Theory of Stellar Oscillations

COURSE 7

LINEAR ADIABATIC STELLAR PULSATION

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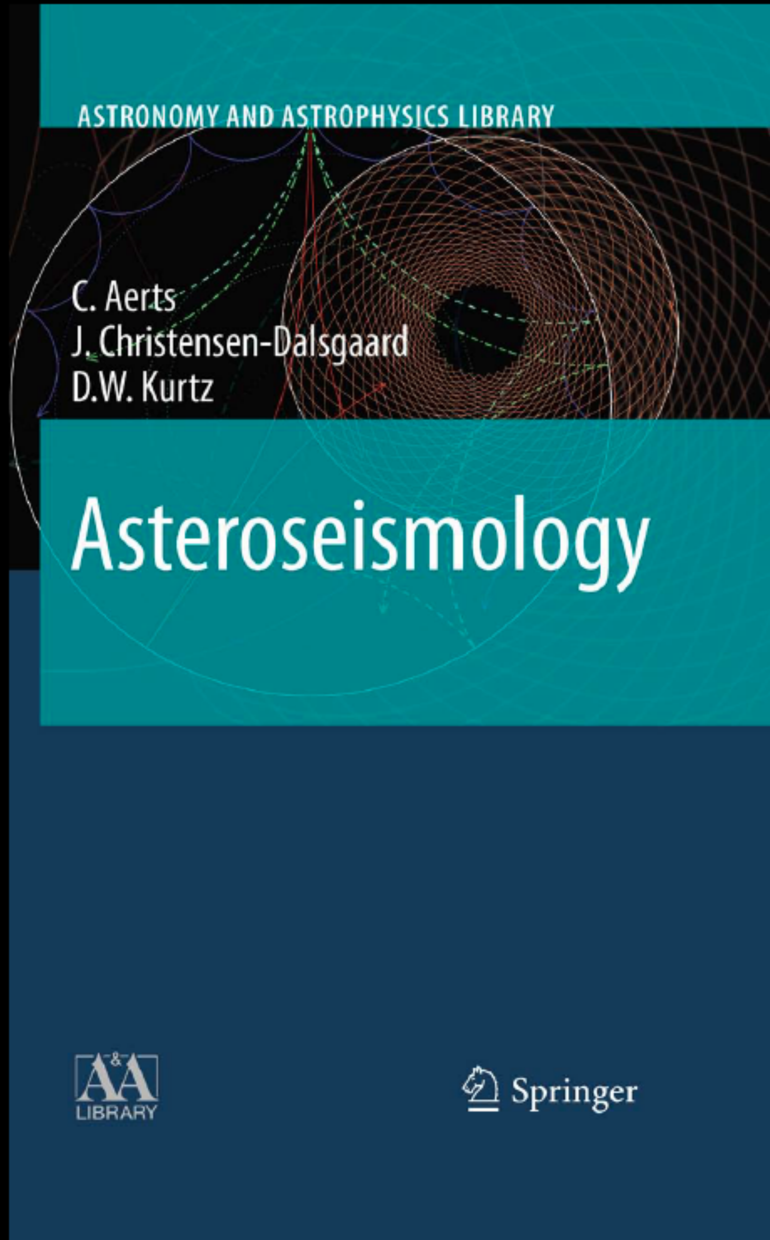
J.-P. Zahn and J. Zinn-Justin, eds.
Les Houches, Session XLVII, 1987
Dynamique des fluides astrophysiques
Astrophysical fluid dynamics



Brief introduction

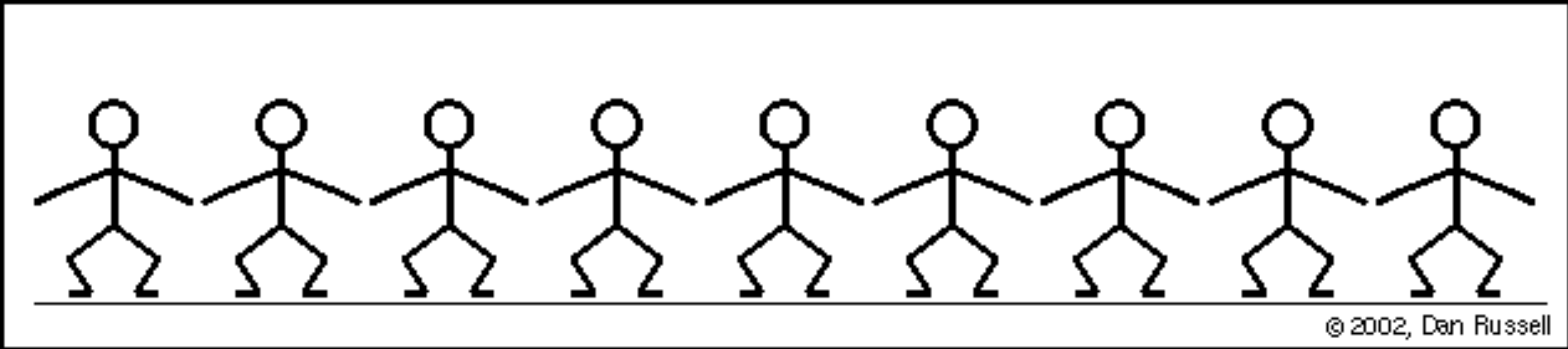
Asteroseismology How does it work?

How would you describe a wave?

Asteroseismology How does it work?

Wave: propagation of information (a perturbation) in space and time

Wave in a supporting medium: material does not need to move from one point of the space to the other to propagate the information



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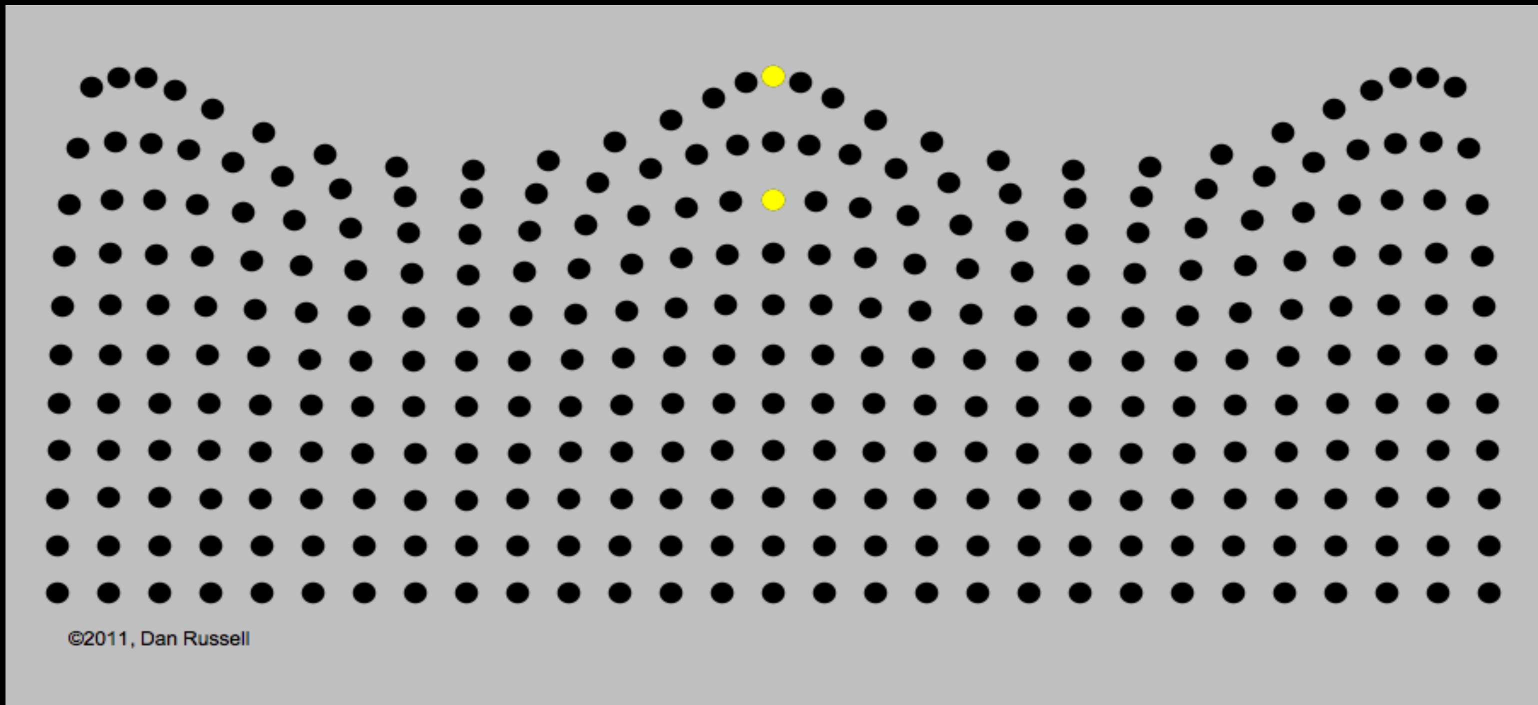

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Waves propagate within stars

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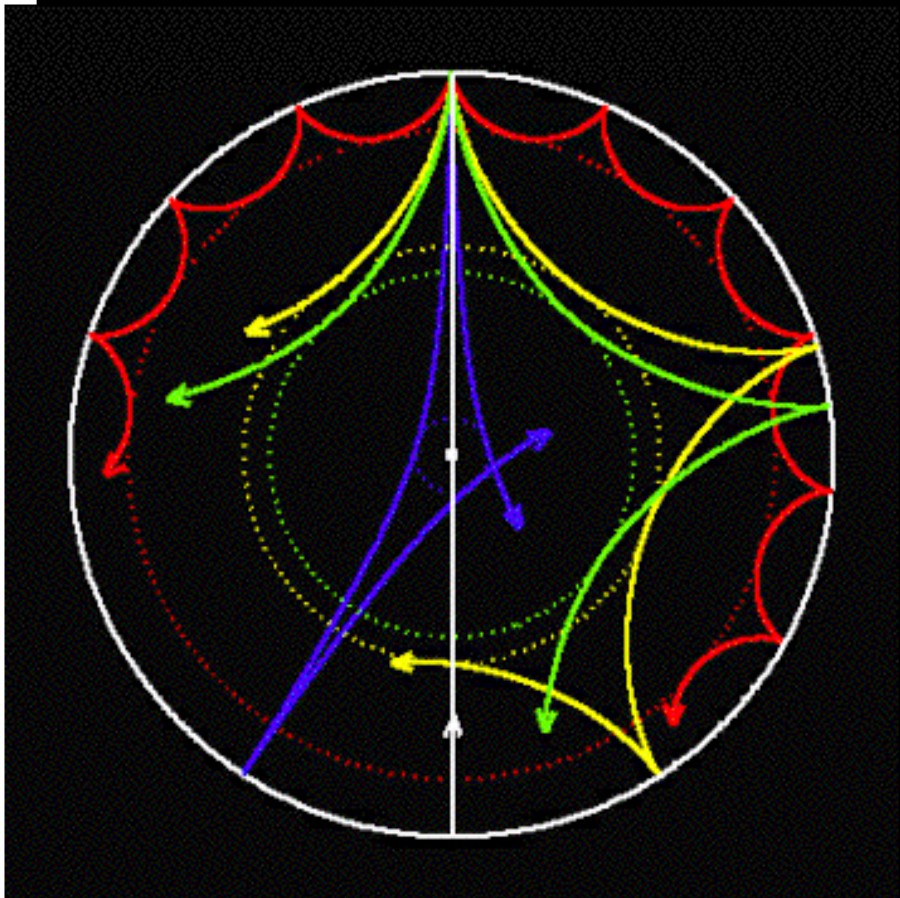

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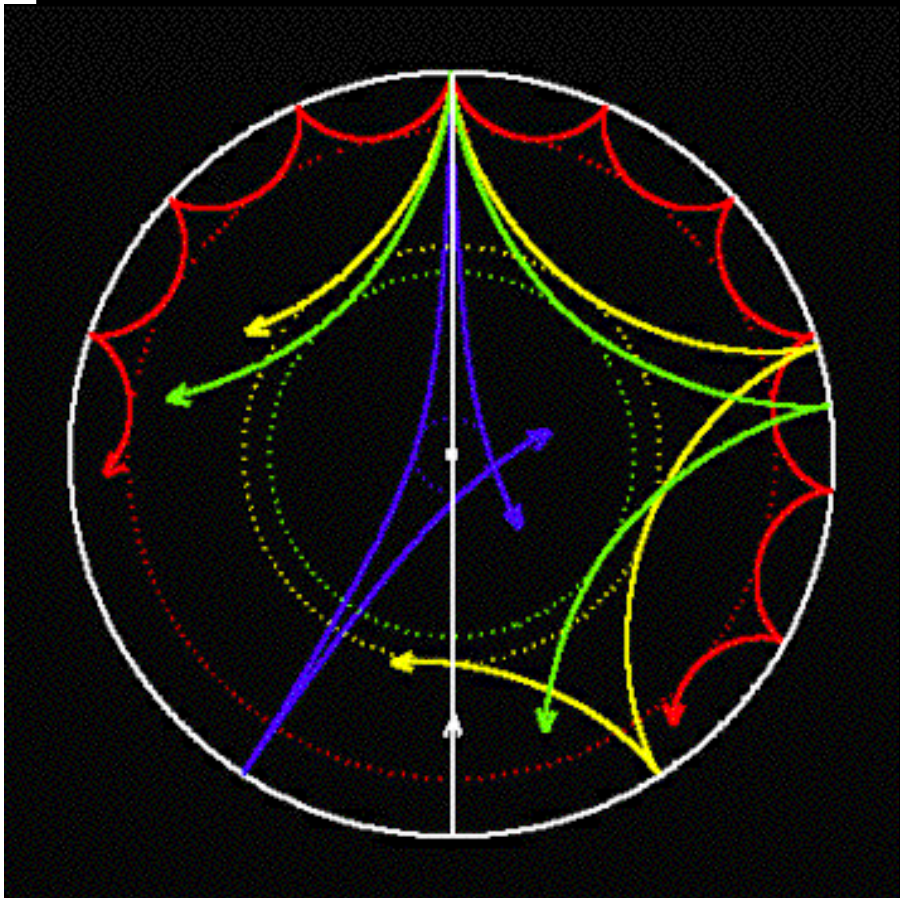


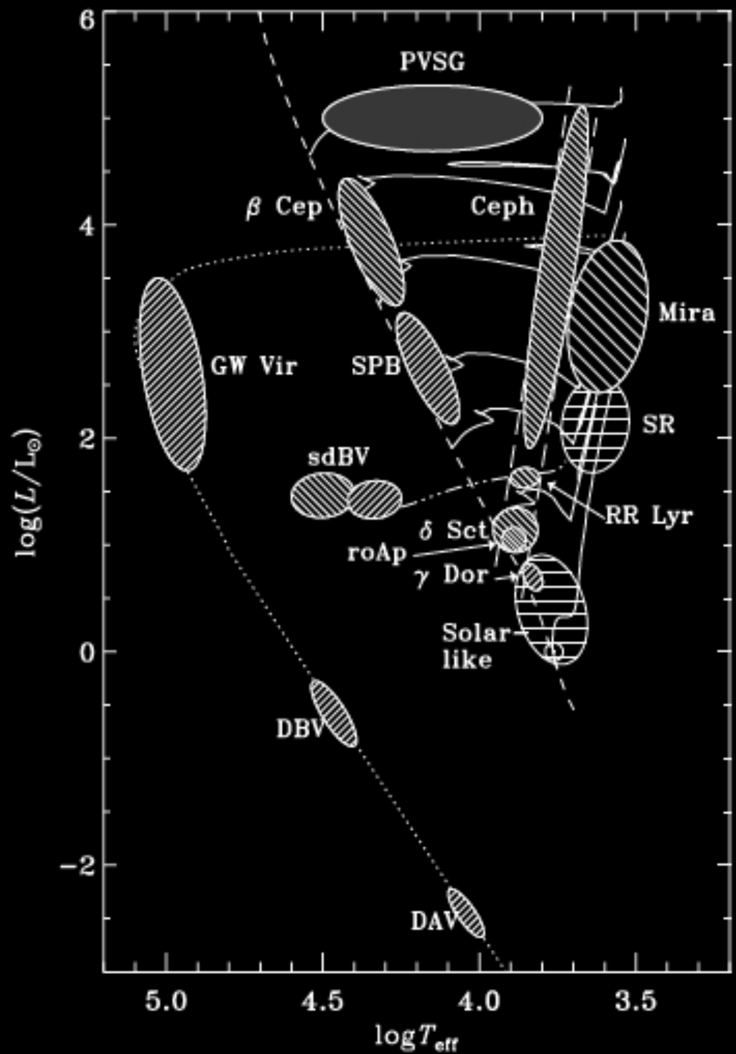
One mode ⇔ one piece of information

- Average information on propagation cavity

- With several modes one can hope to get localized information

Asteroseismology: Across the HR diagram

Kurtz 2010 adapted from Aerts et al. 2010



Asteroseismology: Classification

Origin
- Intrinsically unstable
  - Classical
- Intrinsically stable
  - Solar-like

