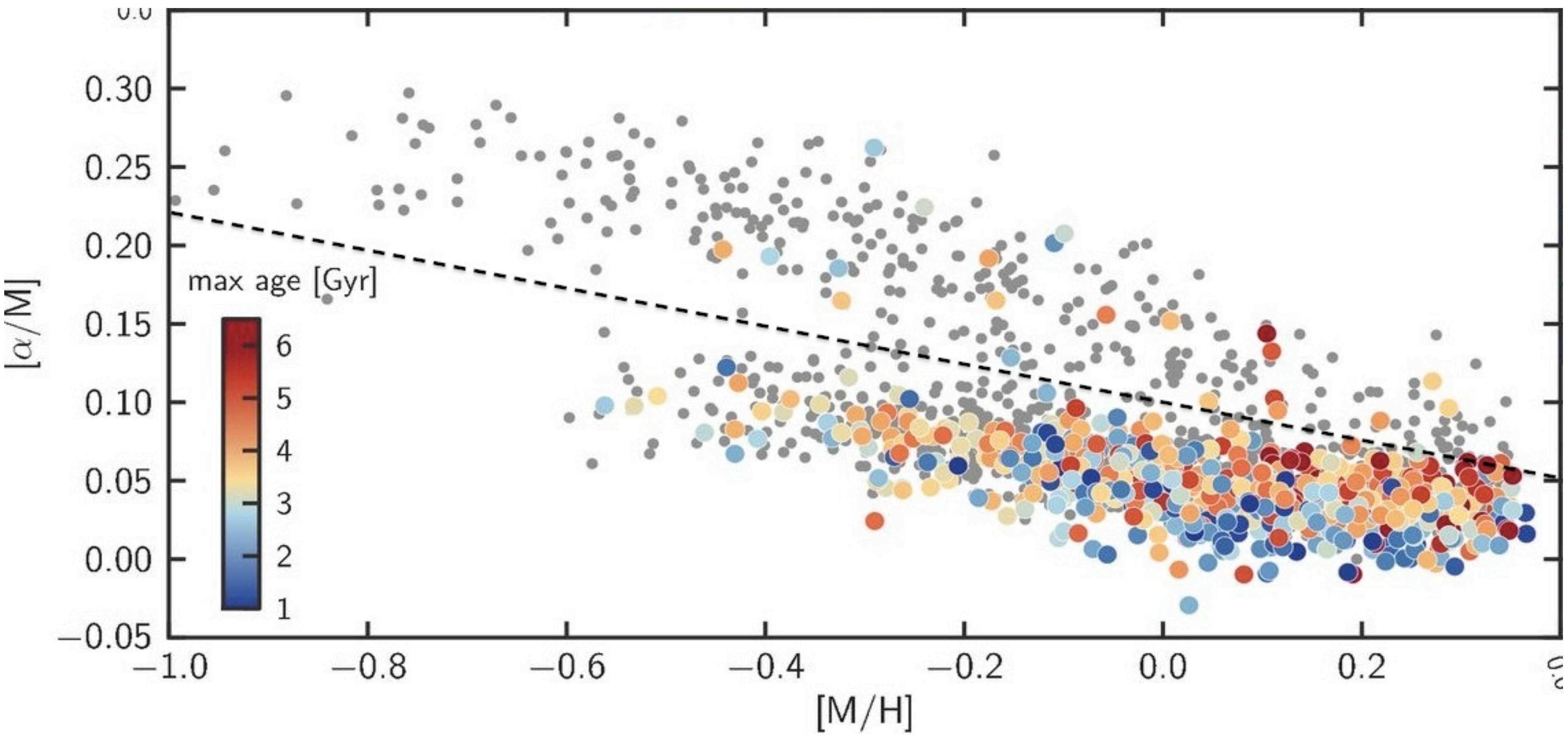


Observations

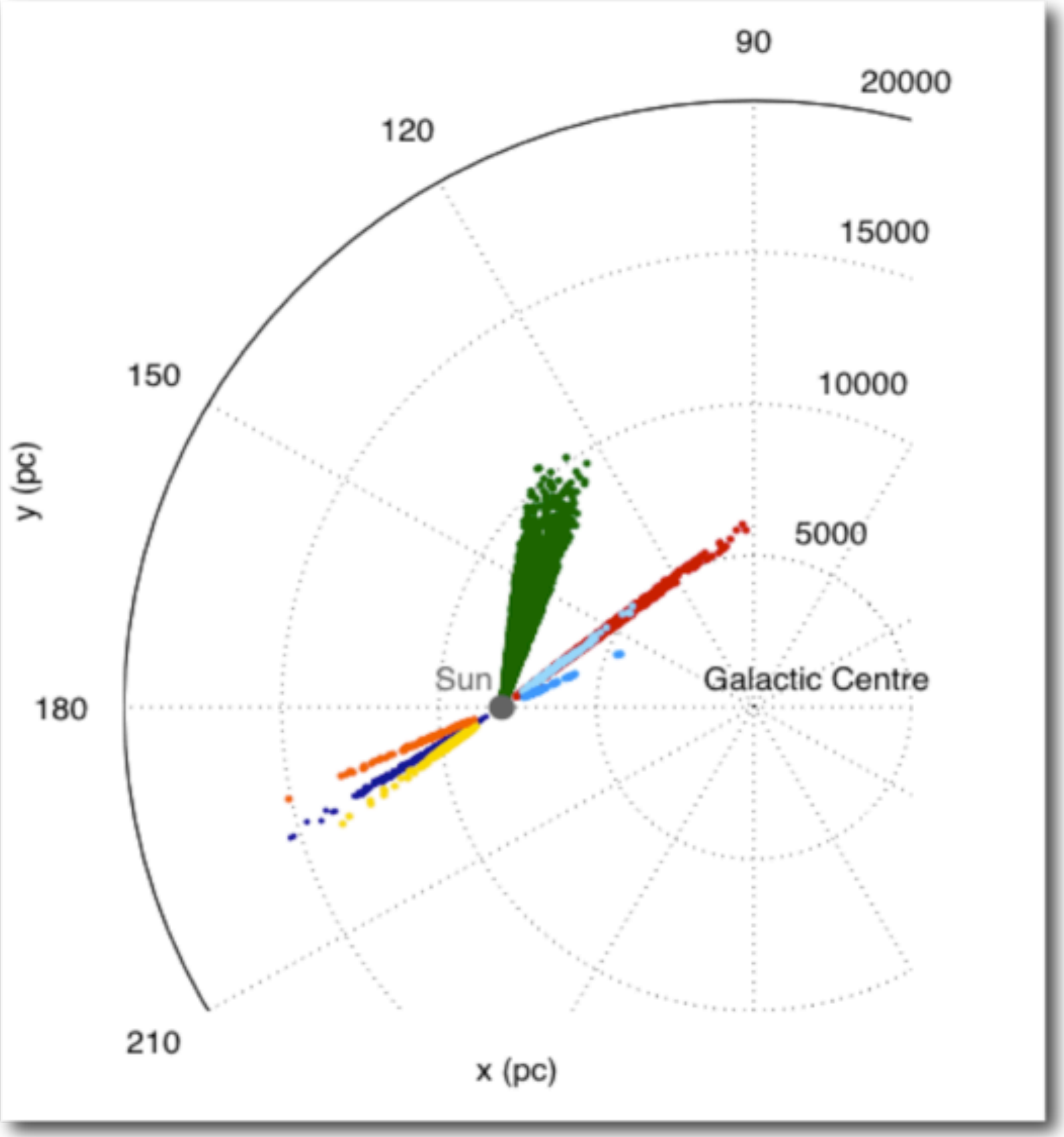
Population synthesis model

Chaplin et al. 2011

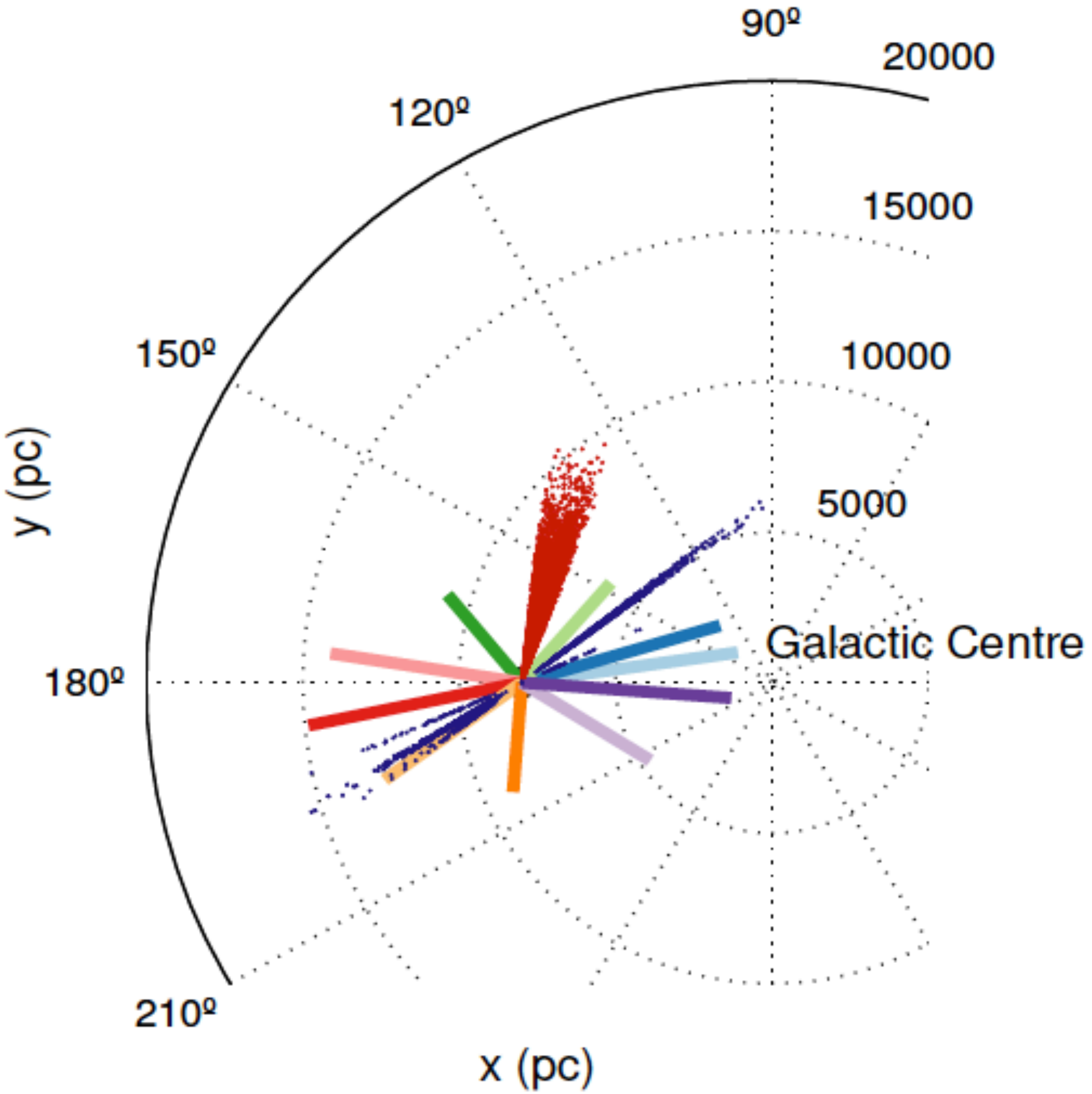
α-rich young stars



adapted from Martig et al. 2015

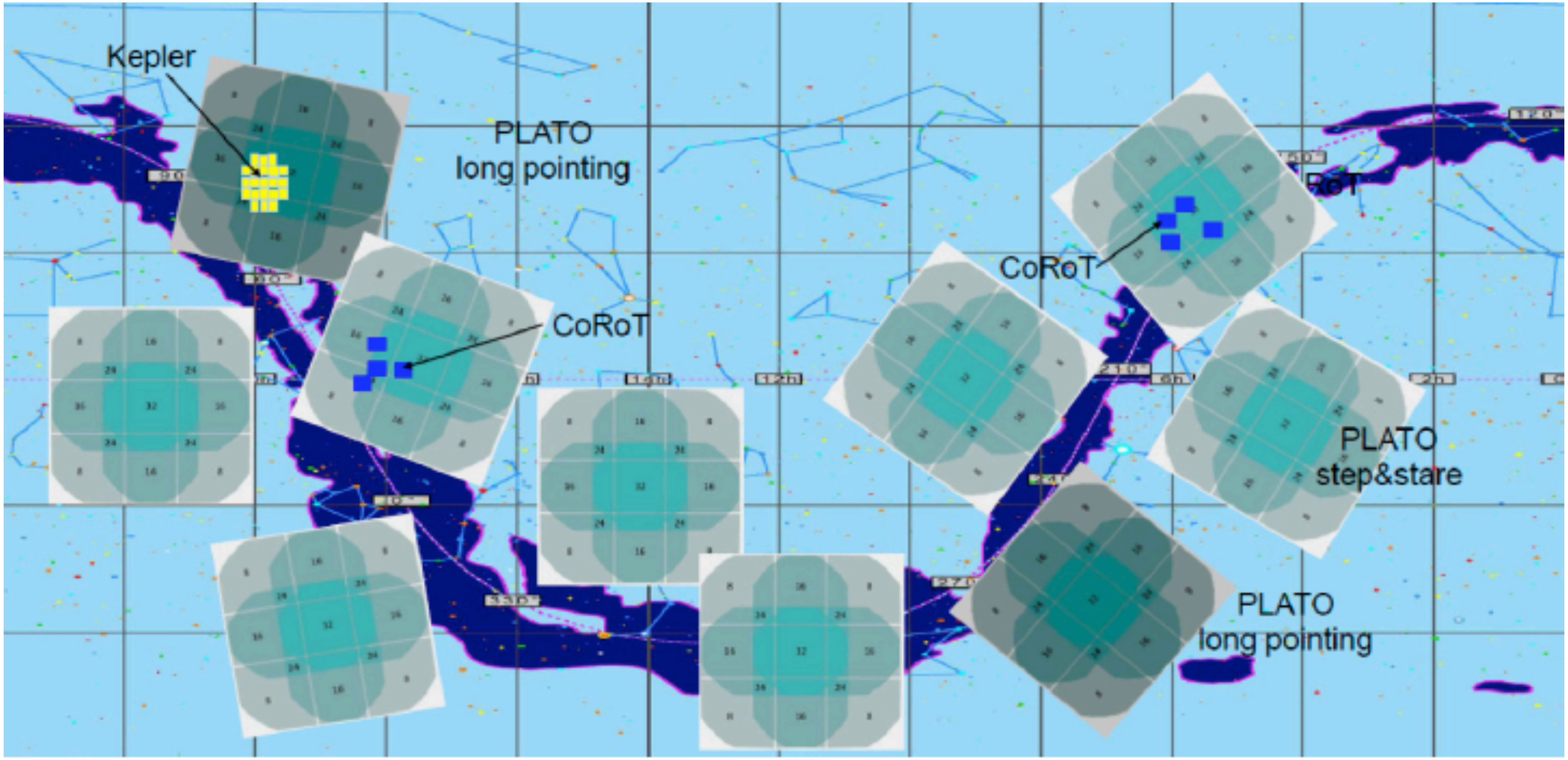


Miglio et al. 2013



Miglio et al. 2014

Plato fields



SUMMARY

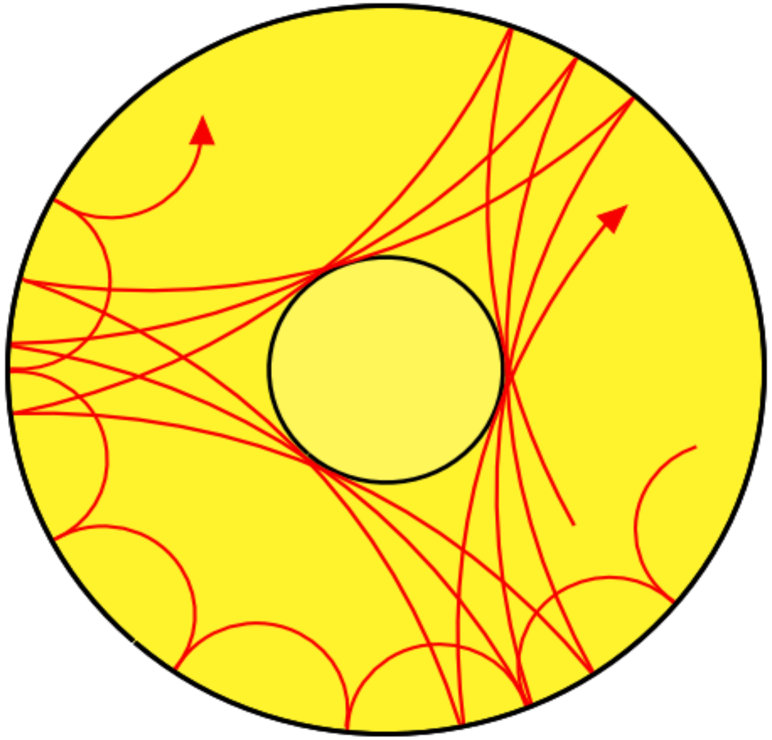
Asteroseismology

aster: star

seismology: oscillations

logos: reasoning

Study of stellar interiors through the analysis of stellar oscillations



Modelling stars: main equations