Nature
- Acoustic waves
  - p modes
- Internal Gravity waves
  - g modes

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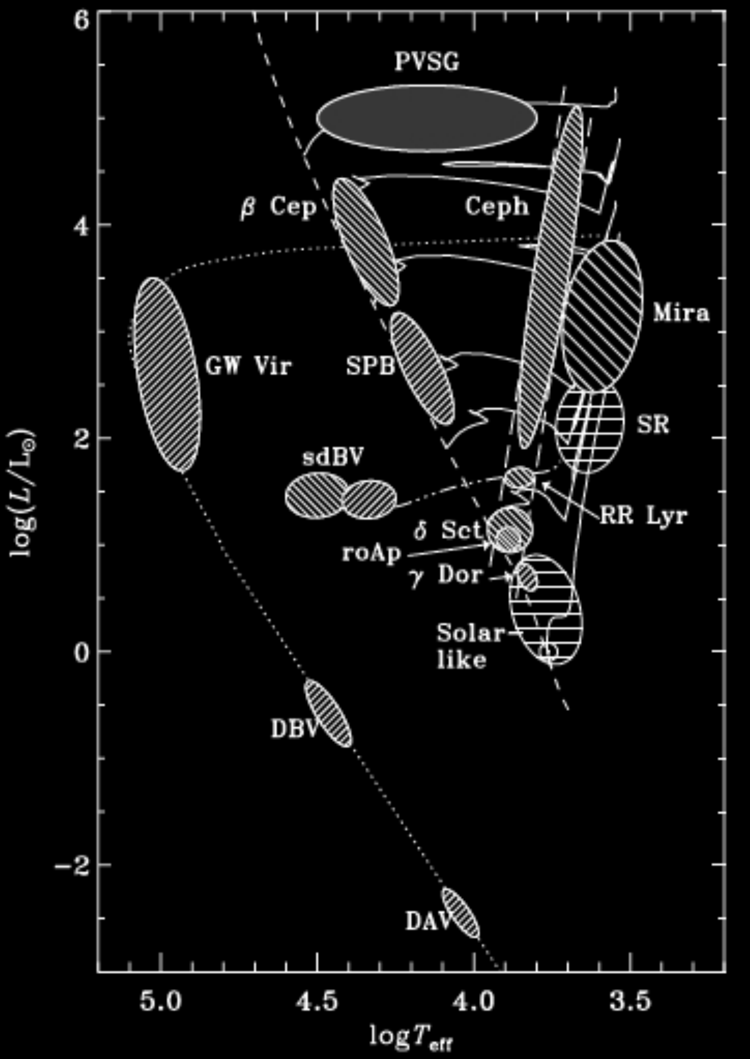
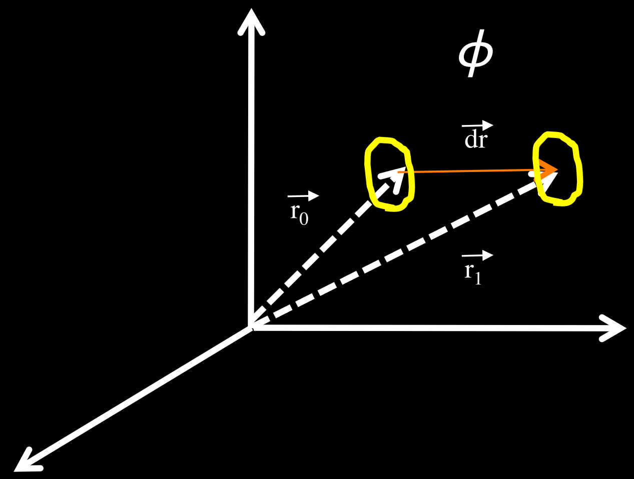


Hydrodynamics

Hydrodynamics

Assume that the gas can be treated as a continuum; Thermodynamic properties well defined at each position $\vec{r}$



Let ϕ be a scalar property of the gas.

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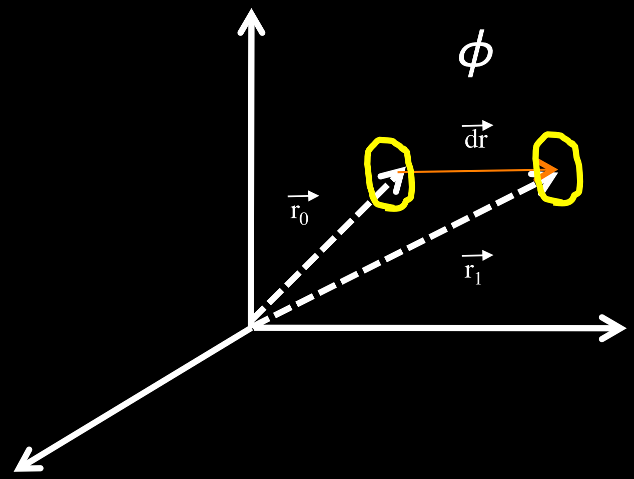

Let ϕ be a scalar property of the gas.

Two ways to look at time evolution of ϕ:

1. At fixed position => Eulerian description
2. Following the motion => Lagrangian description

$$\frac{D\phi}{Dt} = \frac{\partial \phi}{\partial t} + \nabla\phi \cdot \frac{d\vec{r}}{dt}$$
$$= \frac{\partial \phi}{\partial t} + \vec{v} \cdot \nabla\phi$$

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Continuity equation : The mass variation within a given volume V must equal, with opposite sign, the mass crossing the surface S that encloses the volume V.

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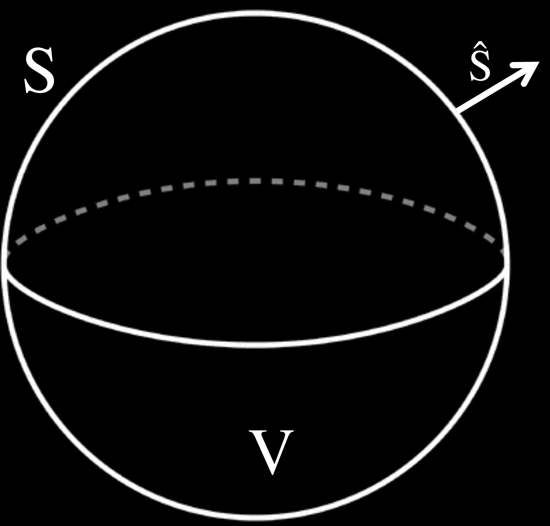

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{v})$$

ρ - density $\vec{v}$ - velocity

Hydrodynamics

Continuity equation
(conservation of mass)

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Following the fluid - Lagrangian description

Continuity equation
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$$\frac{D\rho}{Dt} + \rho\,\nabla\cdot\vec{v} = 0$$

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$$\frac{1}{\rho}\frac{D\rho}{Dt} = -\nabla\cdot\vec{v} \Leftrightarrow \frac{1}{V}\frac{DV}{Dt} = \nabla\cdot\vec{v}$$

ρ - density $\quad \vec{v}$ - velocity $\quad V$ - volume

Rate of
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=> Acoustic waves require div $\vec{v} \neq 0$

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$$\rho \frac{D\vec{v}}{Dt} = -\nabla p + \rho \vec{g} + \vec{F}$$

p - pressure $\quad$ $\vec{g} = -\nabla \phi$ - acceleration of gravity $\quad$ $\vec{F}$ -other body forces

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+ Poisson equation $\quad$ $\nabla^2\phi = 4\pi G\rho$

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Energy equation (first law of thermodynamics): the change in the internal energy of a system equals the heat supplied to the system minus the work done by the system on its surroundings.

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$$\frac{Dq}{Dt} = \frac{DE}{Dt} + p \frac{D(1/\rho)}{Dt}$$

q -heat supplied /mass E -internal energy /mass

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+ Equation of state

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Adiabatic approximation

Characteristic time scale for radiation:
Sun as a whole: 10^7 years
Solar convection zone: 10^3 years

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$$\delta f = f' + \delta \vec{r} . \nabla f_0$$

Summary of perturbed equations

Linear adiabatic pulsation about a static, spherically symmetric equilibrium

$$\rho' + \nabla \cdot (\rho_0 \delta\vec{r}) = 0$$

$$\rho_0 \frac{\partial^2 \delta\vec{r}}{\partial t^2} = -\nabla p' - \rho_0 \nabla \phi' + \rho' \nabla \phi_0$$

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Variables: 4 (ϱ', p', ϕ', $\delta\vec{r}$)

Equations: 4

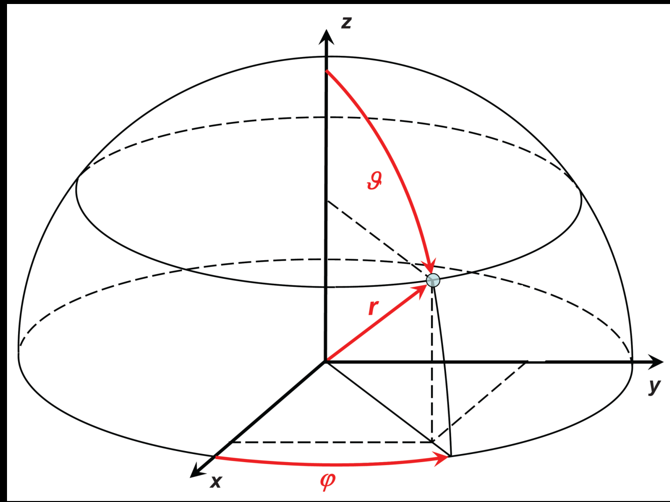
Thus: system of equation is closed, so far as equilibrium quantities are known.

=> can solve it to get solutions for the 4 variables.

Solutions on a sphere

Solutions on a sphere

Consider the spherical coordinates (r,θ,φ)



Variables $(\varrho', p', \phi', \delta\vec{r})$ are function of: r, θ, φ, t

Solutions on a sphere

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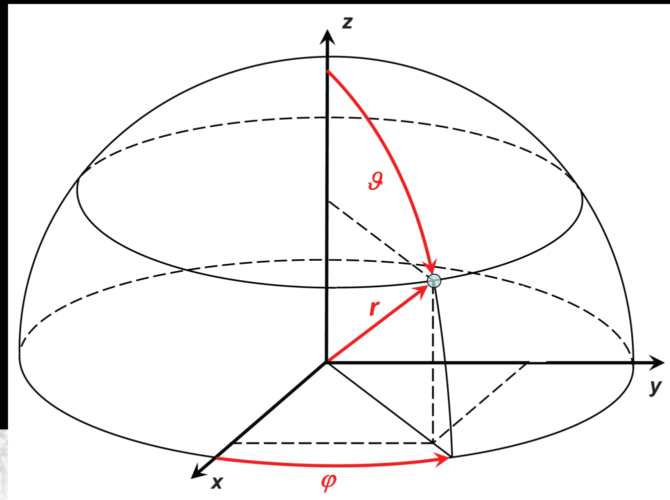
Variables (ϱ', p', ϕ', $\delta\vec{r}$) are function of: r, θ, φ, t

The equations admit solutions of the type:



$$p'(\mathrm{r},\theta,\varphi,t) = \mathrm{Re}[p'(r)Y_l^m(\theta,\varphi)\mathrm{e}^{-i\omega t}]$$

$$\rho'(\mathrm{r},\theta,\varphi,t) = \mathrm{Re}[\rho'(r)Y_l^m(\theta,\varphi)\mathrm{e}^{-i\omega t}]$$

$$\phi'(\mathrm{r},\theta,\varphi,t) = \mathrm{Re}[\phi'(r)Y_l^m(\theta,\varphi)\mathrm{e}^{-i\omega t}]$$

$$\delta\vec{\mathrm{r}}(\mathrm{r},\theta,\varphi,t) = \mathrm{Re}\left\{\left[\xi_r(r)Y_l^m\hat{\mathrm{a}}_\mathrm{r} + \xi_h(r)\left(\frac{\partial Y_l^m}{\partial\theta}\hat{\mathrm{a}}_\theta + \frac{1}{\sin\theta}\frac{\partial Y_l^m}{\partial\phi}\hat{\mathrm{a}}_\phi\right)\right]\mathrm{e}^{-i\omega t}\right\}$$

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Spherical Harmonics Y_l^m

$$k_h = \frac{\sqrt{l(l+1)}}{R}$$

l – angular degree: the number of nodes on the sphere

m - azimuthal order: $|m|$ =number of nodes along the equator => orientation on the sphere

Note: $|m| \leq l$

Spherical Harmonics Y_l^m

l – *angular degree: the number of nodes on the sphere*

$$k_h = \frac{\sqrt{l(l+1)}}{R}$$

m - azimuthal order: $|m|$ =number of nodes along the equator

=> orientation on the sphere

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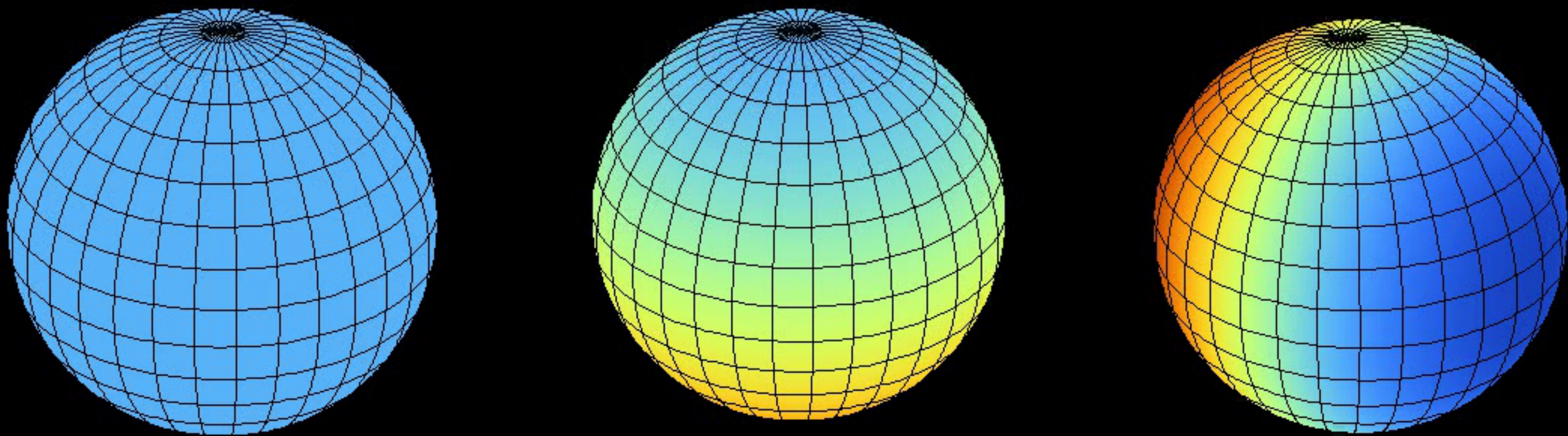

l=0

l=1
m=0

l=1
m=-1

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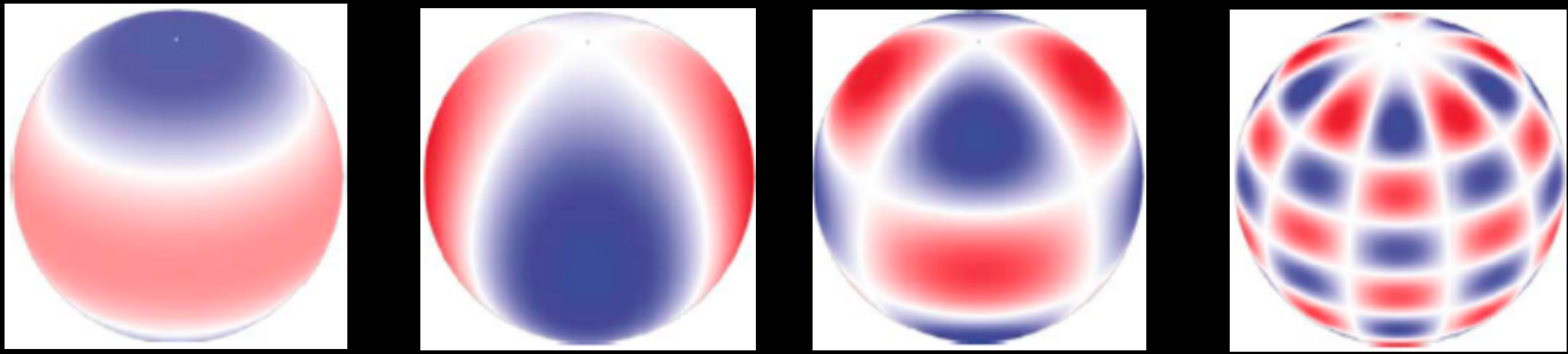
adapted from Aerts et al. 2010



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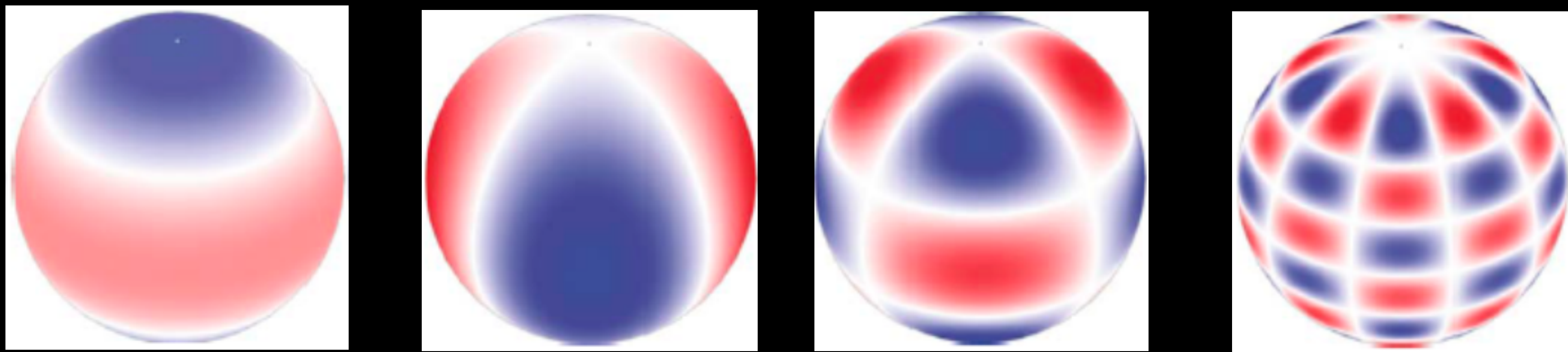

l=2	l=2	l=4	l=10
m=0	$\|m\|$=2	$\|m\|$=2	$\|m\|$=5

Solutions on a sphere

Consider the spherical coordinates (r,θ,φ)

Variables $(\varrho', p', \phi', \delta\vec{r})$ are function of: r, θ, φ, t

The equations admit solutions of the type:

$$p'(\mathrm{r},\theta,\varphi,t) = \mathrm{Re}[p'(r)Y_l^m(\theta,\varphi)\mathrm{e}^{-i\omega t}]$$

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Substituting the solutions on the perturbed equations
… and after significant algebra

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This system, together with the boundary conditions, forms an eigenvalue problem
=> Solving it provide the eigenvalues, ω, and eigenfunctions, ξ_r, p’, ϕ’, dϕ’/dr.