$$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho}$$

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}$$

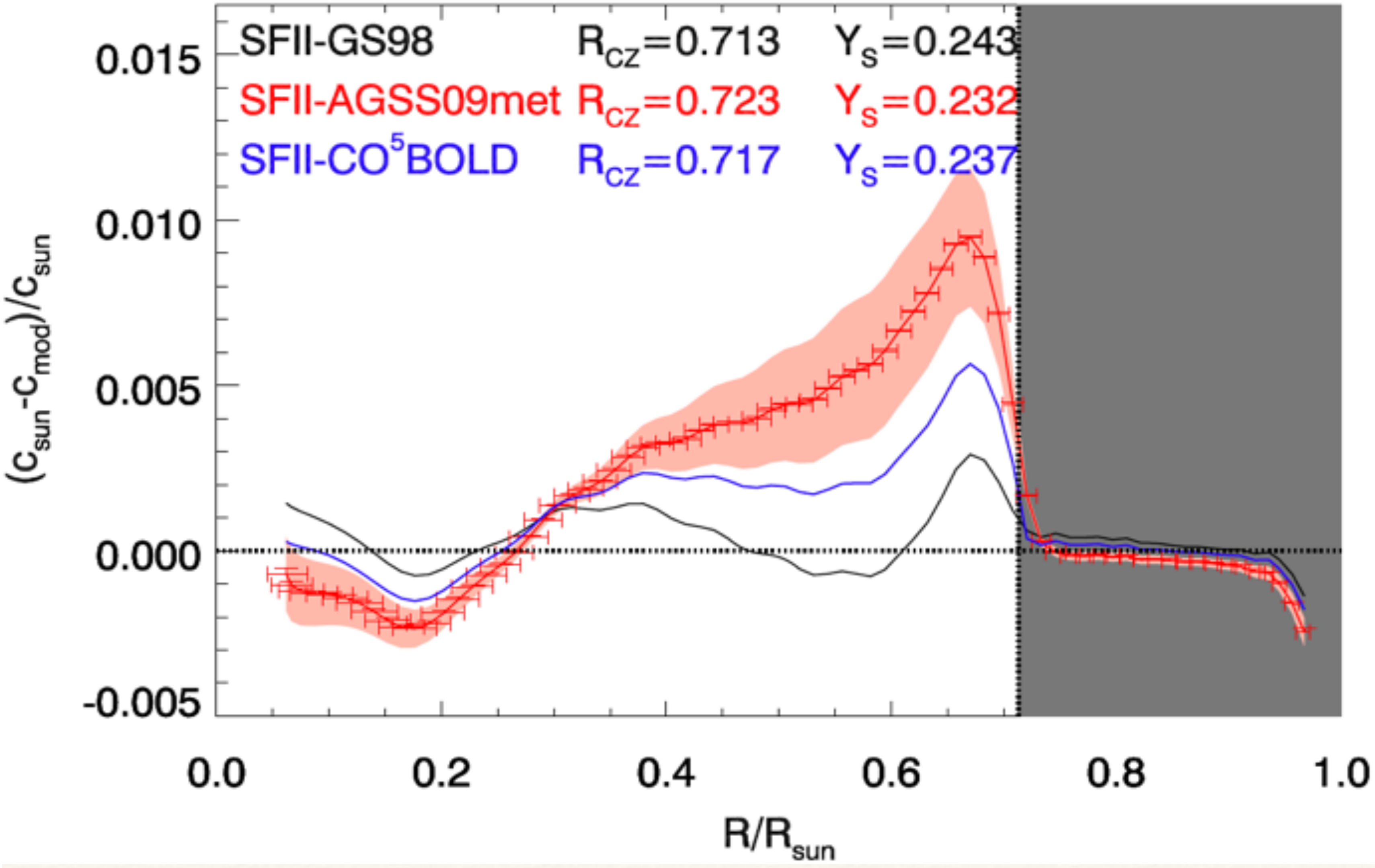
$$\frac{dL}{dm} = \varepsilon - \left[\frac{du}{dt} - \frac{P}{\rho^2}\frac{d\rho}{dt}\right]$$

$$\nabla^2 \Phi = 4\pi G \rho$$

Modelling stars: ingredients

- Select EOS
- Select nuclear reactions
- Select opacity tables
- Choose stability criterion (Ledoux or Schwarzschild)
- Define mixing length parameter
- Select appropriate composition
- Choose any additional mixing processes

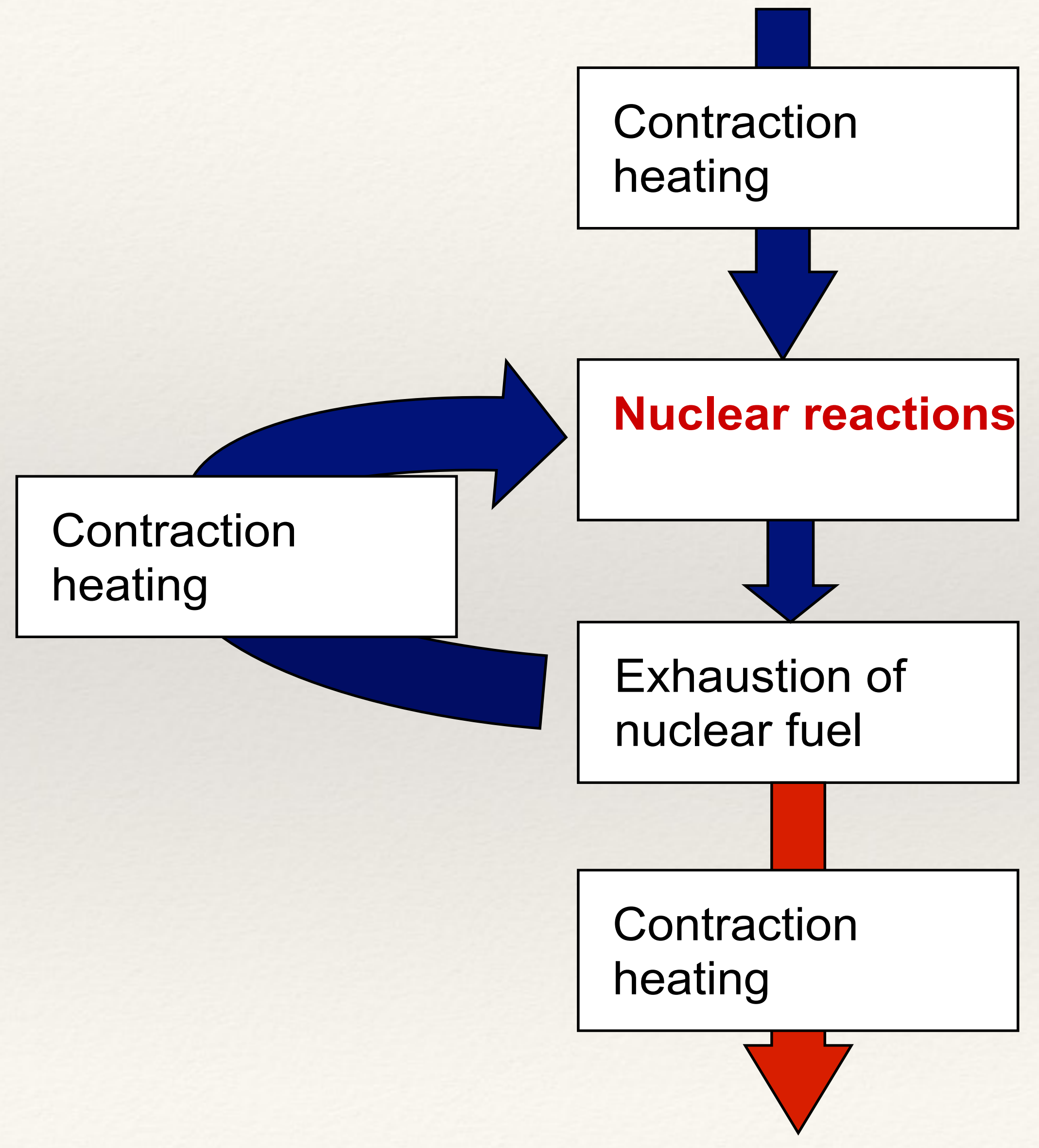
Modelling stars: solar abundance problem



Stellar evolution

- We know the main equations
- We know the constitutive equations
- We know about the mass, composition and additional mixing processes
- Now we can evolve a star….

Stellar evolution



Contraction heating

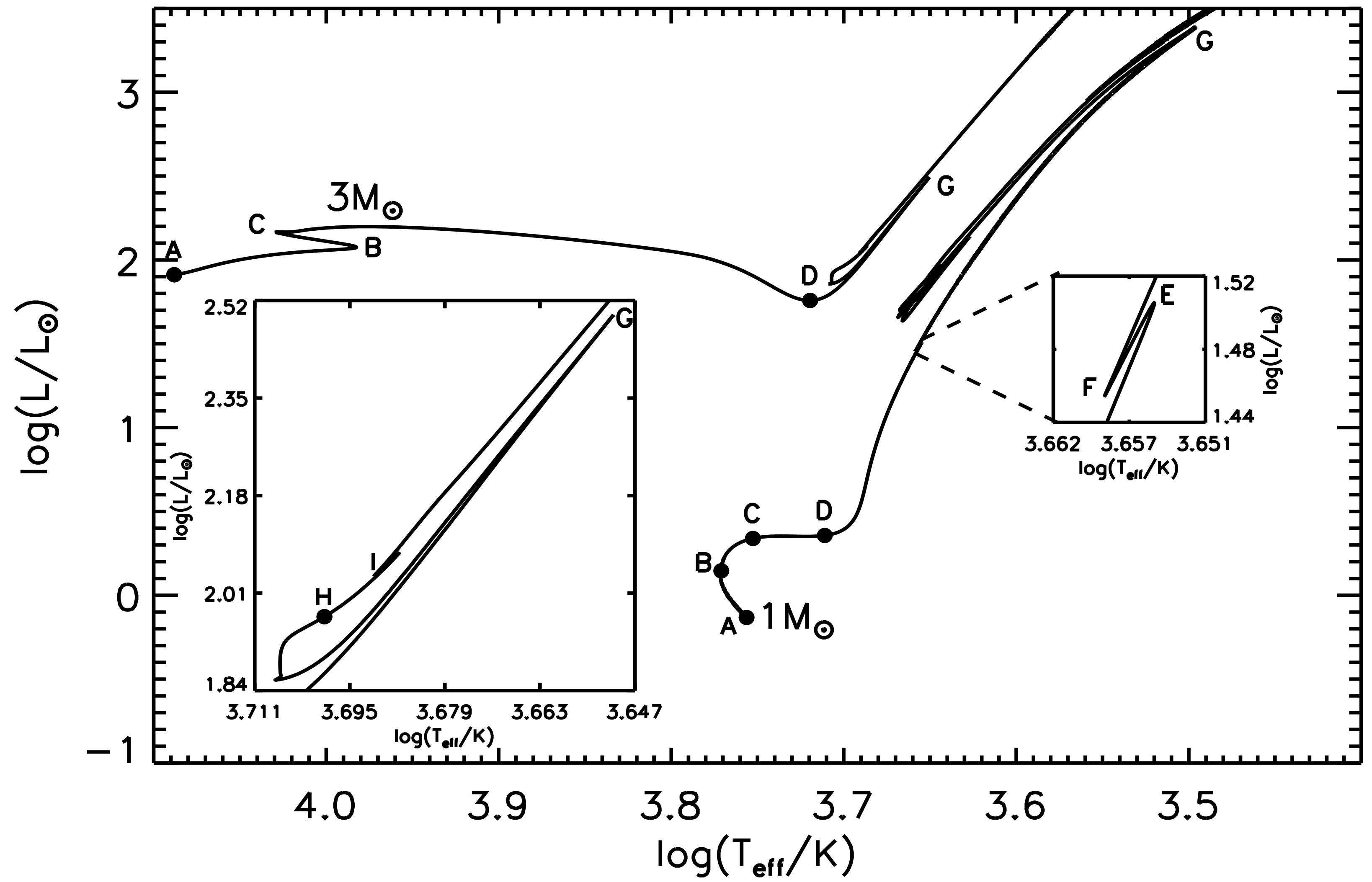
Nuclear reactions

Exhaustion of nuclear fuel

Contraction heating

Contraction heating

Stellar evolution



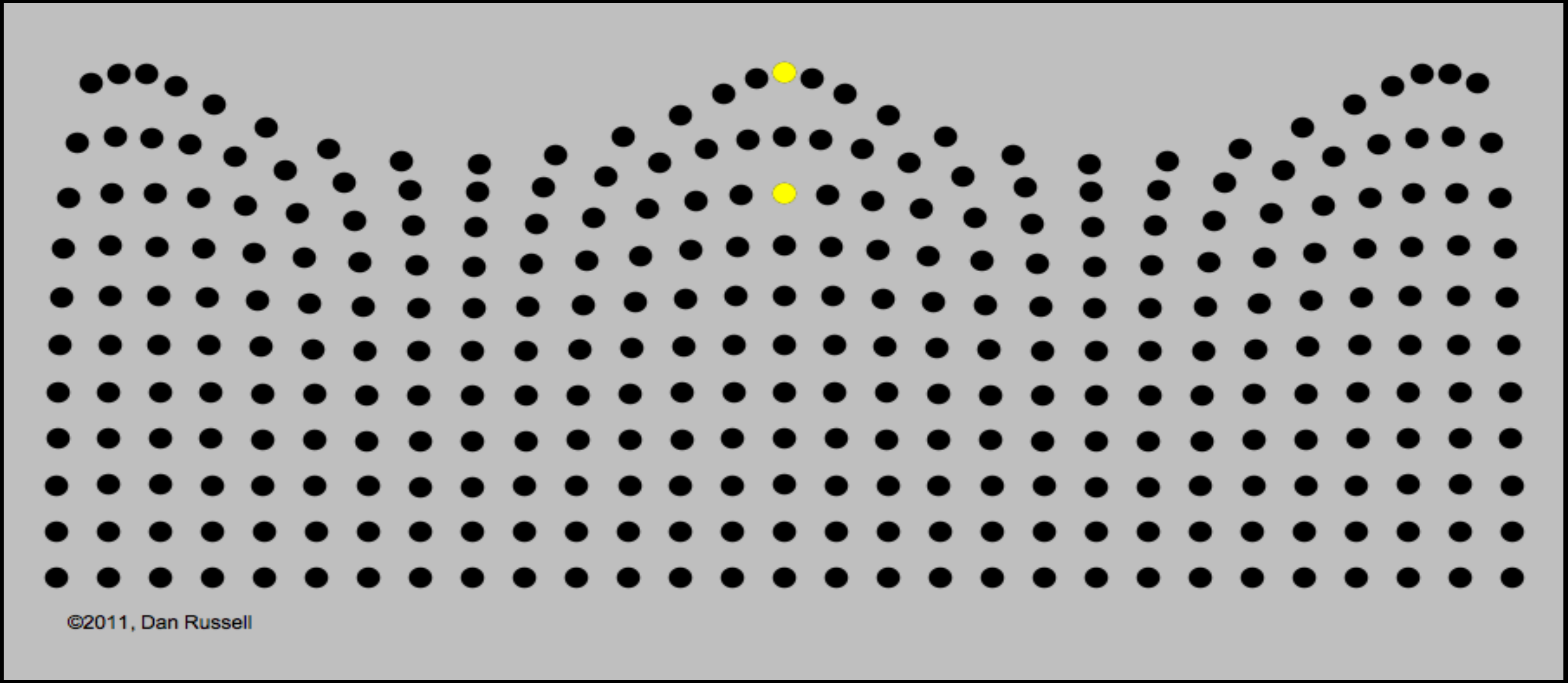
Hekker & Christensen-Dalsgaard 2017

Theory of Stellar Oscillations

Asteroseismology How does it work?

Wave: propagation of information (a perturbation) in space and time

Wave in a supporting medium: material does not need to move from one point of the space to the other to propagate the information



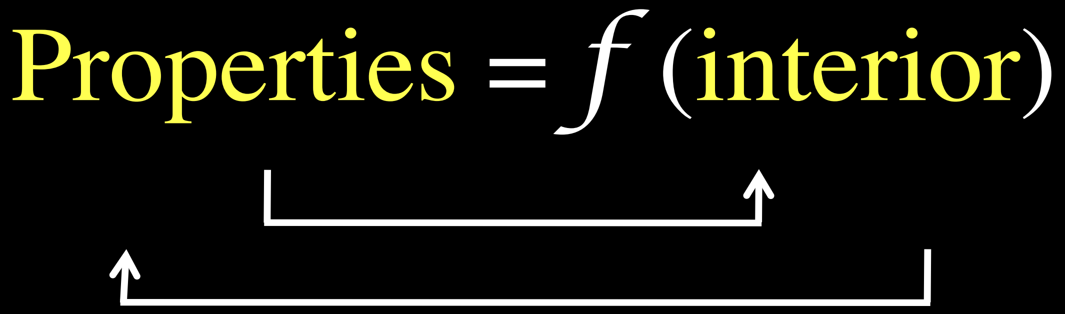
Asteroseismology How does it work?