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$N_0^2 > 0 \Rightarrow \varrho_2^* > \varrho_2$

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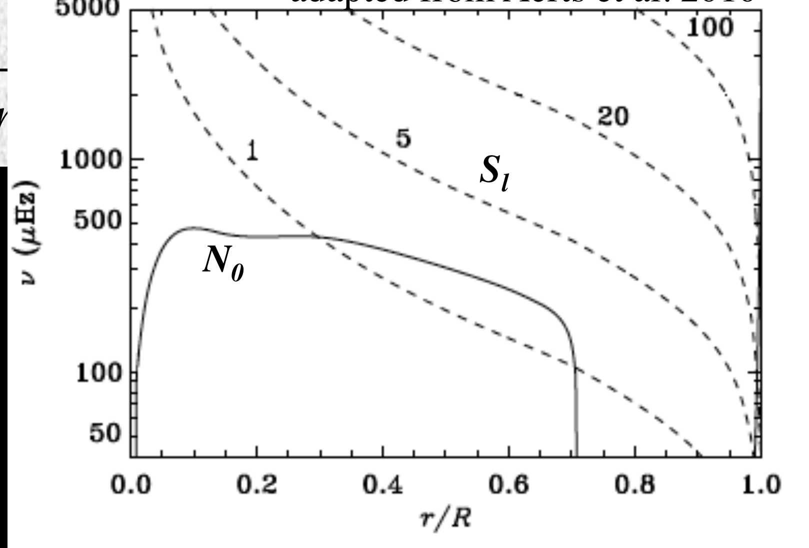

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Fourth order system => 4 boundary conditions required

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- 2 condition at r=R

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Obtained by imposing regularity of the solutions at the centre

displacement must vanish in the centre

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2nd condition: depends on how the atmosphere is treated

e.g. assuming free surface => $\delta p'=0$

$$p' + \xi_r \frac{dp_0}{dr} = 0$$

(But this is not adequate for a real star!)

A better option is to make the numerical solutions match onto the analytical solutions for an isothermal atmosphere.

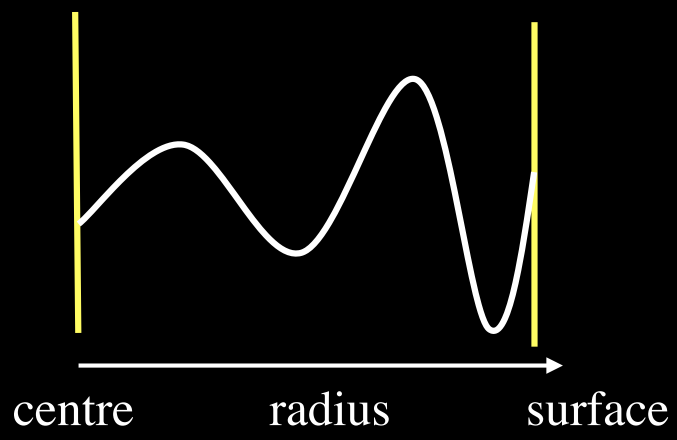
Eigenvalue problem

We reduced the problem to 1D

Equations + boundary conditions

=> admit non-trivial solutions only for a discrete values of frequencies

This set of frequencies is numbered by an integer *n, the radial order*



Eigenvalue problem

In summary: eigenfrequencies are discrete and characterized by three quantum numbers:

$$\boxed{\omega=\omega(n,l,m)}$$

n –*radial order:* |*n*| *related to the number of nodes along the radial direction*

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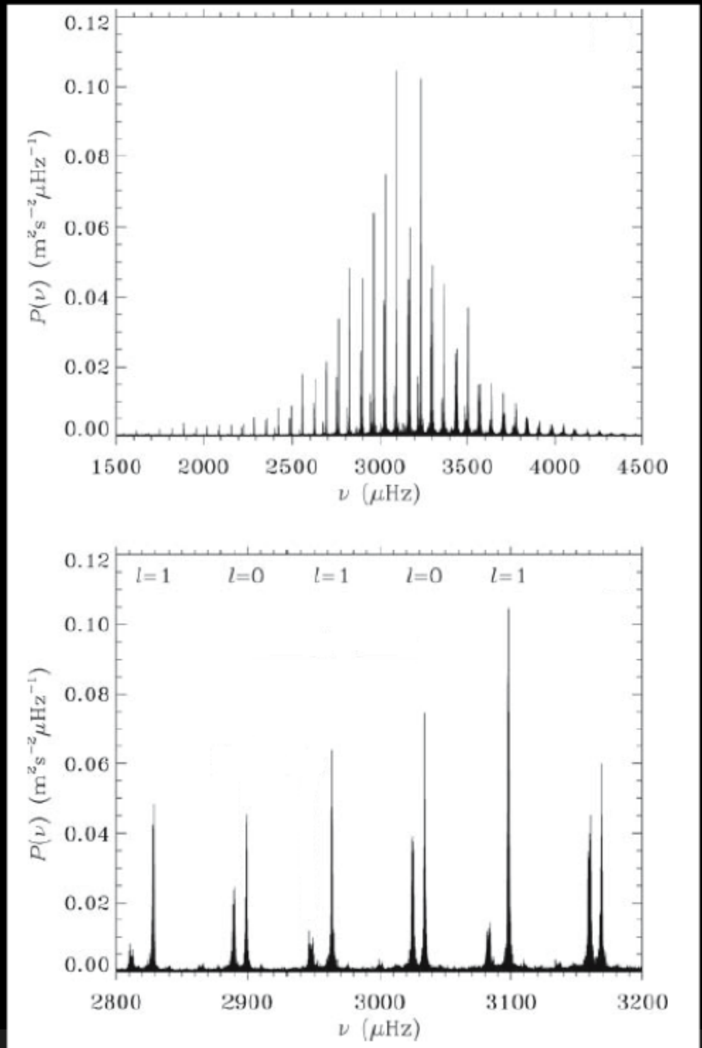
Adapted from Cunha et al 2007 (Bison data)



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$$\boxed{\omega = \omega(n, l, \cancel{m})}$$

Note: That is not the case if the star rotates or has a magnetic field, braking the symmetry.

Trapping of the oscillations

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2 variables: ξ_r, p'
2nd order system

Trapping of oscillations

Following Deubner and Gough 1984

- Work under Cowling approximation
- Assume that locally oscillations can be treated as in a plane-parallel layer under constant gravity (i.e., neglect derivatives of g and r)

(See also, Gough 93)

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In terms of the new variable the 2nd order system of equations can be reduced to a single 2nd order wave equation:

$$\frac{d^2X}{dr^2} + k_r^2 X = 0$$

Where k_r is the local radial wavenember

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Recall the solutions of the wave equation with constant k

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- $k^2 > 0$ => k is real ; $\mathrm{Re}\{y\} = a\cos kx + b\sin kx$

  => *oscillatory behaviour*

- $k^2 < 0$ => $k = i\,|k|$; $\mathrm{Re}\{y\} = ae^{-|k|x} + be^{|k|x}$

  => *exponential grow or decay*

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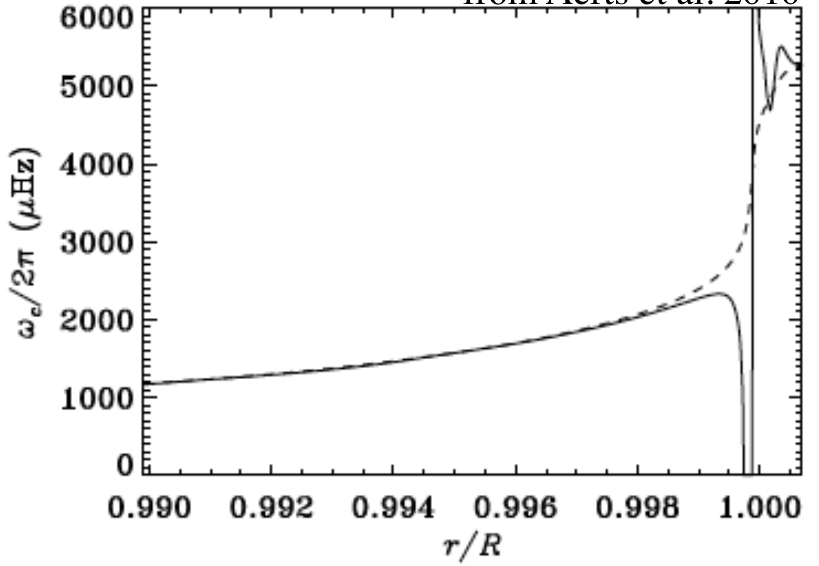
from Aerts et al. 2010



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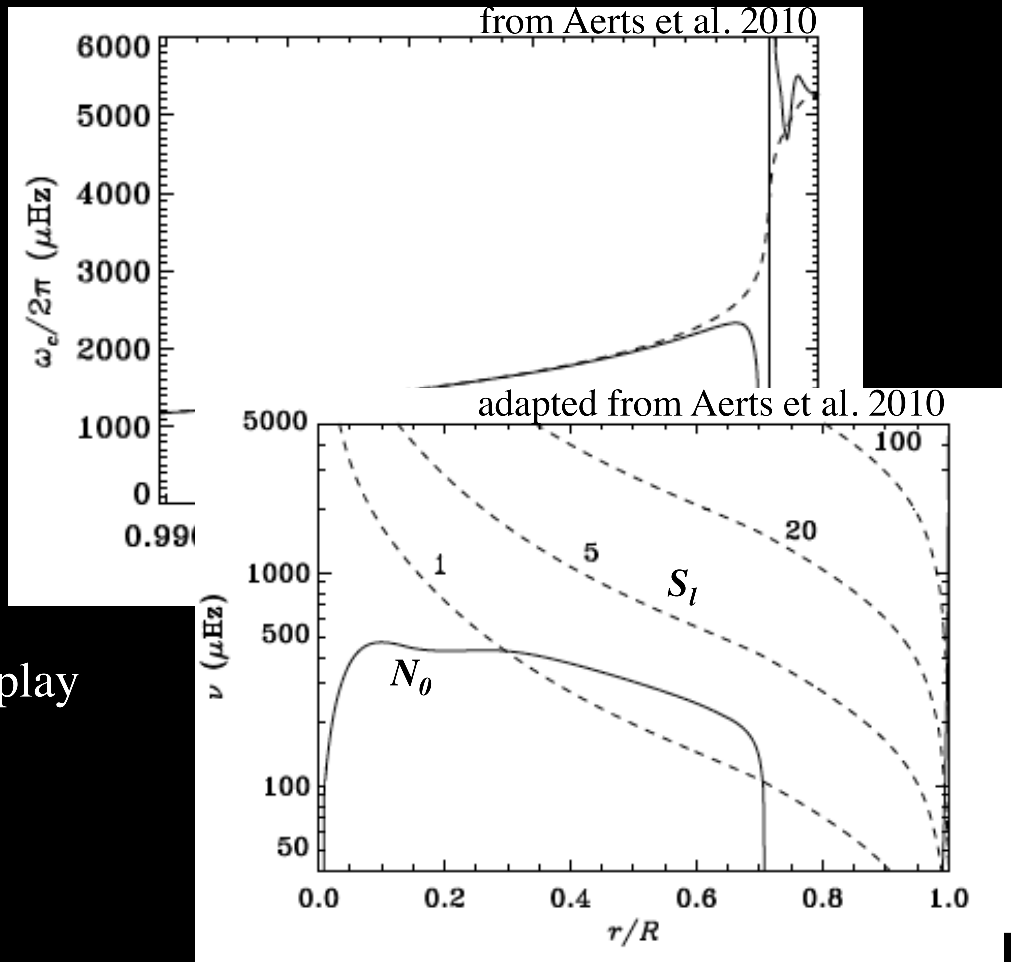

These 3 characteristic frequencies will play a fundamental role in deciding where modes propagate and where they are evanescent.

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$k_r{}^2 < 0$ (exponentially decaying) ?

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Thus, we can rewrite: $$k_r^2 = \frac{1}{{c_0}^2}\left[\omega^2 - \omega_{l+}^2\right]\left[\omega^2 - \omega_{l-}^2\right]$$

Trapping of oscillations

$$\frac{d^2X}{dr^2} + k_r^2 X = 0$$

$$k_r^2 = \frac{1}{c_0{}^2}\left[S_l^2\left(\frac{N_0^2}{\omega^2} - 1\right) + \omega^2 - \omega_c^2\right]$$

What are the regions where: $k_r^2 > 0$ (oscillatory behaviour) ?

$k_r^2 < 0$ (exponentially decaying) ?

Find the turning points of the equation, where $k_r^2 = 0$

$$\omega_{l\pm}^2 = \frac{1}{2}\left(S_l^2 + \omega_c^2\right) \pm \frac{1}{2}\sqrt{\left(S_l^2 + \omega_c^2\right)^2 - 4S_l^2 N_0^2}$$

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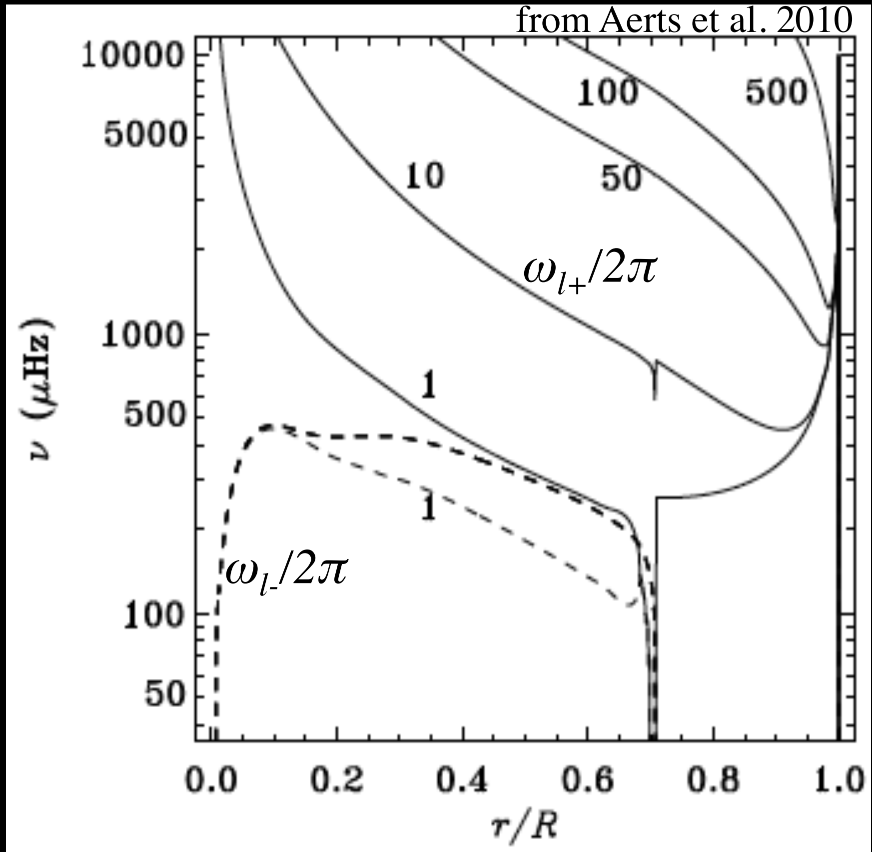
- Modes propagate where $k_r^2 > 0$ => $\omega > \omega_{l+}$ or $\omega < \omega_{l-}$
- Modes are evanescent where $k_r^2 < 0$ => $\omega_{l-} < \omega < \omega_{l+}$

Trapping of oscillations



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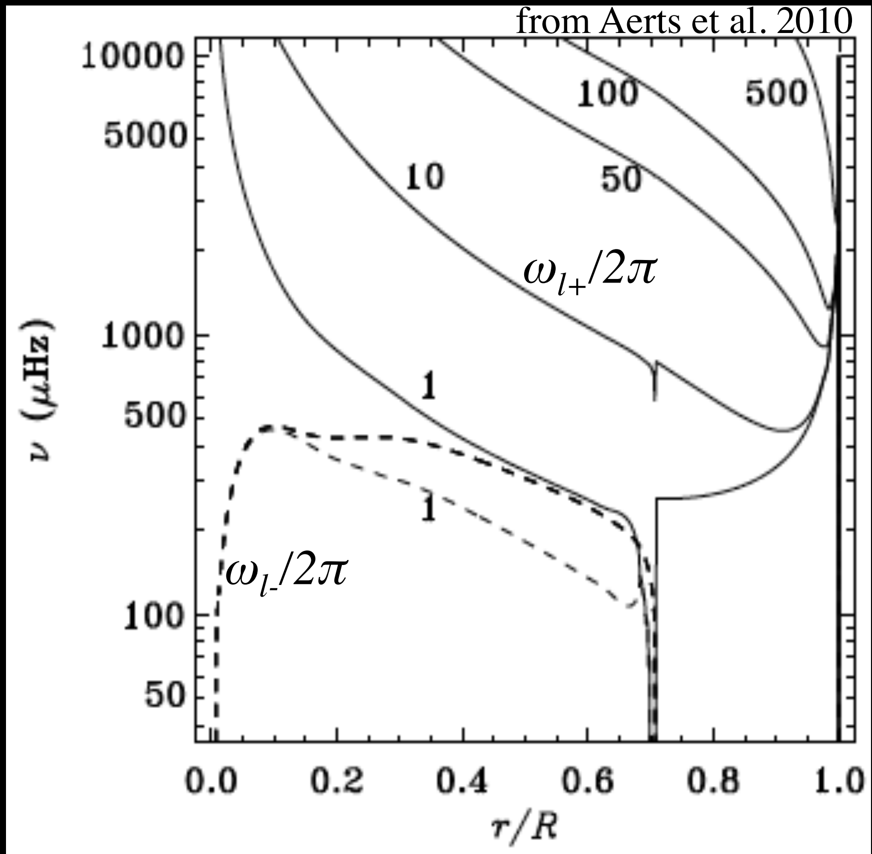

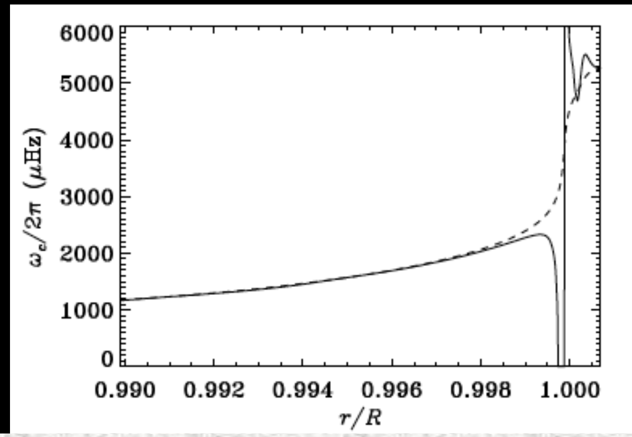

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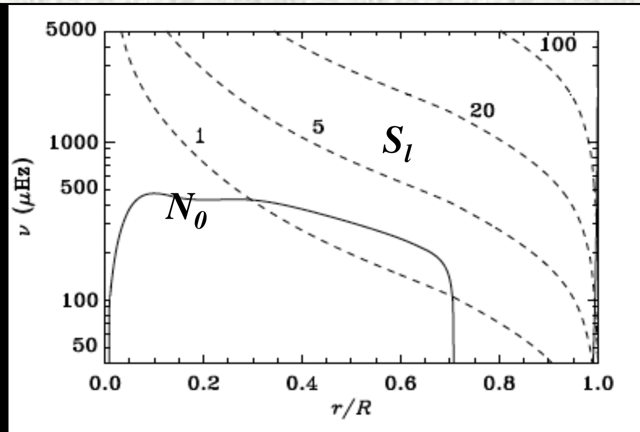

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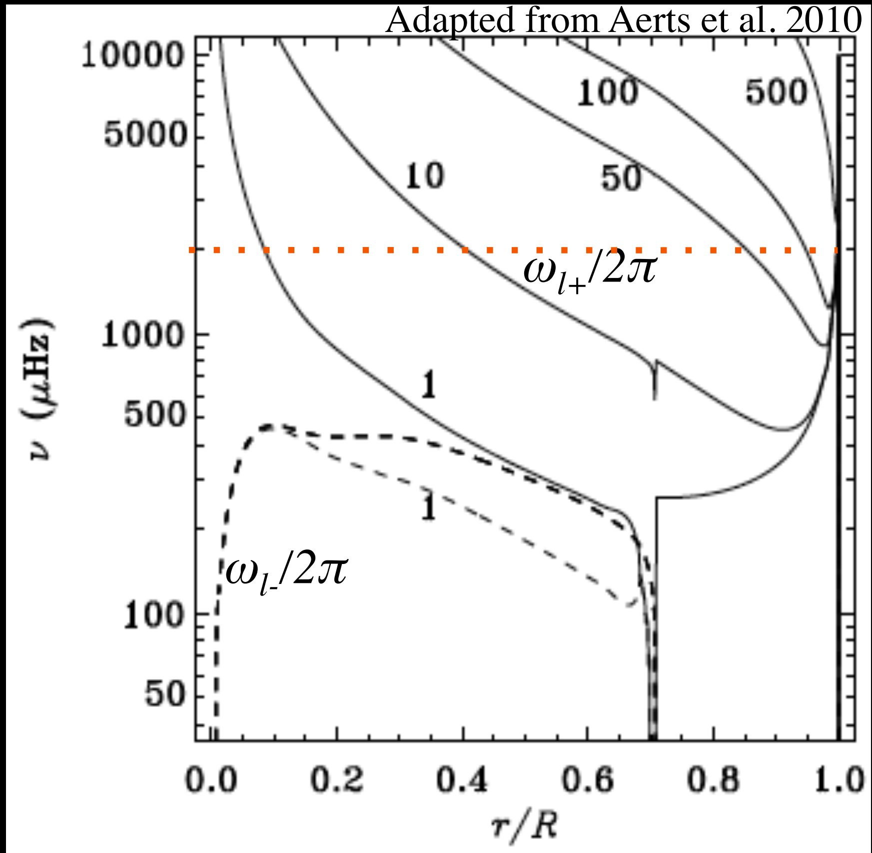

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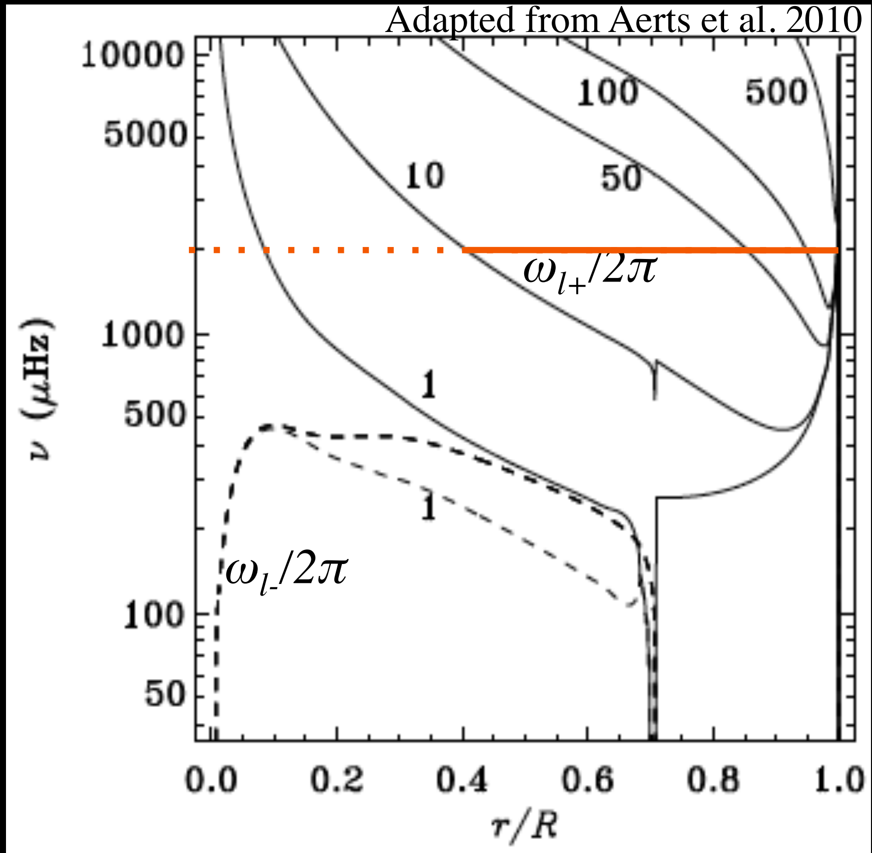

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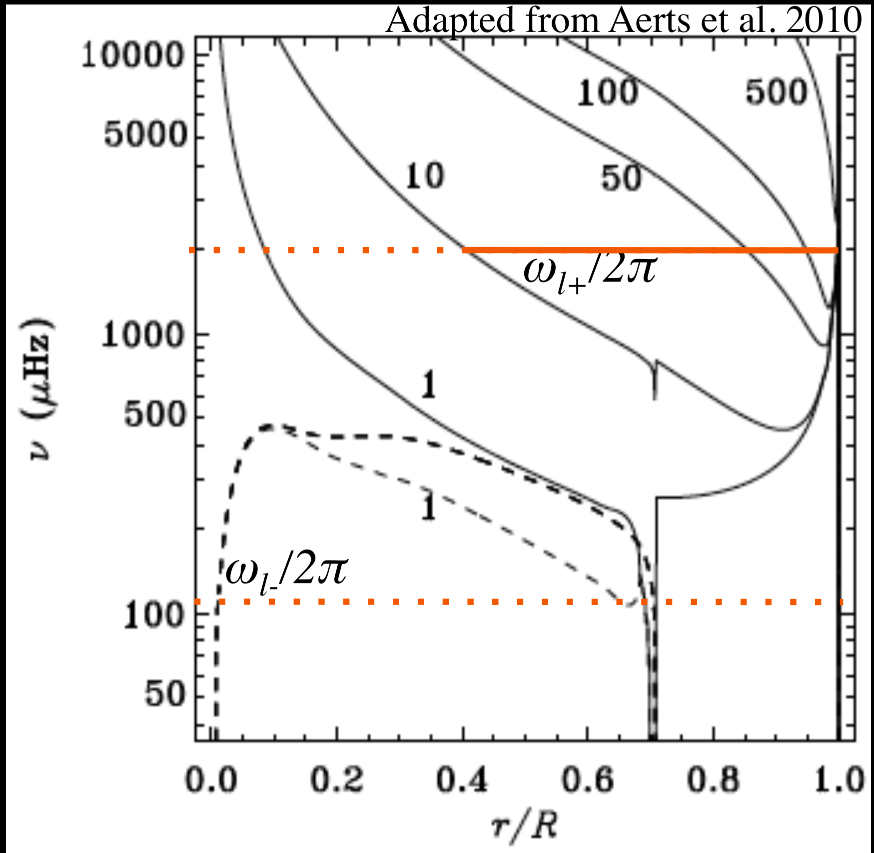

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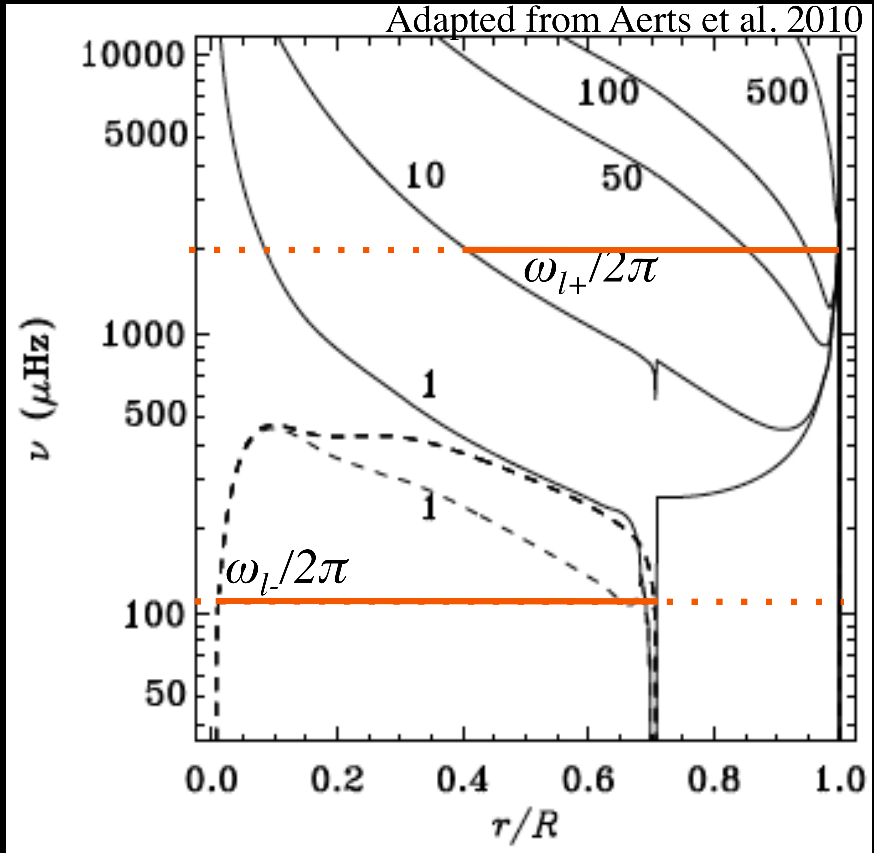

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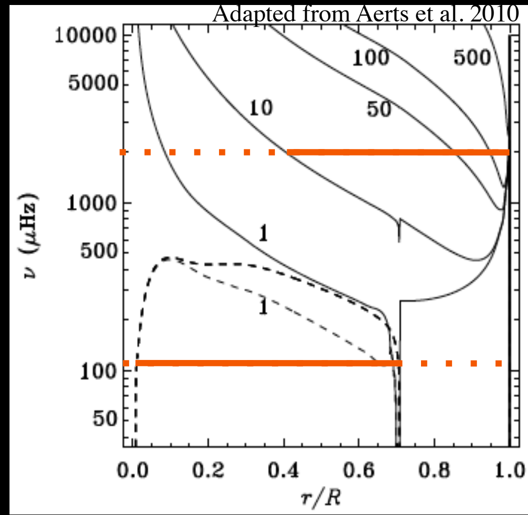
A closer look at the solutions

Trapping of oscillations

A closer look at the two families of solutions

$$k_r^2 = \frac{1}{c_0^{\,2}}\left[S_l^2\left(\frac{N_0^2}{\omega^2}-1\right)+\omega^2-\omega_c^2\right]$$

- High frequency modes $\omega^2 >> N_0^2$



Trapping of oscillations

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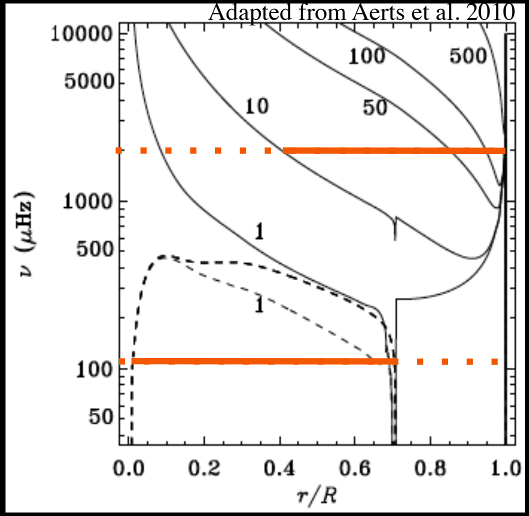
$$k_r^2 \approx \frac{\omega^2 - S_l^2}{c_0^{\,2}} = \frac{\omega^2}{c_0^{\,2}} - \frac{l(l+1)}{r^2}$$



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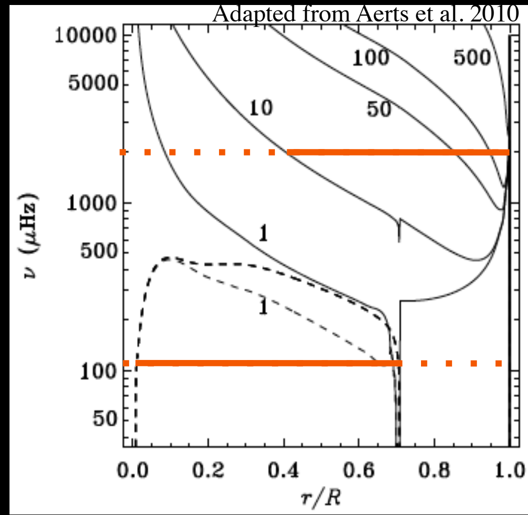

Trapping of oscillations

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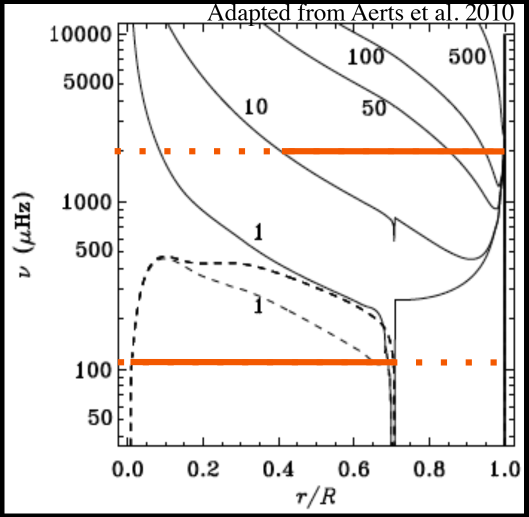
k_h^2

$$k^2 \equiv k_r^2 + k_h^2 \approx \frac{\omega^2}{c_0^2}$$



Trapping of oscillations

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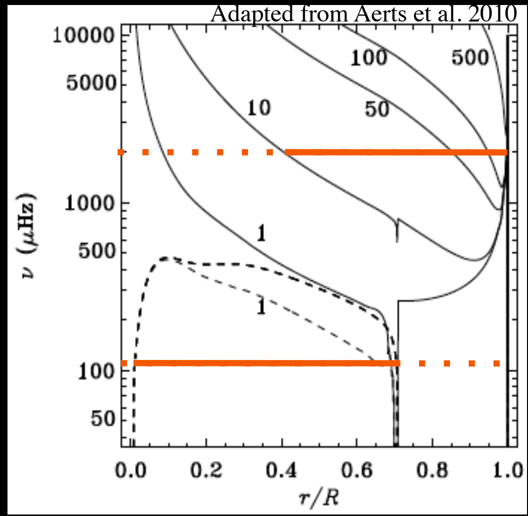
k_h^2

$$k^2 \equiv k_r^2 + k_h^2 \approx \frac{\omega^2}{c_0^2}$$

$$\omega \approx c_0 k$$

Dispersion relation for acoustic wave!