Waves propagate within stars

↓

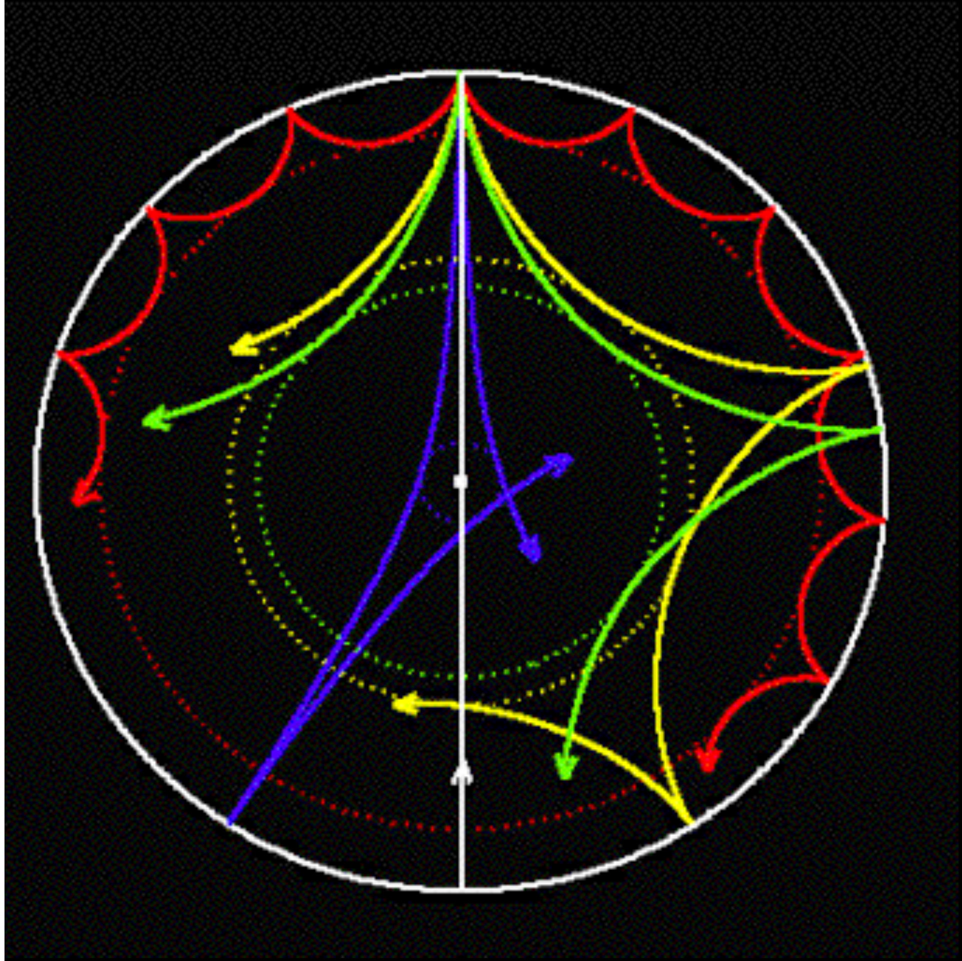
Wave properties (e.g. frequencies) depend on properties of the medium where they propagate (density, pressure, etc.)

↓



$$\text{Properties} = f\,(\text{interior})$$

Asteroseismology How does it work?



One mode ⇔ one piece of information

- ➢ Average information on propagation cavity
- ➢ With several modes one can hope to get localized information

Hydrodynamics

Following the fluid - Lagrangian description

Continuity equation
(conservation of mass)

$$\frac{D\rho}{Dt} + \rho\, \nabla\cdot \vec{v} = 0$$

ρ - density $\vec{v}$ - velocity

Equation of motion (inviscid fluid)
(conservation of linear momentum)

$$\rho \frac{D\vec{v}}{Dt} = -\nabla p - \rho \nabla \phi$$

$$\nabla^2 \phi = 4\pi G \rho$$

p - pressure ϕ - Gravitational potential

Energy equation
(conservation of energy)

$$\Gamma_1 = \left(\frac{\partial \ln p}{\partial \ln \rho}\right)_{ad}$$

$$\frac{Dq}{Dt} = \frac{DE}{Dt} + p\frac{D(1/\rho)}{Dt} = \\ = \frac{1}{\rho(\Gamma_3 - 1)}\left(\frac{Dp}{Dt} - \frac{\Gamma_1 p}{\rho}\frac{D\rho}{Dt}\right)$$

q -heat supplied /mass E -internal energy /mass

$\Gamma_1;\Gamma_3$ - adiabatic exponents

+ Equation of state

Summary of perturbed equations

Linear adiabatic pulsation about a static, spherically symmetric equilibrium

$$\rho' + \nabla \cdot (\rho_0 \delta\vec{\mathrm{r}}) = 0$$

$$\rho_0 \frac{\partial^2 \delta\vec{\mathrm{r}}}{\partial t^2} = -\nabla p' - \rho_0 \nabla \phi' + \rho' \nabla \phi_0$$

$$\nabla^2 \phi' = 4\pi G \rho'$$

$$p' + \delta\vec{r} \cdot \nabla p_0 = \frac{\Gamma_{1,0} p_0}{\rho_0} (\rho' + \delta\vec{r} \cdot \nabla \rho_0)$$

Variables: 4 (ϱ', p', $\boldsymbol{\phi}$', $\delta\vec{\mathrm{r}}$)

Equations: 4

Thus: system of equation is closed, so far as equilibrium quantities are known.

=> can solve it to get solutions for the 4 variables.

Equations for the depth dependent amplitudes

$$\frac{d\xi_r}{dr} = -\left(\frac{1}{\Gamma_{1,0} p_0}\frac{dp_0}{dr} + \frac{2}{r}\right)\xi_r + \left(\frac{S_l^{\,2}}{\omega^2} - 1\right)\frac{1}{c_0^{\,2}\rho_0} p' + \frac{l(l+1)}{r^2\omega^2}\phi'$$

$$\frac{dp'}{dr} = \rho_0(\omega^2 - N_0^2)\xi_r - \rho_0\frac{d\phi'}{dr} + \frac{1}{\Gamma_{1,0} p_0}\frac{dp_0}{dr}p'$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi'}{dr}\right) = 4\pi G\left(\frac{p'}{c_0^2} + \frac{\rho_0 N_0^2}{g_0}\xi_r\right) + \frac{l(l+1)}{r^2}\phi'$$

Equations depend on *l*, but not on *m*

=> In a spherically symmetric star, the eigenvalues are independent of *m*

$$\boxed{\omega=\omega(n,l,\cancel{m})}$$

Note: That is not the case if the star rotates or has a magnetic field, braking the symmetry.

Spherical Harmonics Y_l^m

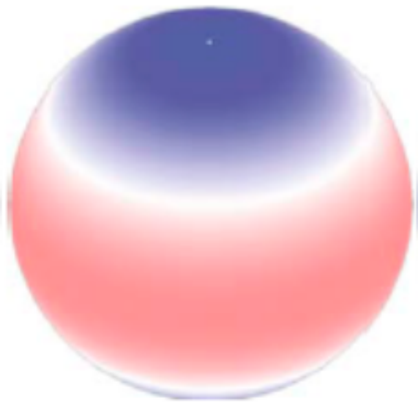
l – *angular degree: the number of nodes on the sphere*

$$k_h = \frac{\sqrt{l(l+1)}}{R}$$

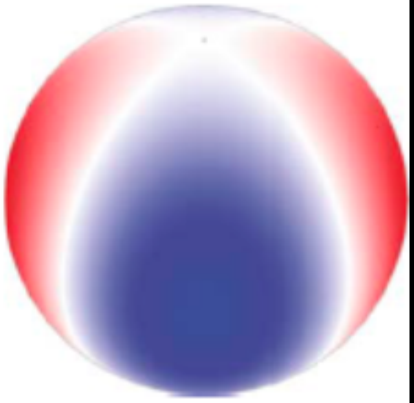
m - azimuthal order: $|m|$ =number of nodes along the equator
=> orientation on the sphere

Note: $|m| \leq l$

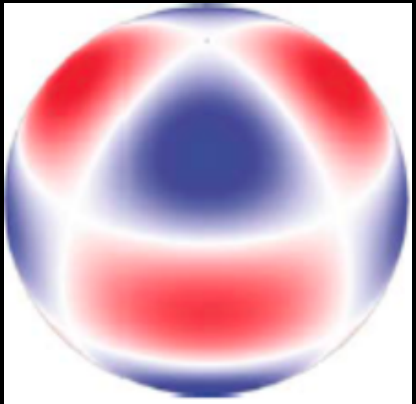
adapted from Aerts et al. 2010



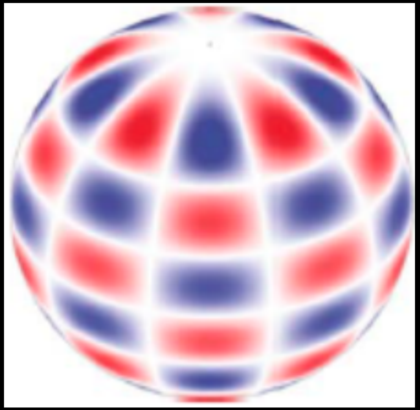
l=2
m=0



l=2
$|m|$=2



l=4
$|m|$=2



l=10
$|m|$=5

Trapping of oscillations

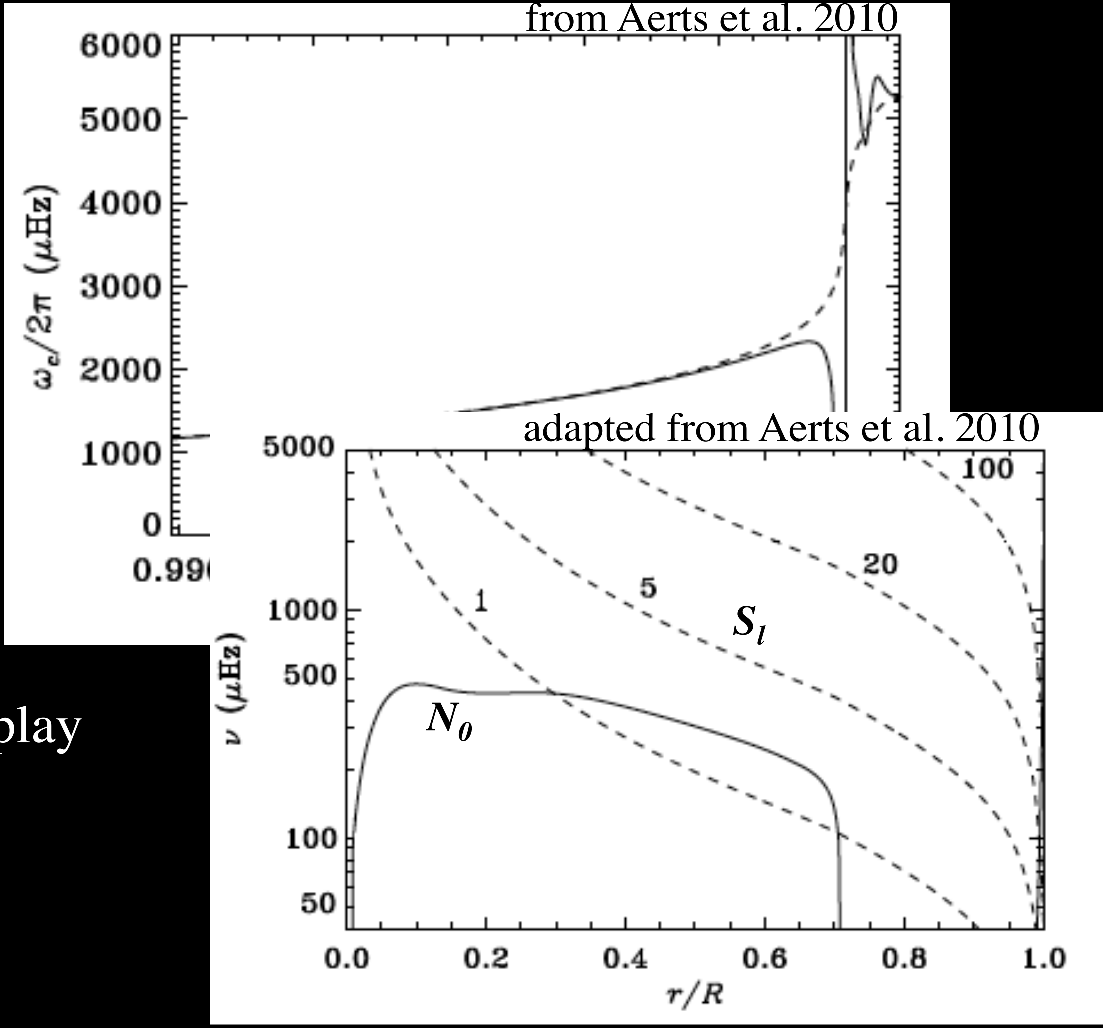
In the star k_r is not constant!

$$\frac{d^2X}{dr^2}+k_r^2X=0$$

$$k_r^2=\frac{1}{c_0^2}\left[\boxed{S_l^2}\left(\frac{\boxed{N_0^2}}{\omega^2}-1\right)+\omega^2-\boxed{\omega_c^2}\right]$$

$$\omega_c^2=\frac{c_0^2}{4H^2}\left(1-2\frac{dH}{dr}\right)$$

$$H^{-1}=-\frac{d\ln\rho}{dr}$$


from Aerts et al. 2010

adapted from Aerts et al. 2010

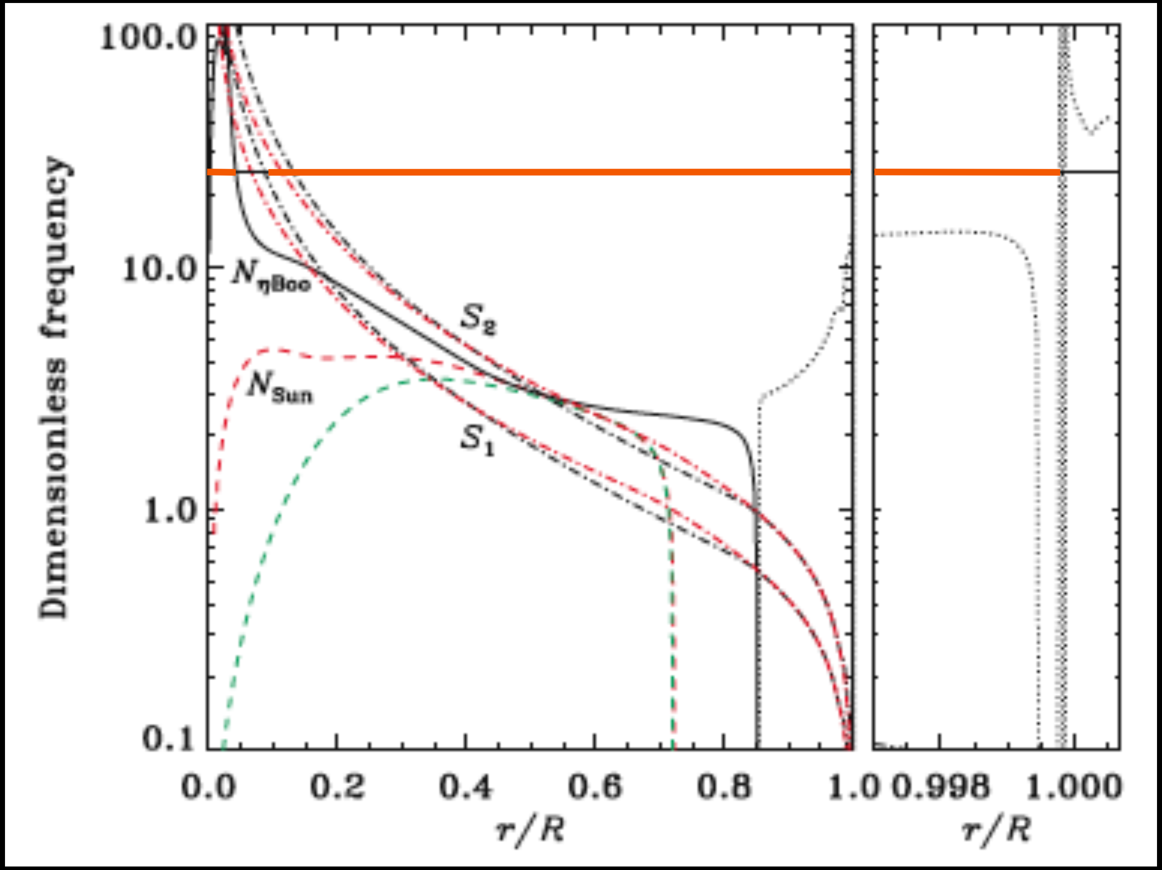
These 3 characteristic frequencies will play a fundamental role in deciding where modes propagate and where they are evanescent.

Trapping of oscillations

The case of an evolved star

- Propagation diagram for the sun and a subgiant star

Cunha et al. 2007



Why do stars oscillate?

- convective outer layers in which stochastic excitation of oscillations takes place
- some outer layers act as a heat engine:

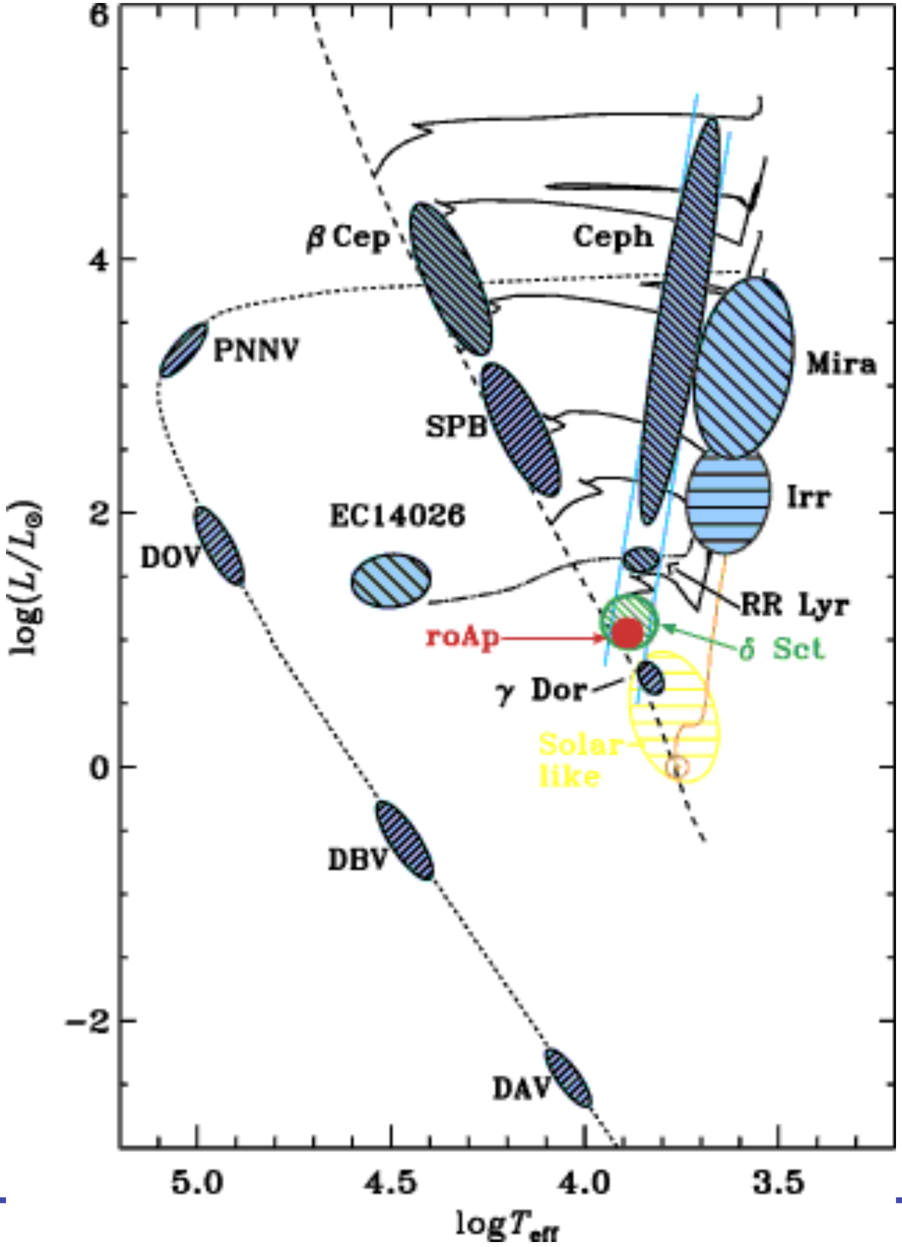
  partial ionisation zones absorb and accumulate

  energy generated in the stellar interior

  (opacity mechanism)
- forced oscillations may occur due to tidal effects

  in close binaries

Oscillating stars: rapidly oscillating Ap stars



Excitation mechanism:
κ mechanism

Restoring force: pressure

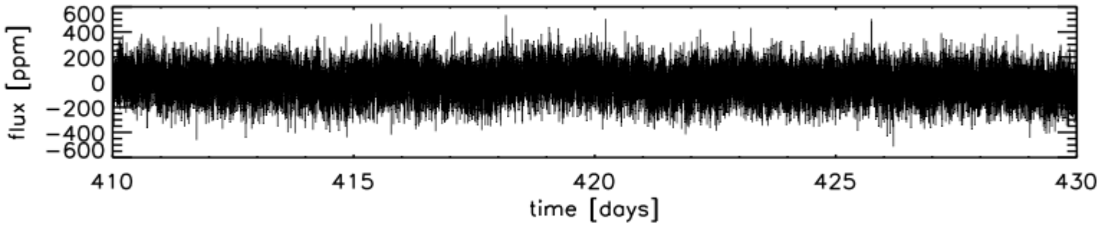
Typical periods: 5-20 mins

Evolutionary phase: MS

Mass range: 1.5-2.0 M_{Sun}

Highly magnetic stars

Important “timescales”