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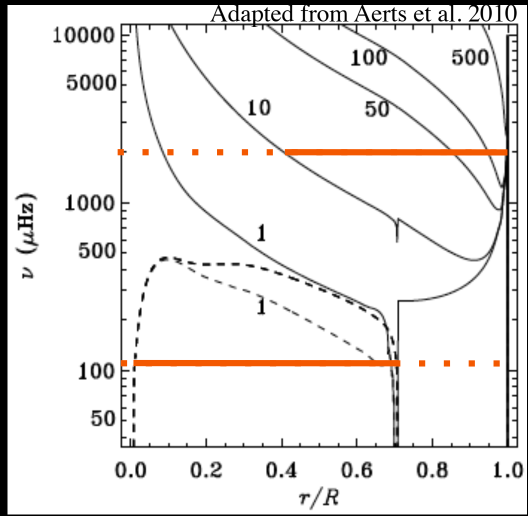

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k_h^2

$$k^2 \equiv k_r^2 + k_h^2 \approx \frac{\omega^2}{c_0^2}$$

$$\omega \approx c_0 k$$

Dispersion relation for acoustic wave!

ω increases as k increases

=> the radial order n increases with the frequency

Trapping of oscillations

$$k_r^2 = \frac{1}{c_0^{\ 2}}\left[S_l^2\left(\frac{N_0^2}{\omega^2}-1\right)+\omega^2-\omega_c^2\right]$$

A closer look at the two families of solutions

- High frequency modes $\omega^2 >> N_0^2$

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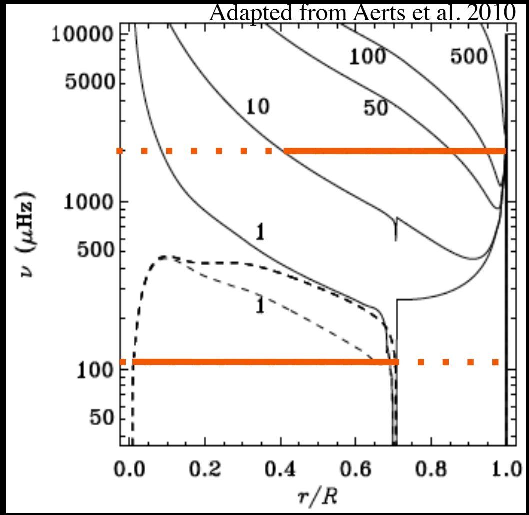

Lower turning point

Trapping of oscillations

A closer look at the two families of solutions

$$k_r^2 = \frac{1}{c_0^{\,2}}\left[S_l^2\left(\frac{N_0^2}{\omega^2}-1\right)+\omega^2-\omega_c^2\right]$$

- High frequency modes $\omega^2 >> N_0^2$

Except near the surface

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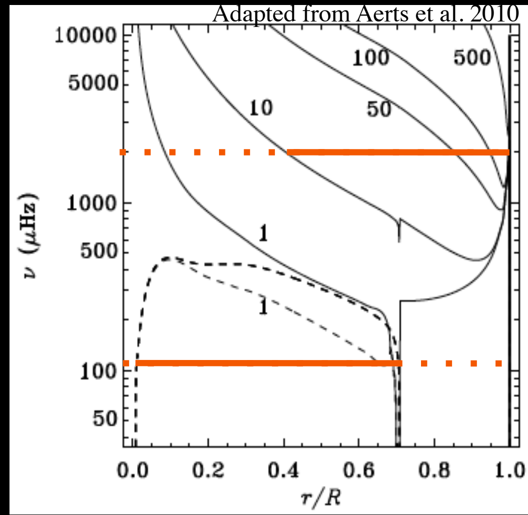
Lower turning point $\omega^2 = S_l^2$

$$r_{1,l} = \frac{\sqrt{l(l+1)}\,c_0}{\omega}$$



Adapted from Aerts et al. 2010

Trapping of oscillations

A closer look at the two families of solutions

$$k_r^2 = \frac{1}{c_0^{\,2}}\left[S_l^2\left(\frac{N_0^2}{\omega^2}-1\right)+\omega^2-\omega_c^2\right]$$

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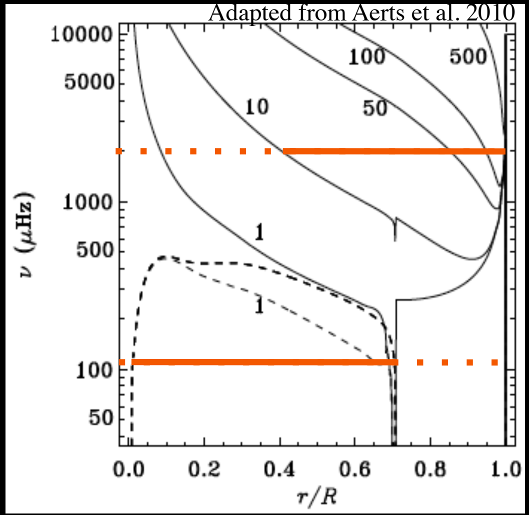

Lower turning point $\omega^2 = S_l^2$

$$r_{1,l} = \frac{\sqrt{l(l+1)}\,c_0}{\omega}$$

$r_{1,l}$ increases as l increases

=> larger degree modes have shallower cavities

For fixed l: $r_{1,l}$ increases as ω increases

=> higher frequency modes propagate deeper, for fixed degree

Trapping of oscillations

A closer look at the two families of solutions

- High frequency modes $\omega^2 >> N_0^2$

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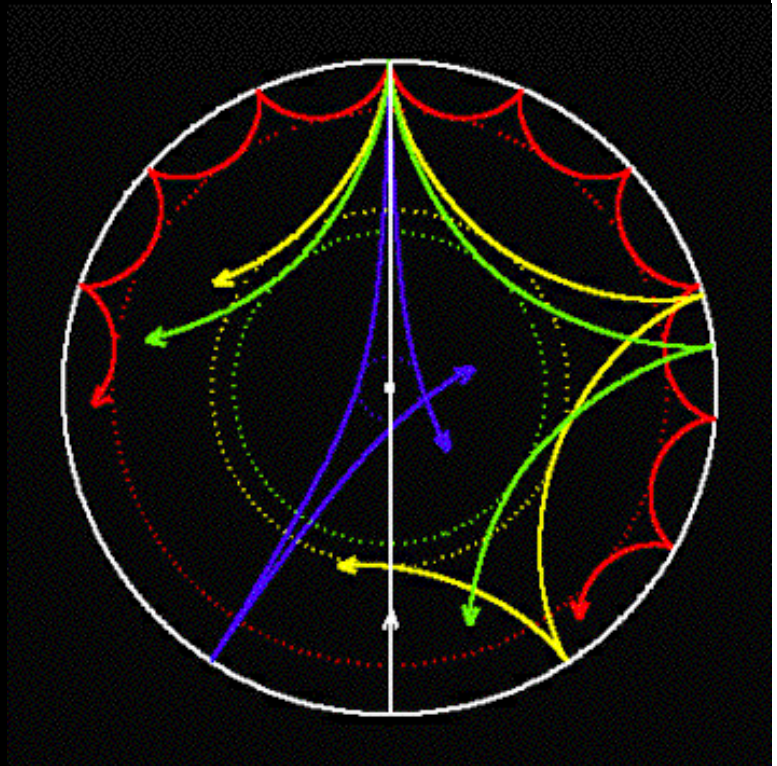

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Trapping of oscillations

A closer look at the two families of solutions

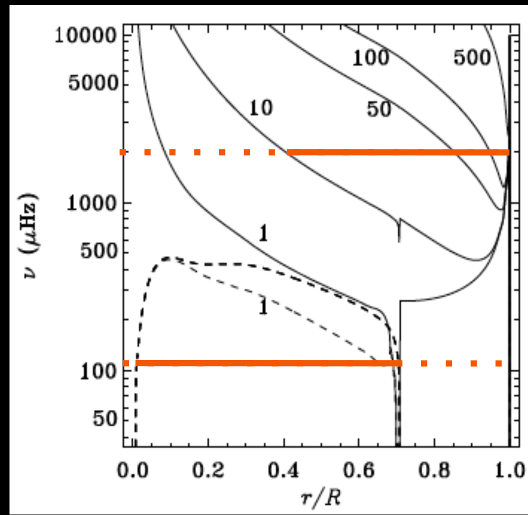
$$k_r^2 = \frac{1}{c_0^{\;2}}\left[S_l^2\left(\frac{N_0^2}{\omega^2}-1\right)+\omega^2-\omega_c^2\right]$$

- High frequency modes $\omega^2 >> N_0^2$

Near the surface

$$k_r^2 \approx \frac{\omega^2-\omega_c^2}{c_0^{\;2}}$$

Upper turning point



Trapping of oscillations

A closer look at the two families of solutions

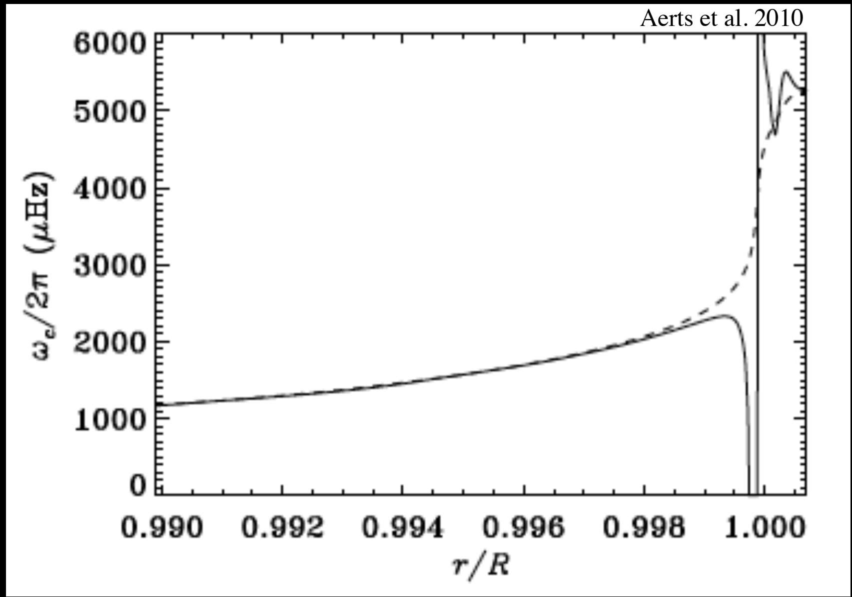
$$k_r^2 = \frac{1}{c_0^{\ 2}}\left[S_l^2\left(\frac{N_0^2}{\omega^2}-1\right)+\omega^2-\omega_c^2\right]$$

- High frequency modes $\omega^2 >> N_0^2$

Near the surface

$$k_r^2 \approx \frac{\omega^2-\omega_c^2}{c_0^{\ 2}}$$

Upper turning point $\omega^2 = \omega_c^2$



Trapping of oscillations

A closer look at the two families of solutions

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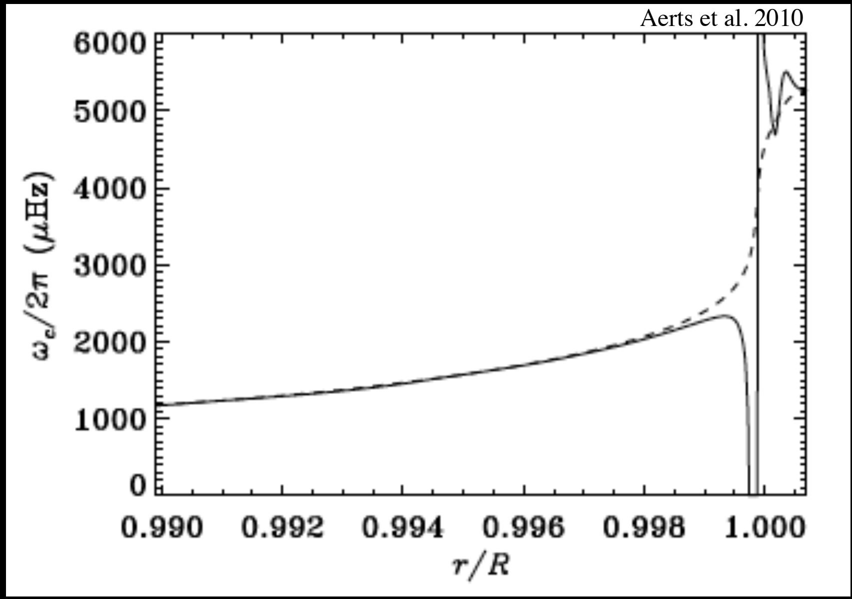
Near the surface

$$k_r^2 \approx \frac{\omega^2-\omega_c^2}{c_0^2}$$

Upper turning point $\omega^2 = \omega_c^2$

$$\omega \approx \frac{c_0}{2H}\left[1-2\frac{dH}{dr}\right]$$



Trapping of oscillations

A closer look at the two families of solutions

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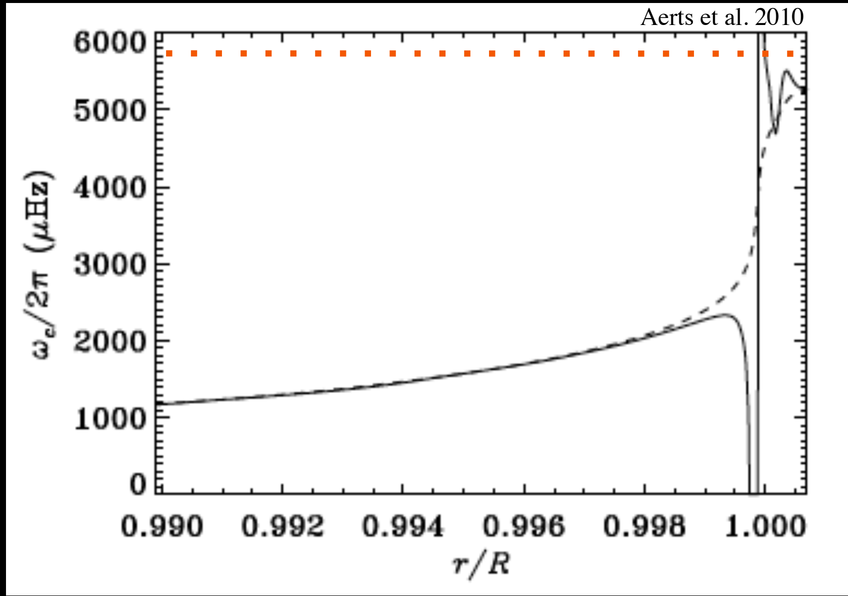

Trapping of modes occurs up to ~ 5.3 mHz in the sun
… but partial reflection occurs at even higher frequencies

Trapping of oscillations

A closer look at the two families of solutions

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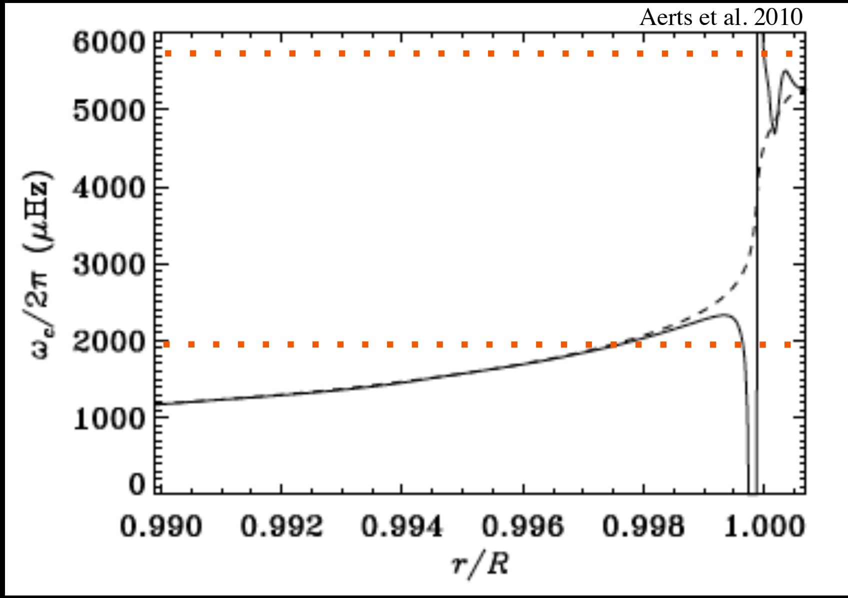