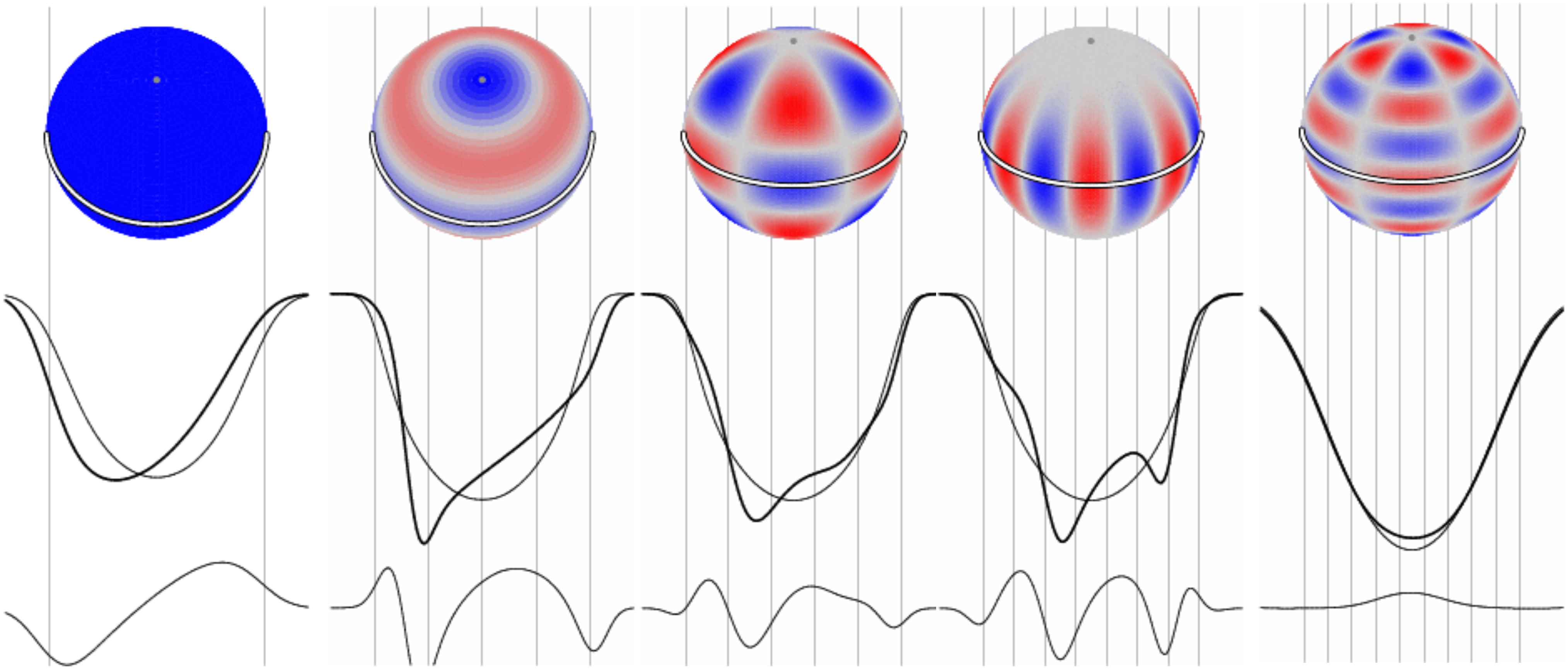
- Frequency resolution in the Fourier power power spectrum is reciprocal of total timespan T of timeseries:

$$\delta\nu = \frac{1}{T}$$

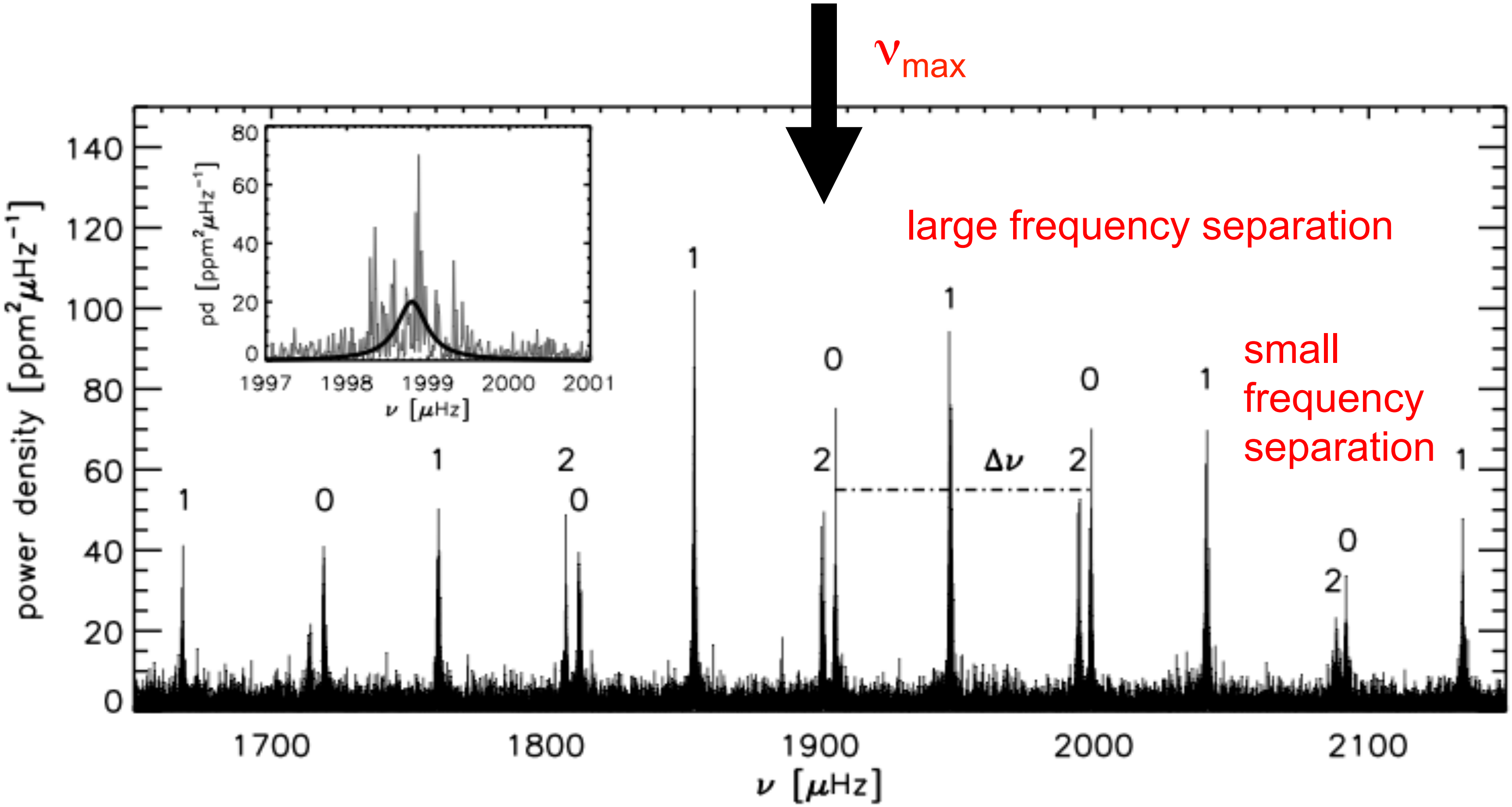
- Nyquist frequency: highest frequency at which one can reliably obtain results depends on the time sampling δt:

$$\nu_{Nyq} = \frac{1}{2\delta t}$$

Mode identification through line profile variations



Solar-like oscillations



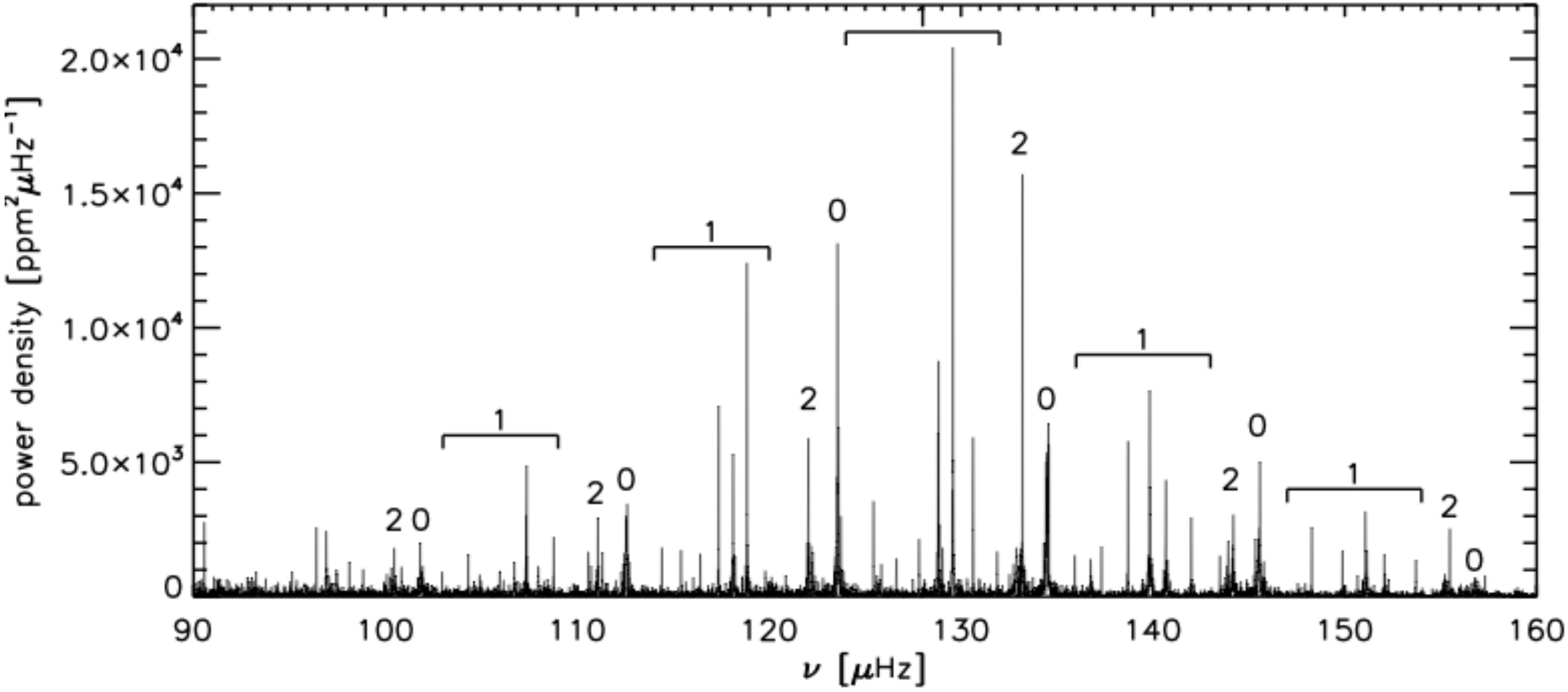
ν_{max}

large frequency separation

small frequency separation

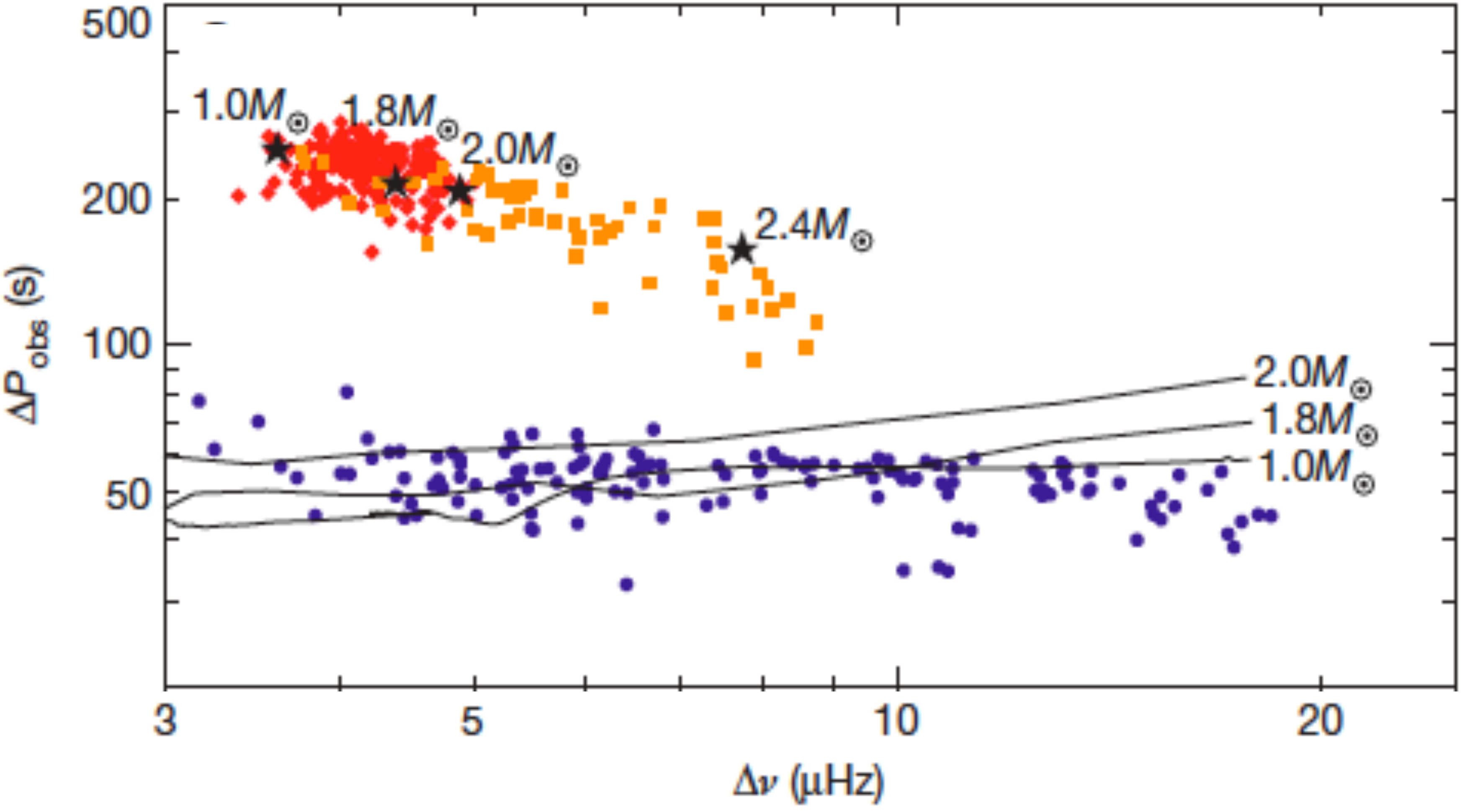
Hekker & Mazumdar 2014

Red giant



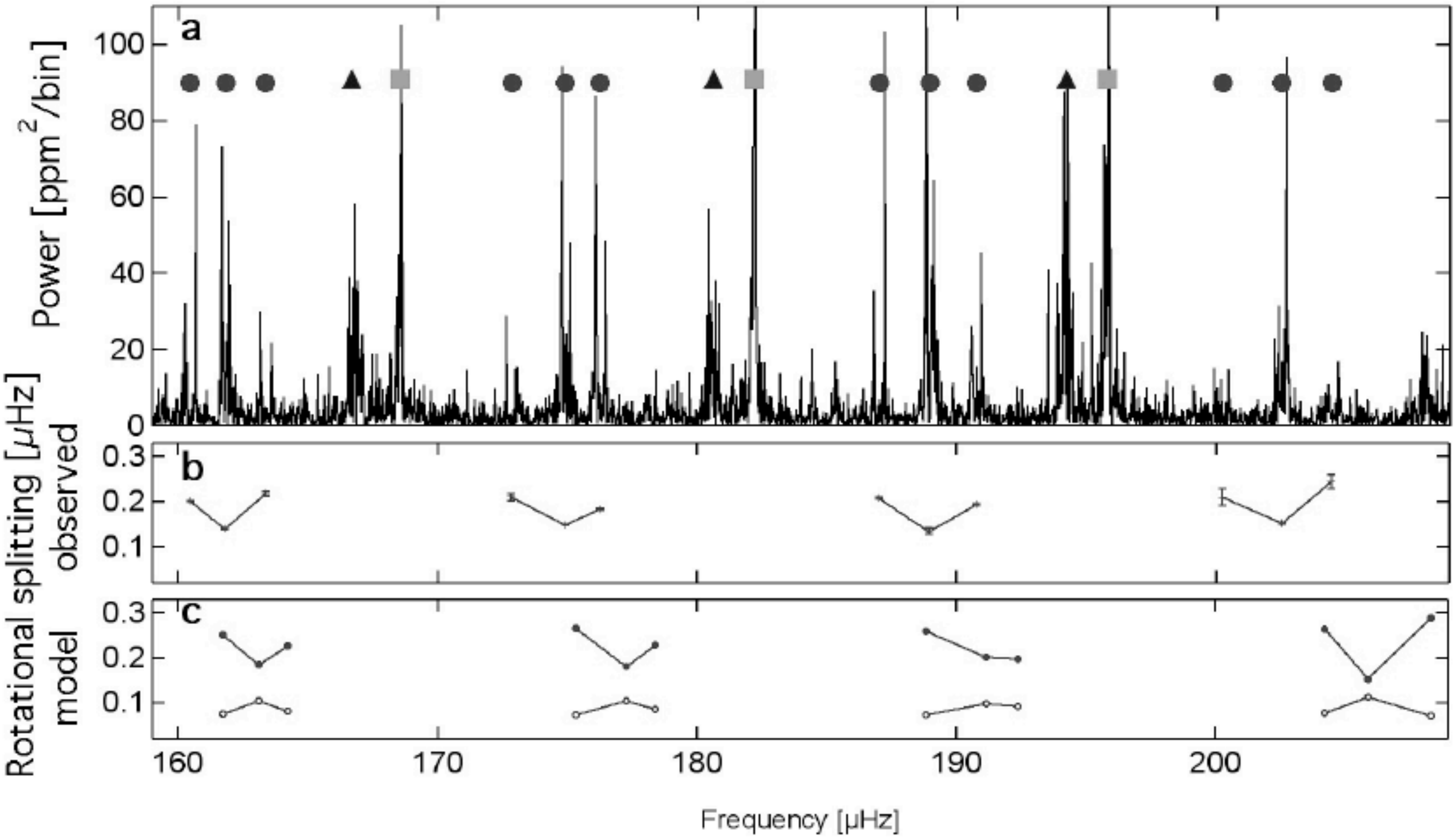
Hekker & Mazumdar 2014

Period spacing



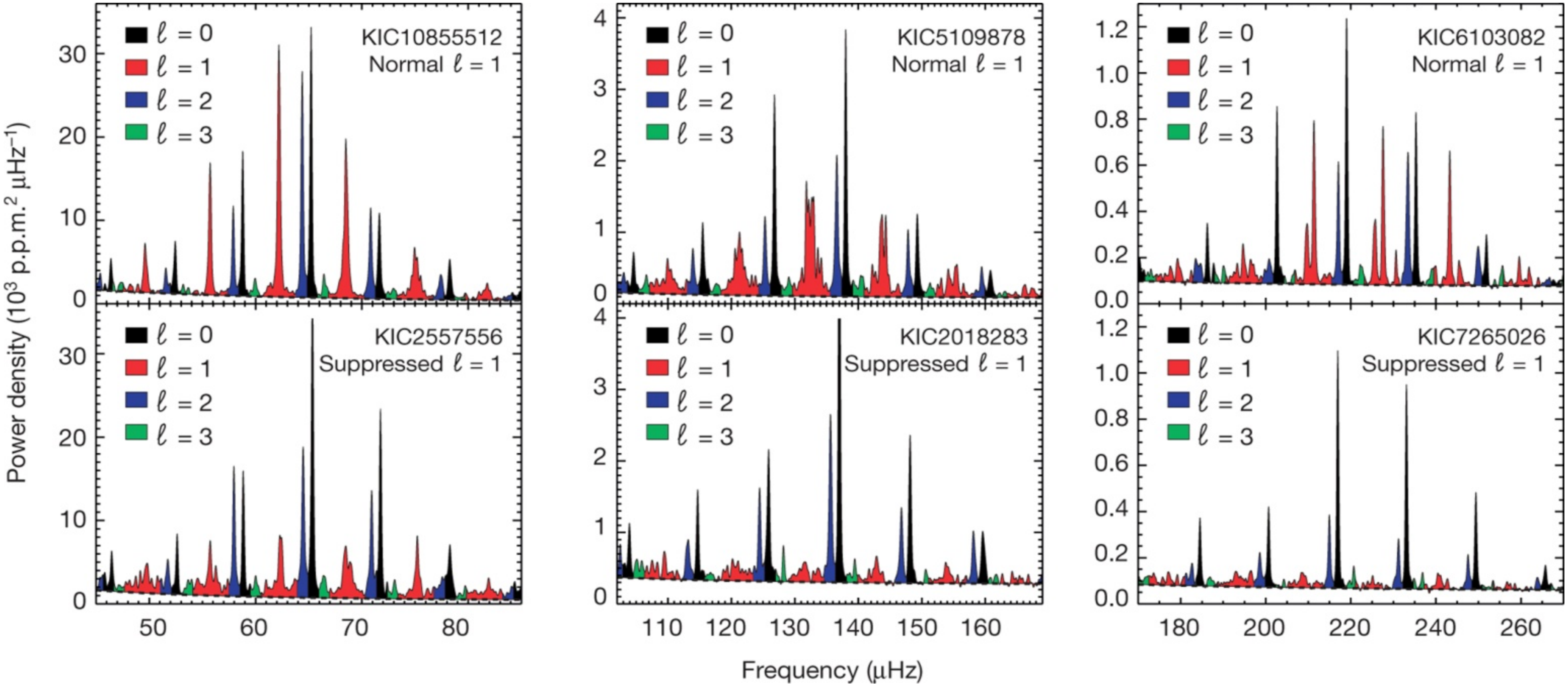
Bedding.. Hekker et al. 2011, Nature

Rotation in red giants



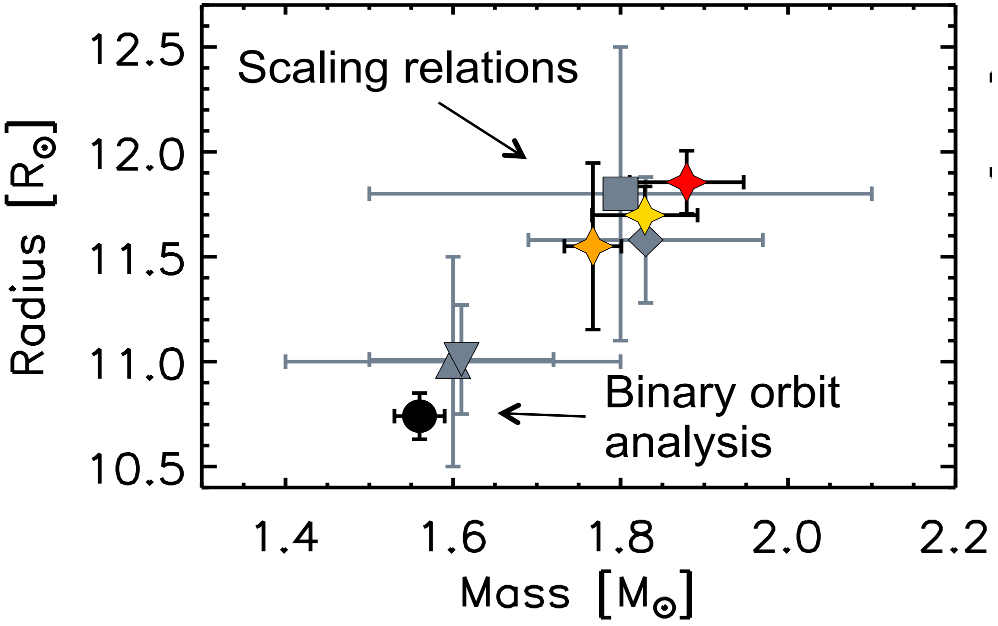
Beck et al. 2011, Nature

Suppressed dipole modes



Stello et al. 2016

Testing scaling relations

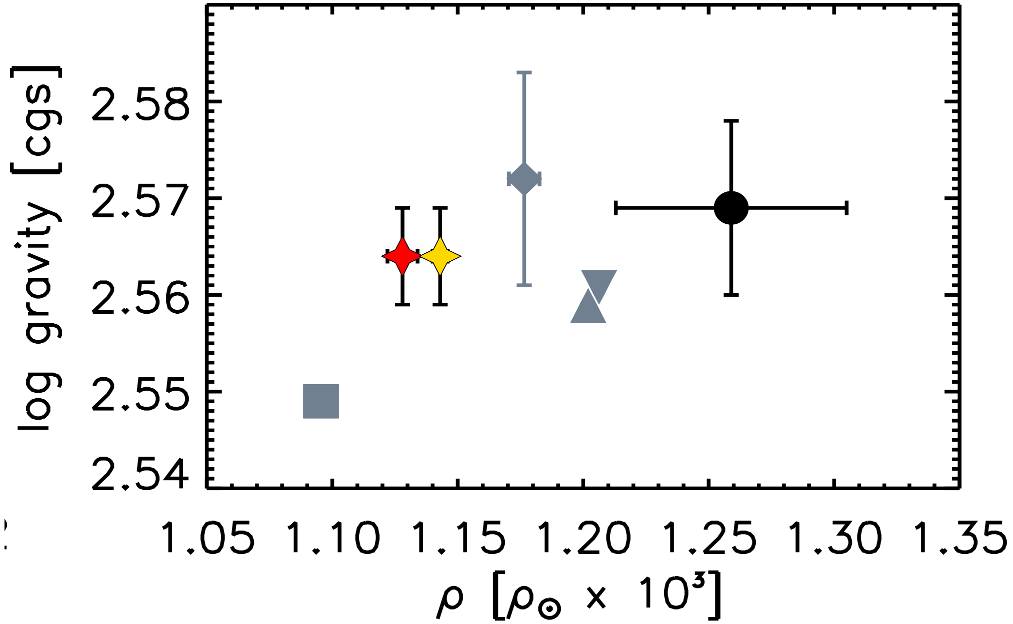


orbital solution

scaling relation + $\Delta\nu_\odot$

scaling relation + $\Delta\nu_{ref}$

grid-based modeling



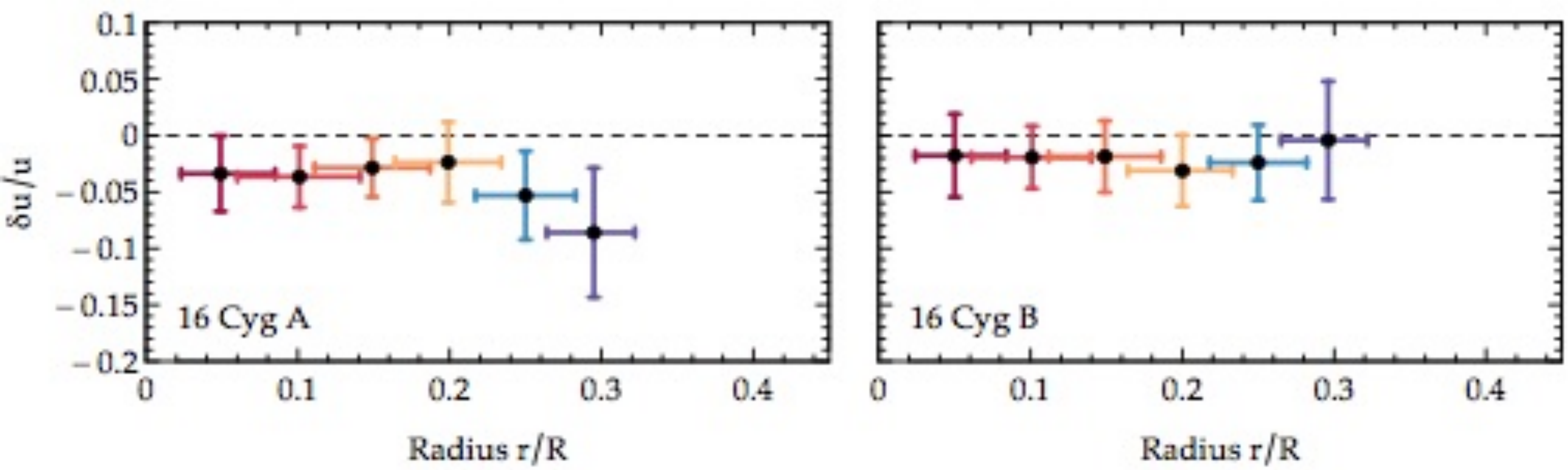
Themeßl et al. in prep.

What is the intrinsic accuracy of age determinations?

Parameters		$\mu(\epsilon)$ [Gyr]
$\langle r_{02} \rangle$	ν_{max}	0.642