Trapping of modes occurs up to ~ 5.3 mHz in the sun
… but partial reflection occurs at even higher frequencies

Modes with frequencies lower than ~2 mHz in the sun are reflected below the photosphere
=> not so affected by the details of the outermost layers

Trapping of oscillations

A closer look at the two families of solutions

$$k_r^2 = \frac{1}{c_0^{\,2}}\left[S_l^2\left(\frac{N_0^2}{\omega^2} - 1\right) + \omega^2 - \omega_c^2\right]$$

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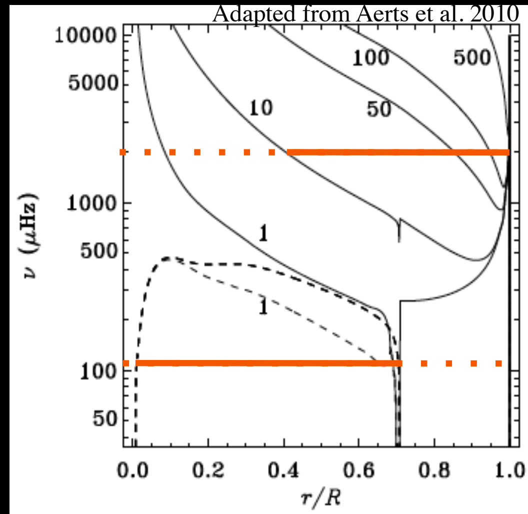

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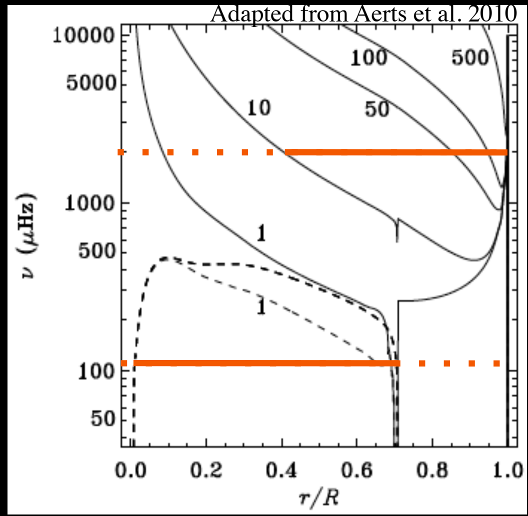

Trapping of oscillations

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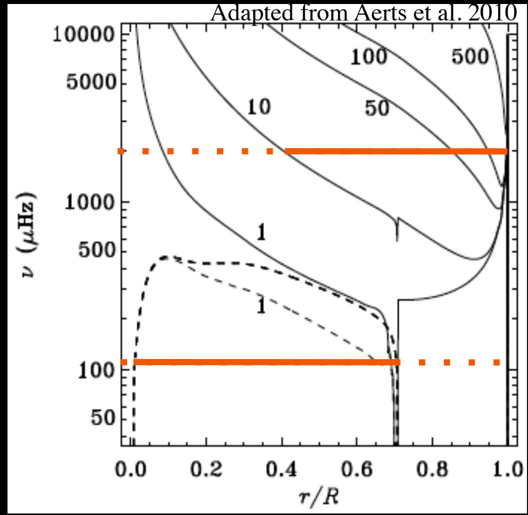
$k_h^{\,2}$



Trapping of oscillations

A closer look at the two families of solutions

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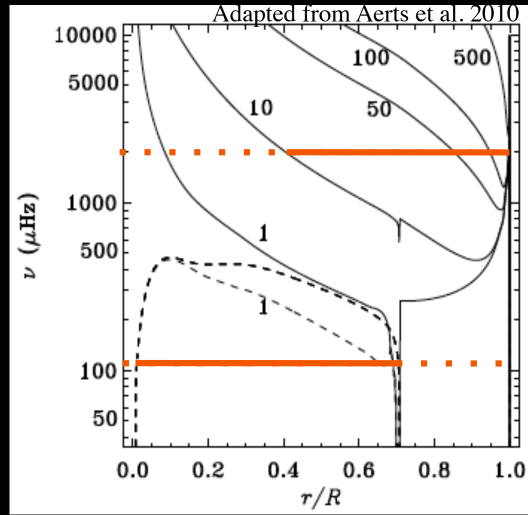
$k_h{}^2$

$$\omega^2 \approx \frac{N_0^2}{1 + \dfrac{k_r^2}{k_h^2}}$$



Dispersion relation for gravity wave.

Trapping of oscillations

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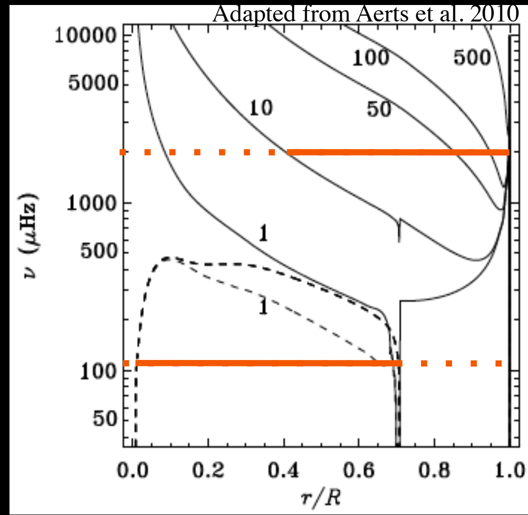
A closer look at the two families of solutions



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$$\omega^2 \approx \frac{N_0^2}{1+\dfrac{k_r^2}{k_h^2}}$$

Dispersion relation for gravity wave.

$\omega < N_0$

ω decreases as k_r increases

=> $|n|$ increases as frequency decreases

Trapping of oscillations

A closer look at the two families of solutions

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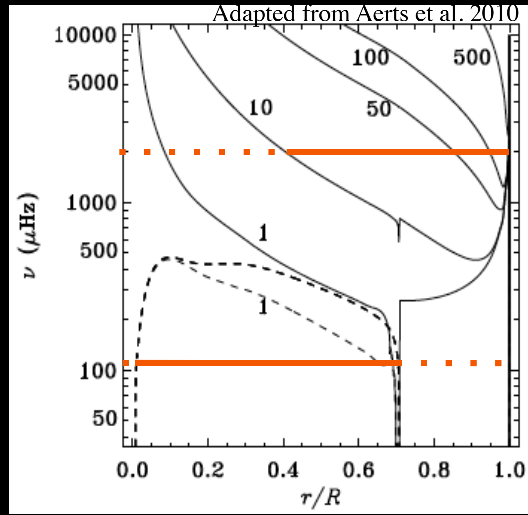

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$$\omega^2 \approx \frac{N_0^2}{1+\dfrac{k_r^2}{k_h^2}}$$

Dispersion relation for gravity wave.

Smaller k_r/k_h => Larger λ_r/λ_h => larger ω
=> larger frequencies for "needle-like" motion

The frequency of a gravity wave is always smaller that N_0

Trapping of oscillations

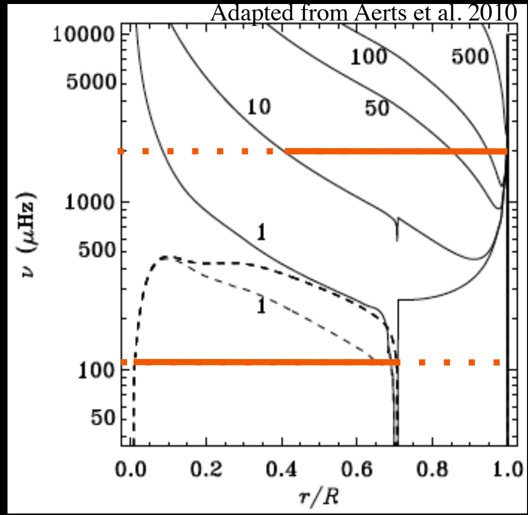
A closer look at the two families of solutions

$$k_r^2 = \frac{1}{c_0^{\ 2}}\left[S_l^2\left(\frac{N_0^2}{\omega^2}-1\right)+\omega^2-\omega_c^2\right]$$

- Low frequency modes $\omega^2 << S_l^2$

$$k_r^2 \approx \frac{S_l^2}{c_0^{\ 2}}\left[\frac{N_0^{\ 2}}{\omega^2}-1+\frac{\omega^2}{S_l^2}-\frac{\omega_c^2}{S_l^2}\right] \approx \frac{l(l+1)}{r^2}\left[N_0^{\ 2}-\omega^2\right]\frac{1}{\omega^2}$$

$k_h^{\ 2}$



Turning points

Trapping of oscillations

A closer look at the two families of solutions

$$k_r^2 = \frac{1}{c_0^{\,2}}\left[S_l^2\left(\frac{N_0^2}{\omega^2}-1\right)+\omega^2-\omega_c^2\right]$$

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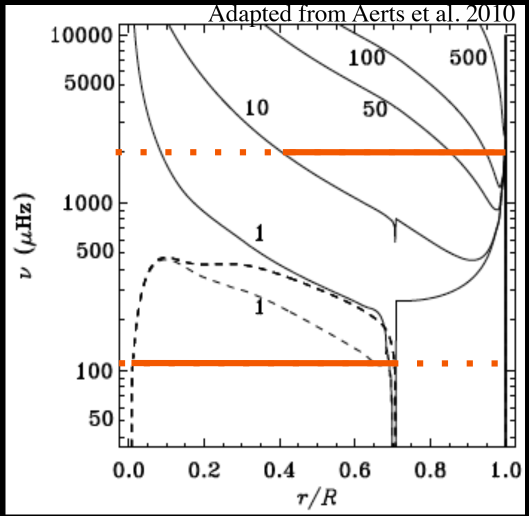
Turning points $\omega^2 = N_0^2$



Gravity waves propagate only in convectively stable regions!

Trapping of oscillations

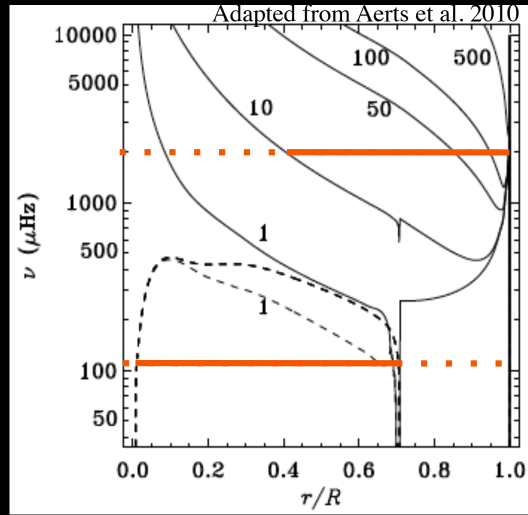
A closer look at the two families of solutions

$$k_r^2 = \frac{1}{c_0^{\,2}}\left[S_l^2\left(\frac{N_0^2}{\omega^2}-1\right)+\omega^2-\omega_c^2\right]$$

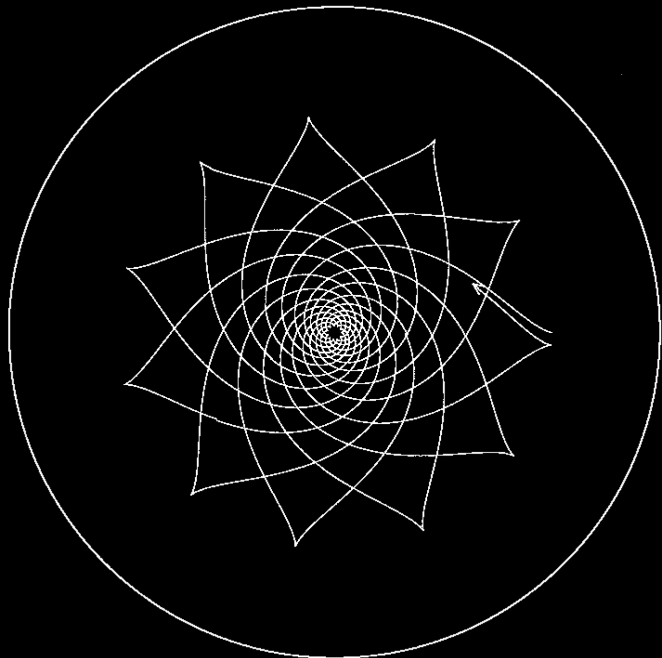
➢ Low frequency modes $\omega^2 << S_l^2$

$$k_r^2 \approx \frac{S_l^2}{c_0^{\,2}}\left[\frac{N_0^{\,2}}{\omega^2}-1+\frac{\omega^2}{S_l^2}-\frac{\omega_c^2}{S_l^2}\right]\approx\frac{l(l+1)}{r^2}\left[N_0^{\,2}-\omega^2\right]\frac{1}{\omega^2}$$

$k_h^{\,2}$



Turning points $\omega^2 = N_0^2$



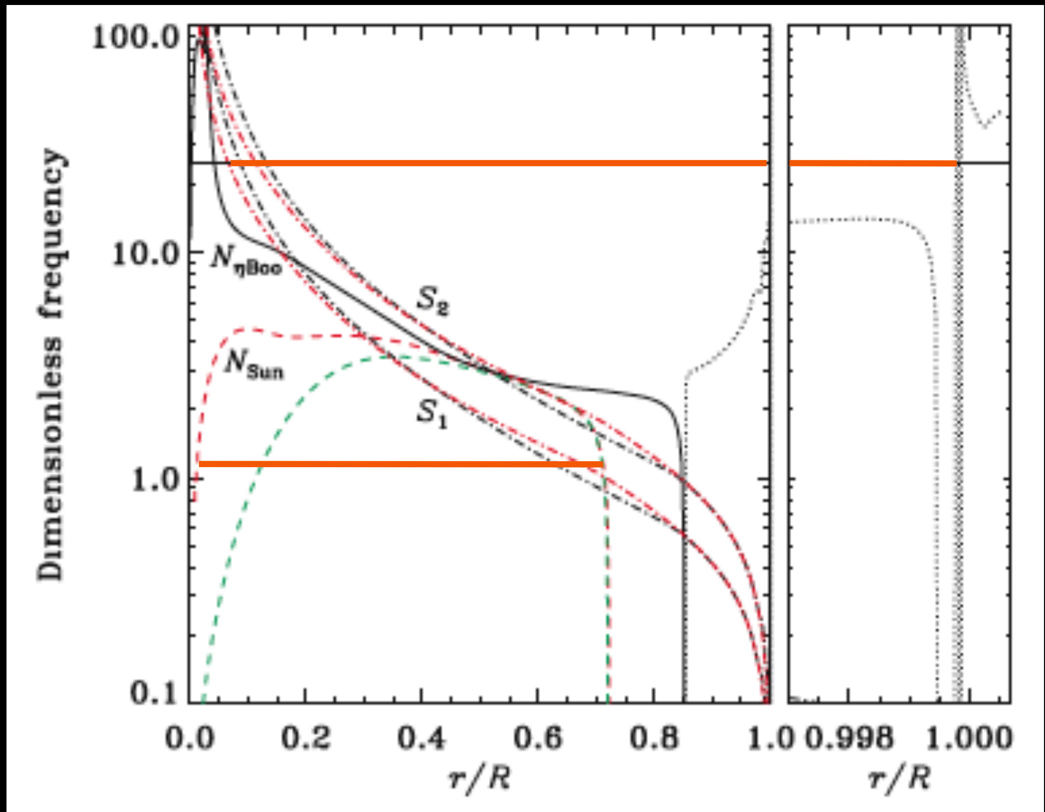
The case of an evolved star

Trapping of oscillations

The case of an evolved star

- Propagation diagram for the sun and a subgiant star

Cunha et al. 2007



Trapping of oscillations

The case of an evolved star

- Propagation diagram for the sun and a subgiant star

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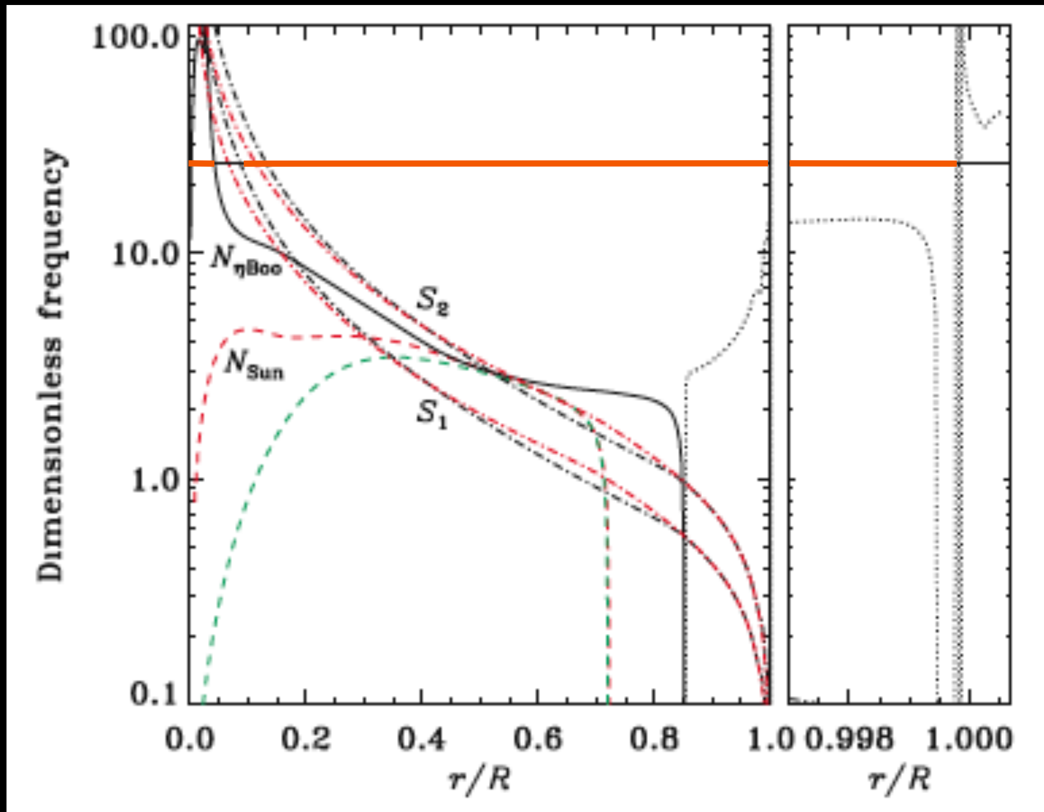