$\langle r_{02} \rangle$	$\log \mathrm{g}$	0.683
$\langle r_{13} \rangle$	ν_{max}	0.711
$\langle r_{02} \rangle$	$\langle \Delta\nu_0 \rangle$	0.694
$\langle \Delta\nu_0 \rangle$	$\langle \delta\nu_{02} \rangle$	0.701
$\langle r_{02} \rangle$	$\langle \delta\nu_{02} \rangle$	0.701
PC_2	PC_8	0.767
PC_2	PC_4	0.762
$\log \mathrm{g}$	$\langle \Delta\nu_0 \rangle$	1.29
$\log \mathrm{g}$	T_{eff}	1.53

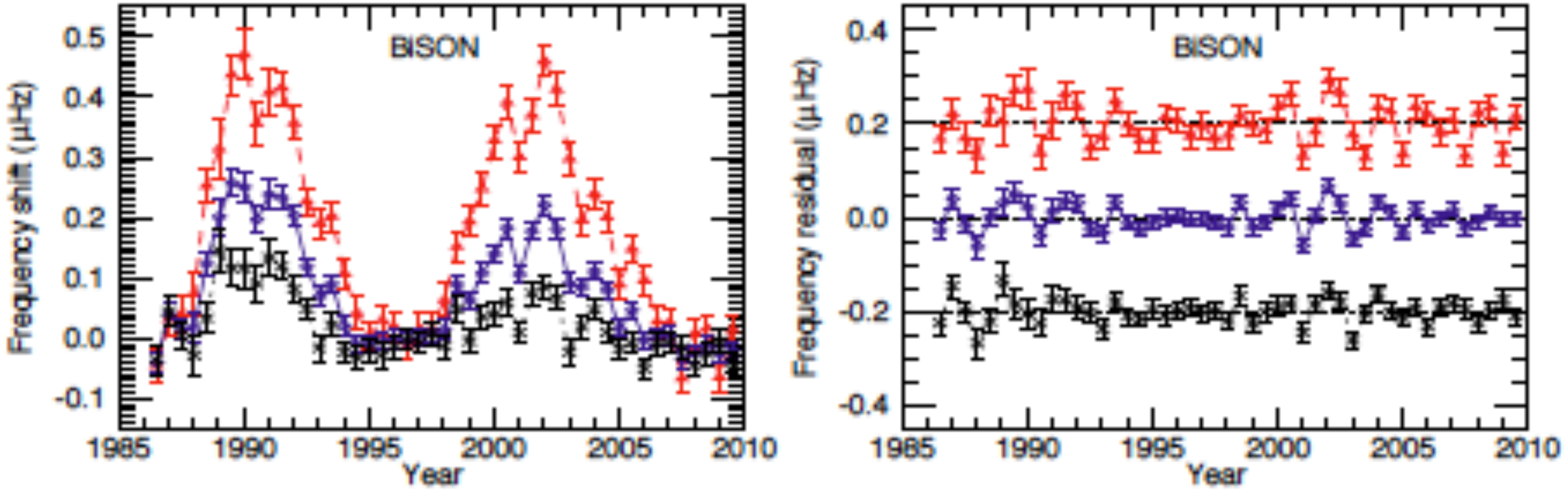
Angelou et al. (2017)

Stellar inversion



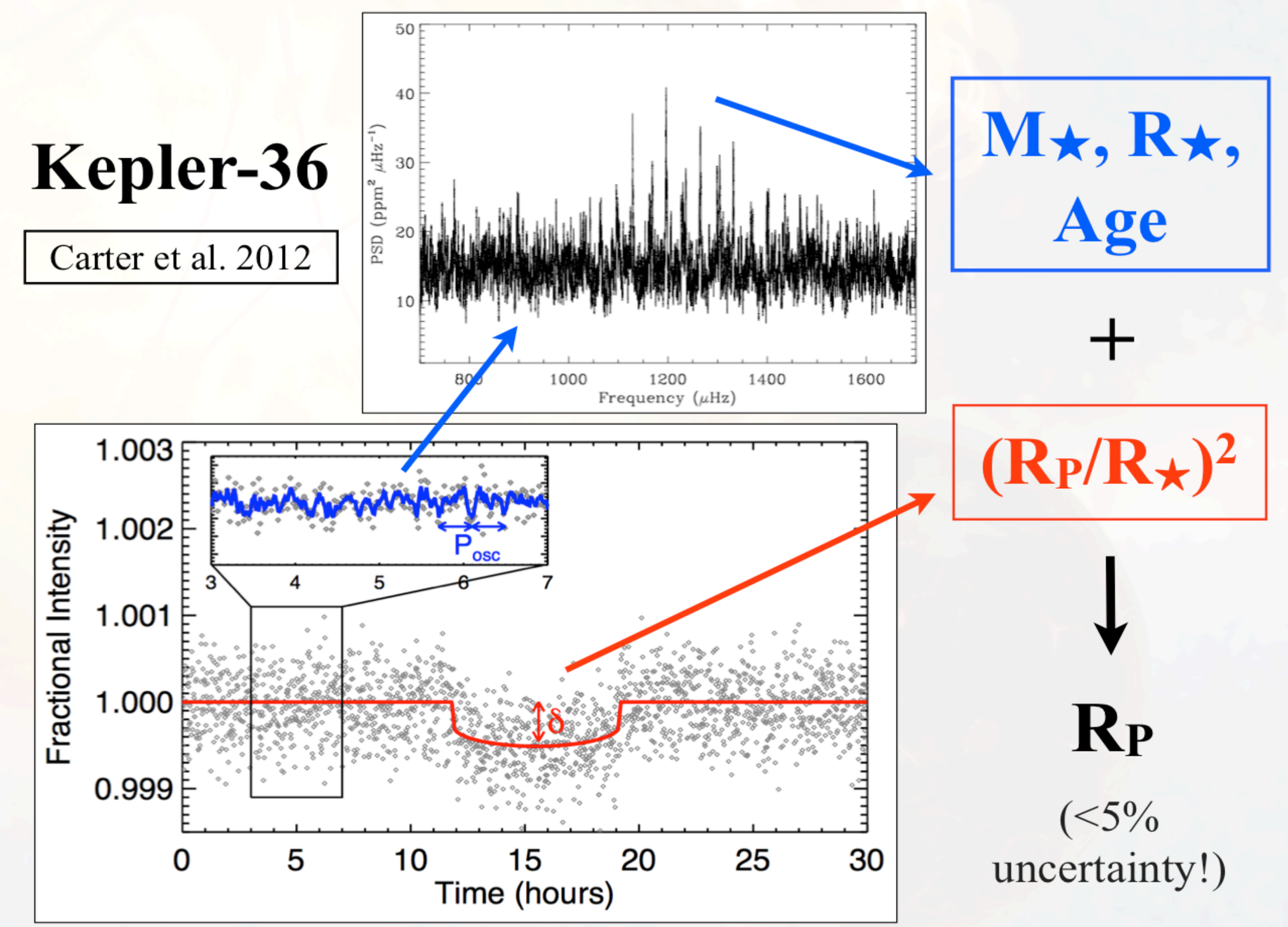
Bellinger et al. submitted

Sun: second dynamo...?



Broomhall et al. 2012, MNRAS 420, 1405

Planet radius



Kepler-36

Carter et al. 2012

$M_\star$, $R_\star$, Age

+

$(R_P/R_\star)^2$

↓

R_P

(<5% uncertainty!)

Galactic Archeology

"the study of the formation and evolution of the Milky Way by reconstructing its past from its current constituents"

important parameters:

- position
- distance
- velocity
- chemical composition
- age / evolutionary phase

Impact of asteroseismology