Acoustic and internal gravity waves

Acoustic and gravity waves

Summary

Acoustic waves

- Maintained by gradient of pressure fluctuation;
- Radial or non-radial;
- Propagate in convectively stable or non-stable regions

Internal gravity waves

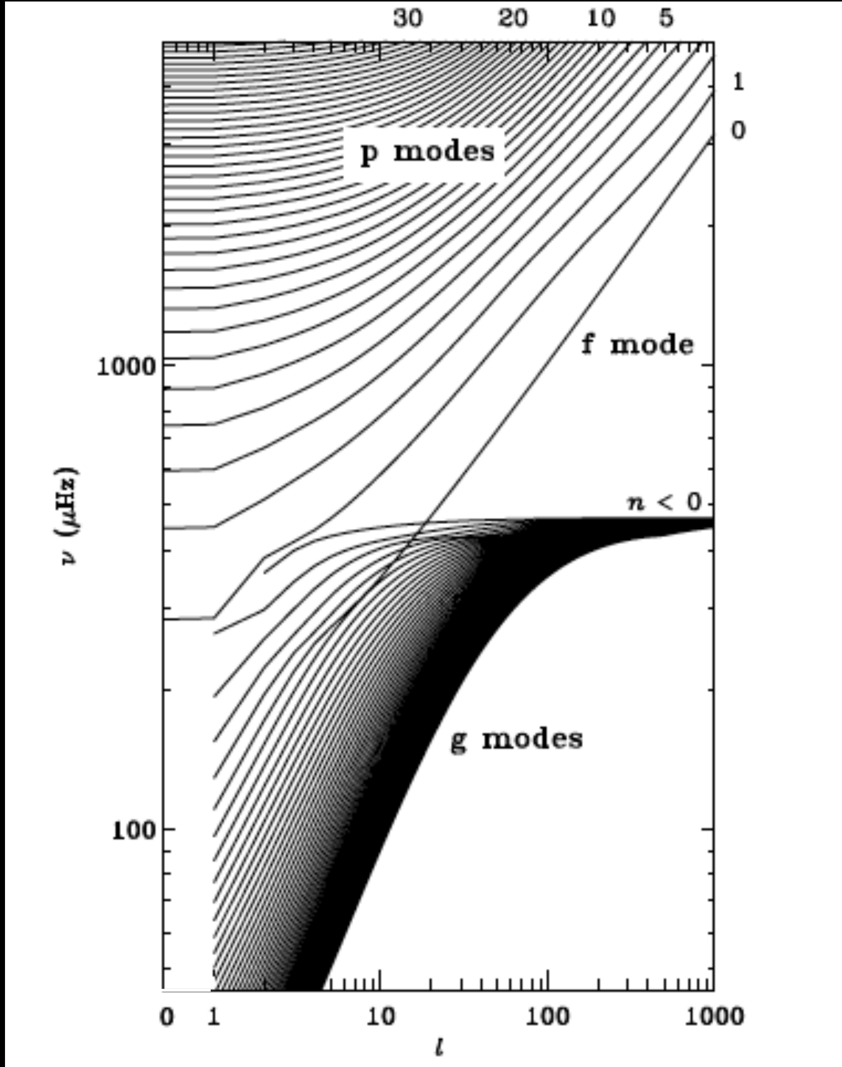
- Maintained by gravity acting on density fluctuation;
- Always non-radial;
- Propagate in convectively stable regions only

Numerical solutions

Numerical results

Eigenfrequencies

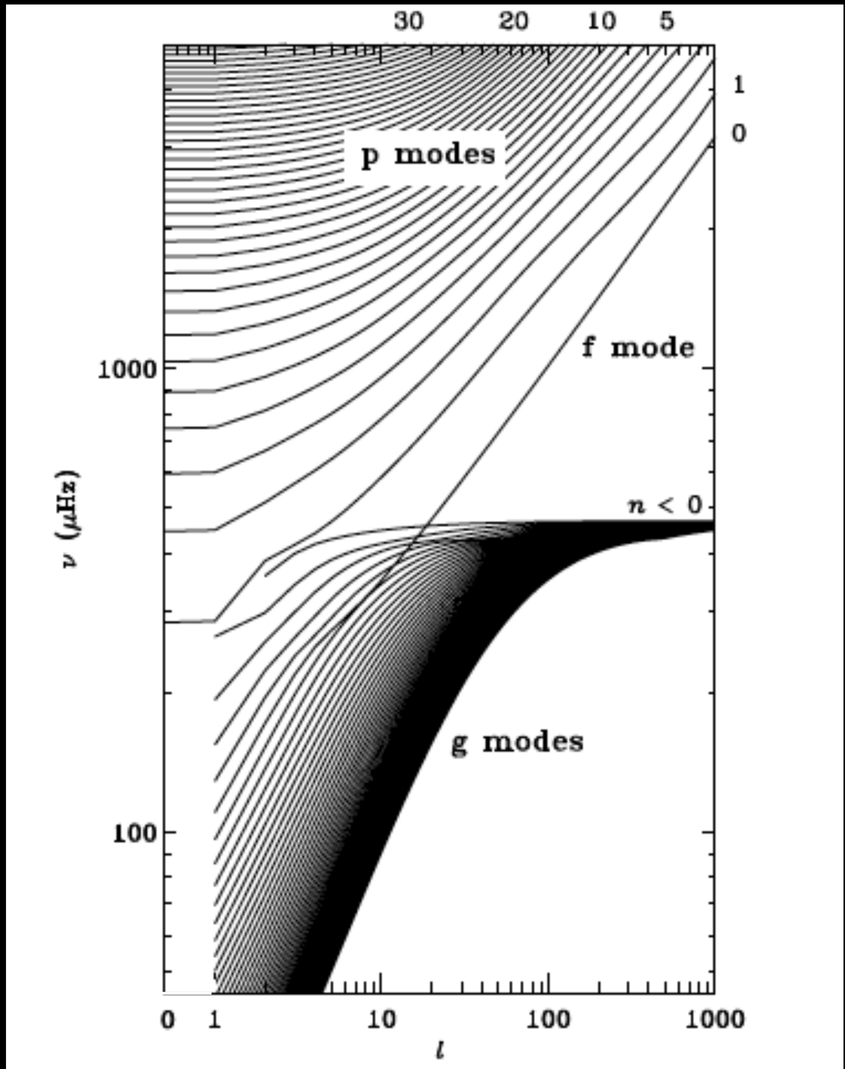
Aerts et al. 2010

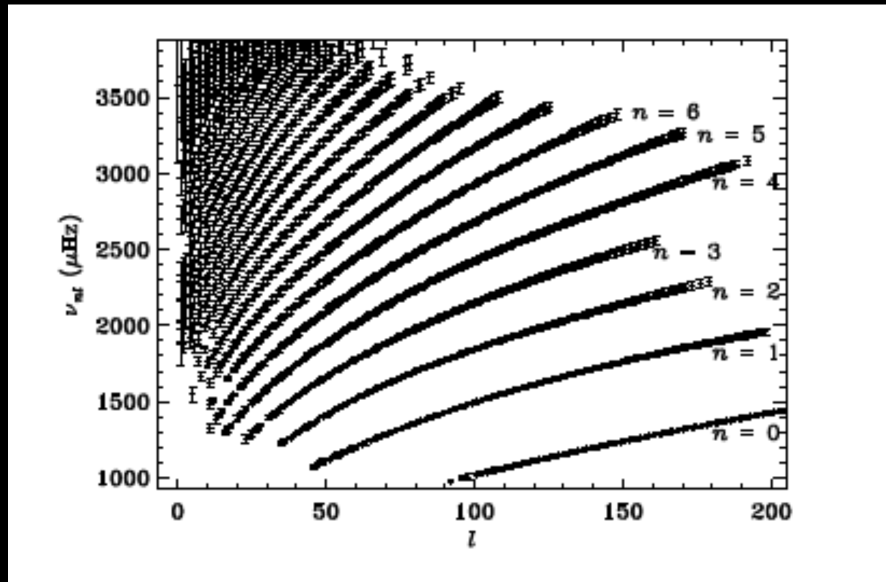


Numerical results

Eigenfrequencies

Aerts et al. 2010



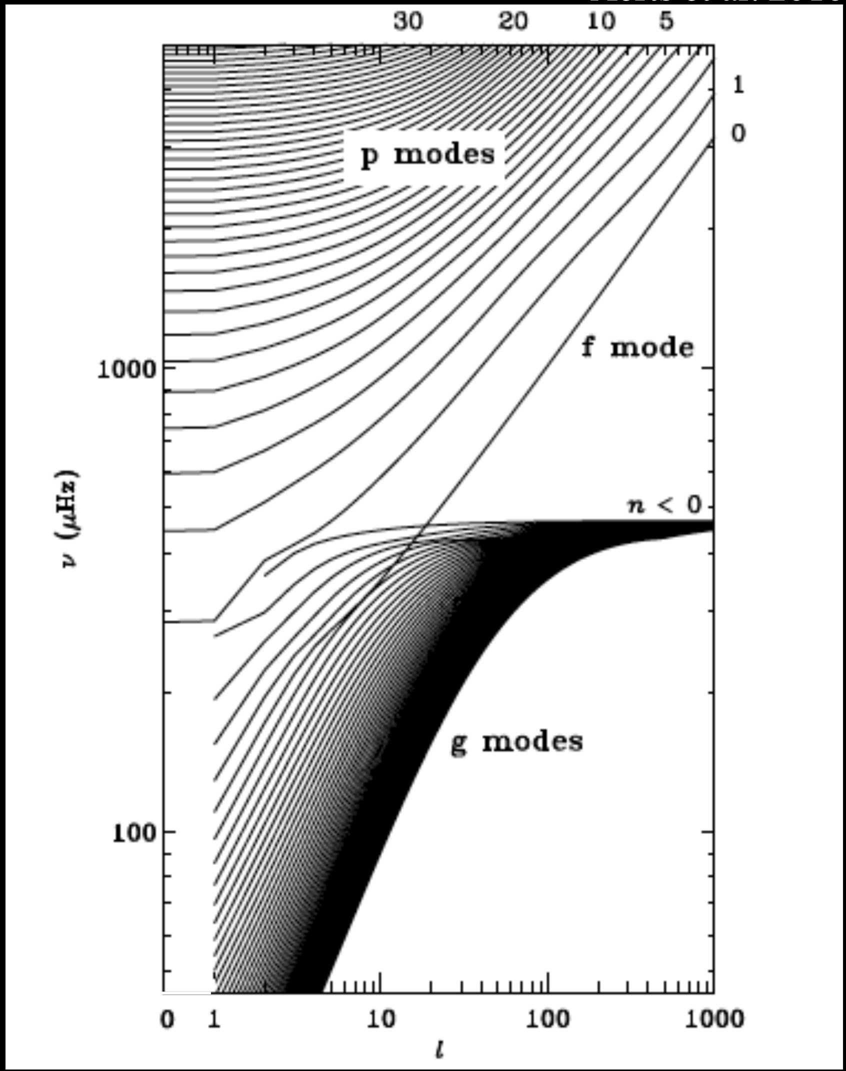


MDI observations

Numerical results

Eigenfrequencies

Aerts et al. 2010



Acoustic modes: $n > 0$

Gravity modes: $n < 0$

Numerical results

Eigenfrequencies

Remember

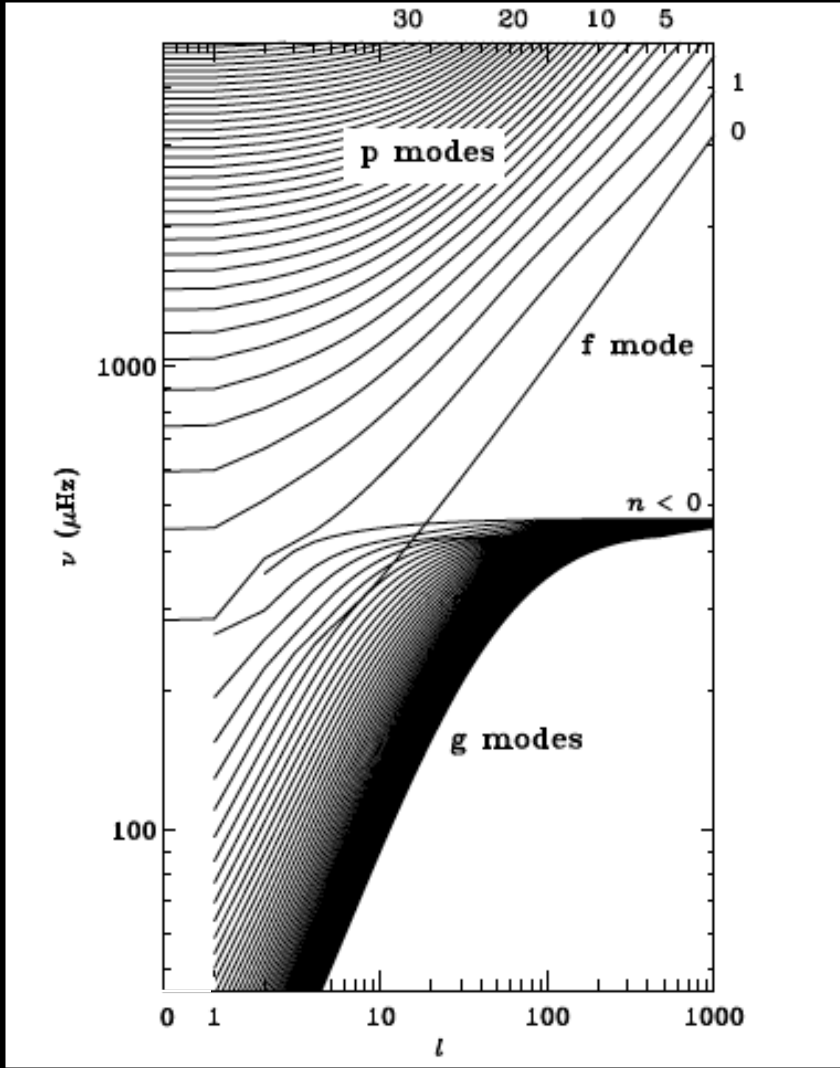
Acoustic waves

$$\omega \approx c_0 k$$

$$\omega^2 \approx \frac{N_0^2}{1 + \frac{k_r^2}{k_h^2}}$$

Gravity waves

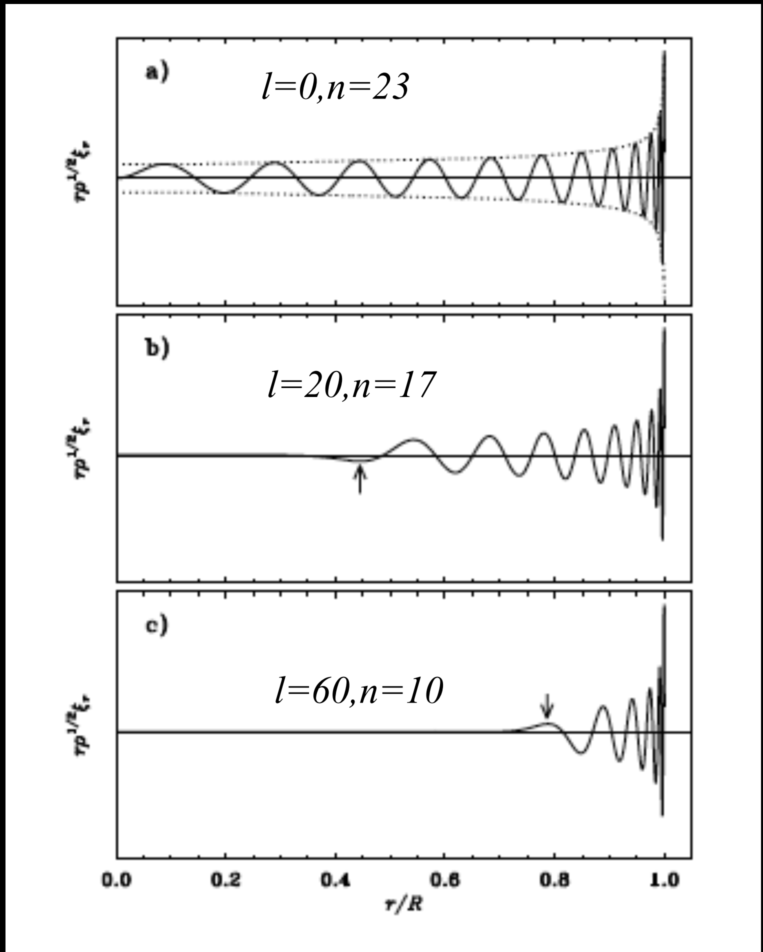
Aerts et al. 2010

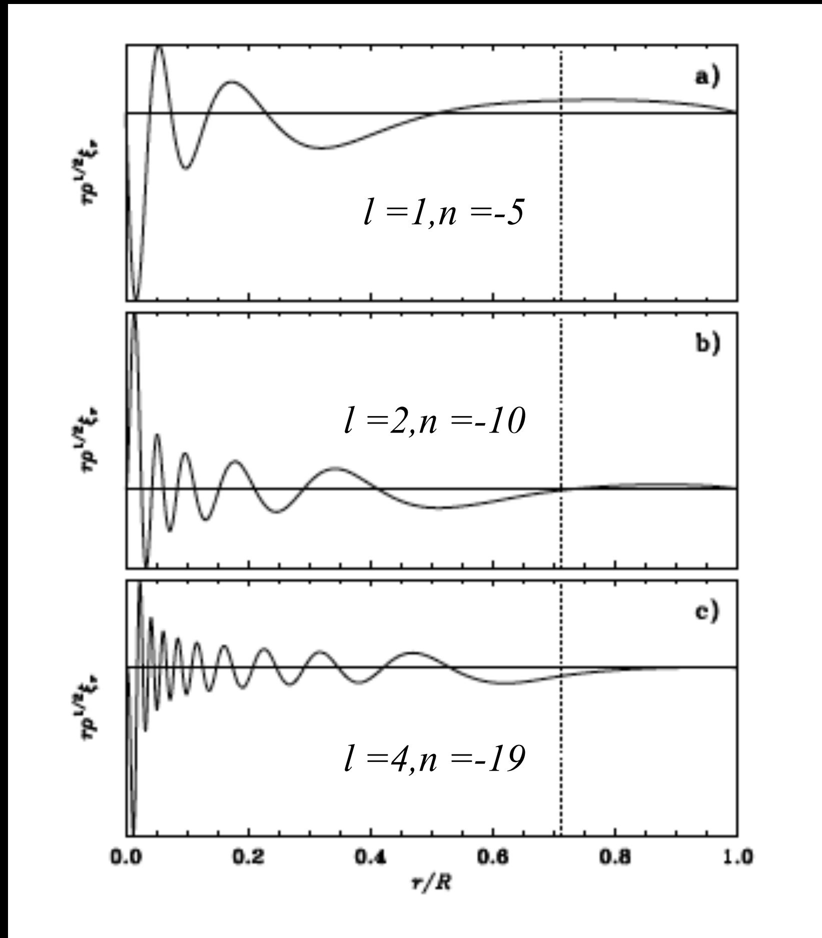


Numerical results

Eigenfunctions

Aerts et al. 2010

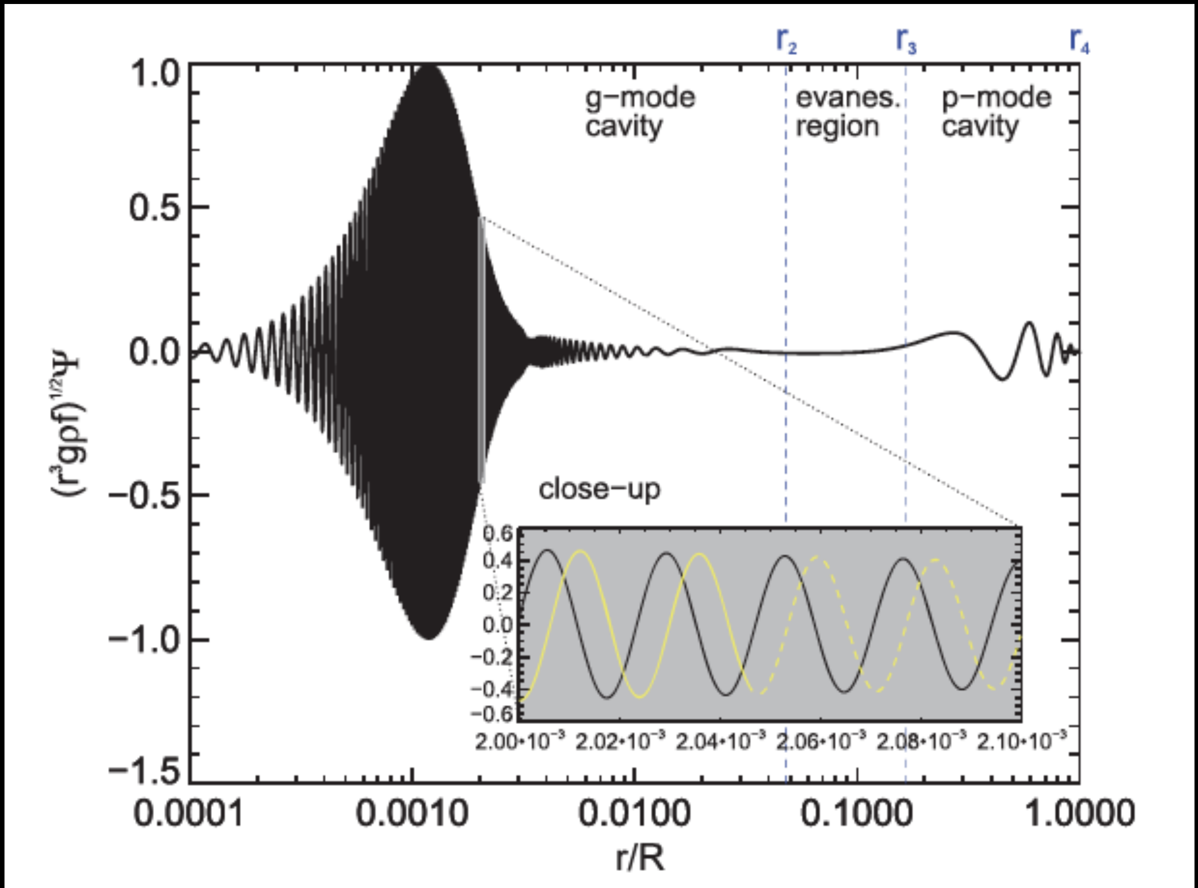




Numerical results

Eigenfunctions

Cunha et al. 2015



A number of important things that were left out

- The actual asymptotic analysis:

  => analytical solutions for the eigenfunctions
  and eigenfrequencies

- Frequency combinations (large separation, small separations, ratios, etc)

- Inference methodologies (forward modelling, inverse modelling, glitches, etc)

- Deviations from spherical symmetry (rotation, magnetic effects, application of the variational principle)

- Mode excitation (stochastic, coherent)

- etc…

Asymptotic analysis

Linear, adiabatic oscillations in the Cowling approximation.

High *n*, low *l*, acoustic oscillations:

$$\nu_{nl} \approx \left(n + \frac{l}{2} + \alpha \right) \Delta\nu_0 + \textit{higher order terms}$$

where $$\Delta\nu_0 = \left(2\int_0^R \frac{dr}{c} \right)^{-1}$$

- $\Delta\nu_0$ prop $(M/R^3)^{1/2}$
- α function of ν and is due to surface effects
- Note: $\nu=\omega/2\pi$

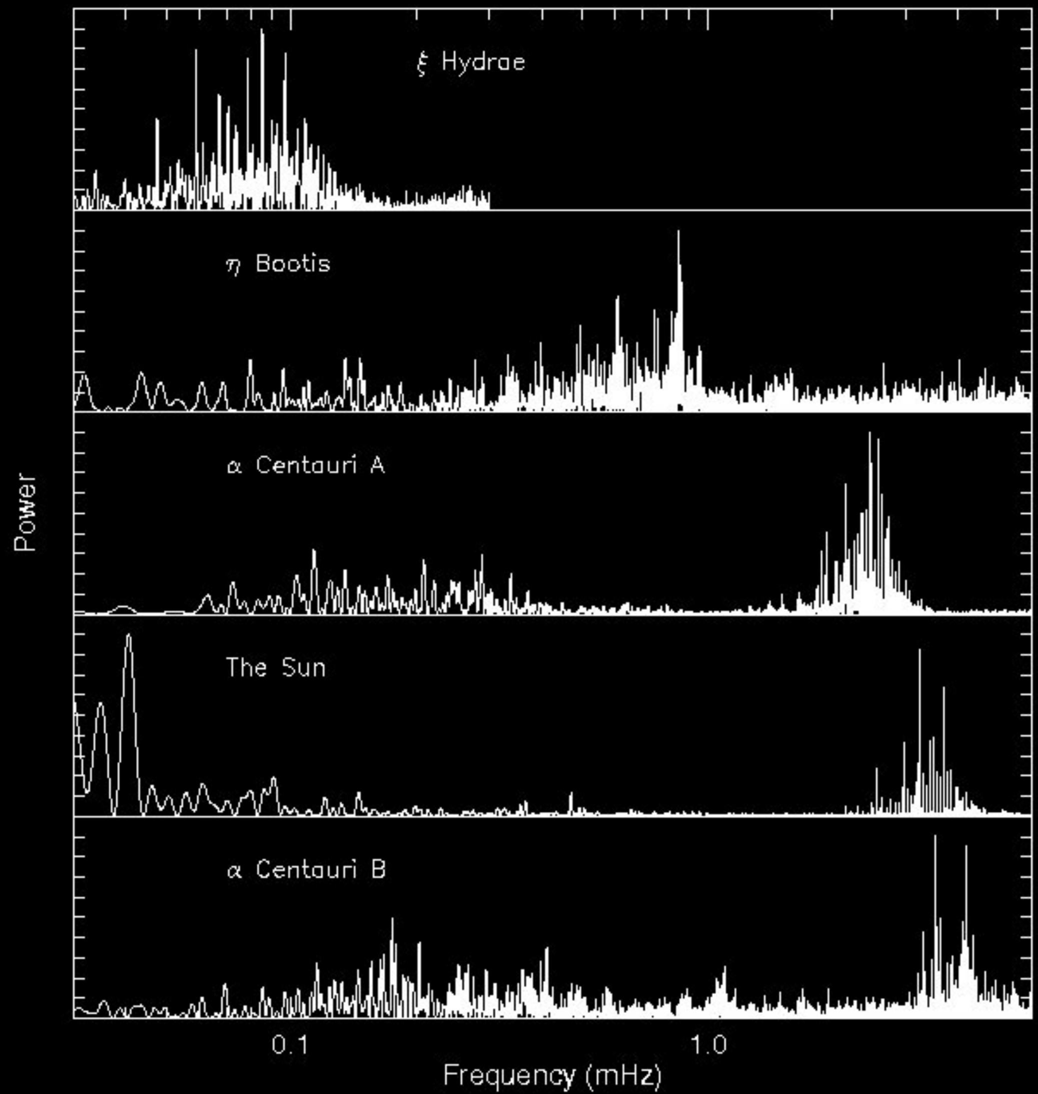
Asymptotic analysis

Adiabatic oscillations in the Cowling approximation.

High *n*, low *l*, acoustic oscillations:

$$\nu_{nl} \approx \left(n + \frac{l}{2} + \alpha\right)\Delta\nu_0 + \ldots$$

$\Delta\nu_0$ prop $(M/R^3)^{1/2}$



Asymptotic analysis

Large separations $\Delta\nu_{nl}$

$$\nu_{nl} \approx \left(n + \frac{l}{2} + \alpha \right) \Delta\nu_0 + higher\ order\ terms$$

Asymptotic analysis

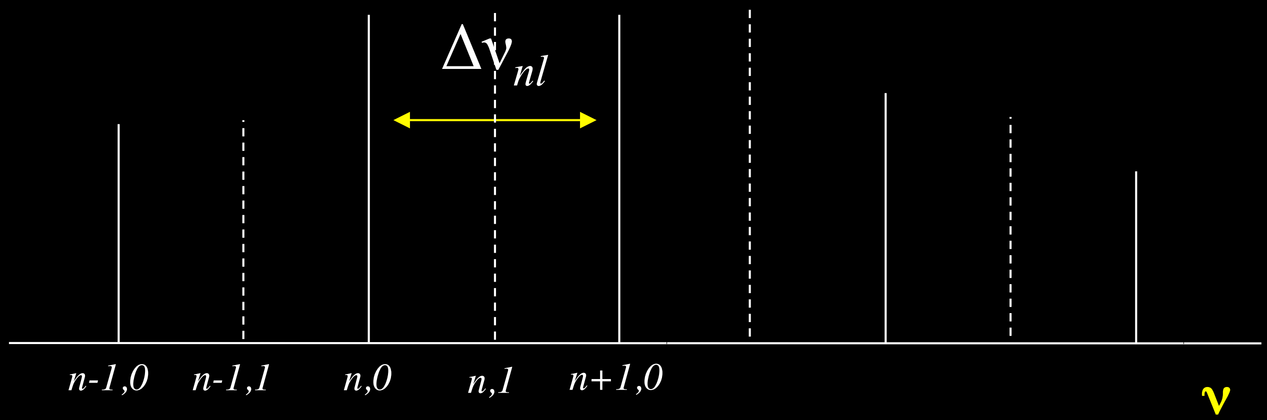
Large separations $\Delta\nu_{nl}$

$$\nu_{nl} \approx \left(n + \frac{l}{2} + \alpha \right) \Delta\nu_0 + higher\ order\ terms$$

$$\Delta\nu_{nl} = \nu_{n+1,l} - \nu_{n,l} \approx \Delta\nu_0$$

$\alpha\ (M/R^3)^{1/2}$

Schematic
Power
Spectrum



Asymptotic analysis

Adiabatic oscillations in the Cowling approximation.

High n, low l, acoustic oscillations:

$$\nu_{nl} \approx \left(n + \frac{l}{2} + \alpha \right) \Delta\nu_0 - \left[Al(l+1) - \delta \right] \frac{\Delta\nu_0}{\nu_{nl}} + \ldots$$

where $$A = \frac{1}{4\pi^2 \Delta\nu_0} \left[\frac{c(R)}{R} - \int_0^R \frac{dc}{r} \right]$$

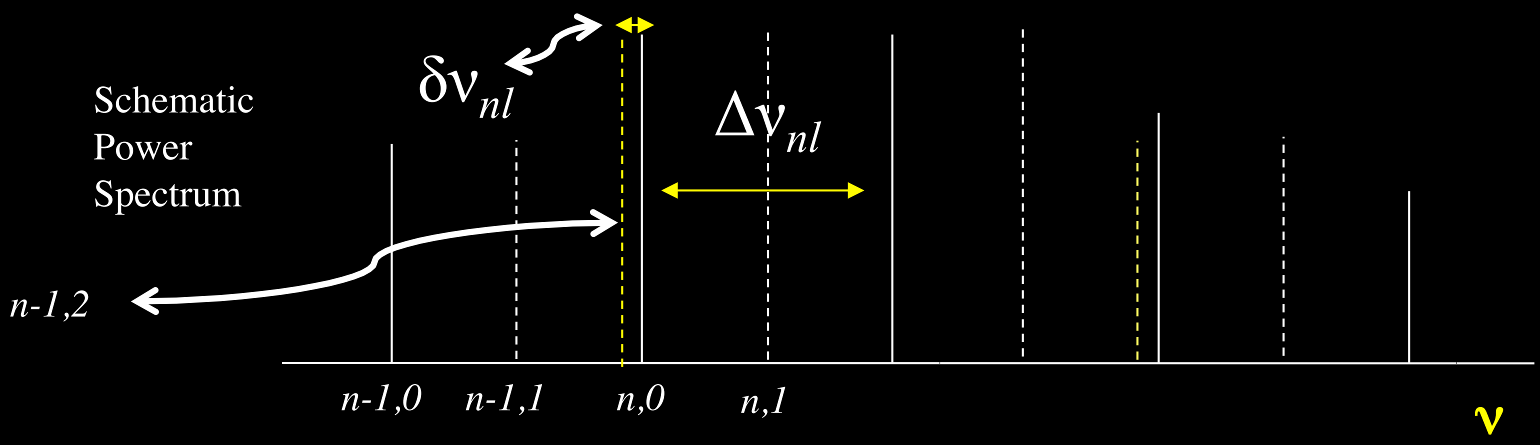
Asymptotic analysis

small separations $\delta\nu_{nl}$

$$\nu_{nl} \approx \left(n + \frac{l}{2} + \alpha\right)\Delta\nu_0 - \left[Al(l+1) - \delta\right]\frac{\Delta\nu_0}{\nu_{nl}} + \ldots$$

where $$A = \frac{1}{4\pi^2\Delta\nu_0}\left[\frac{c(R)}{R} - \int_0^R \frac{dc}{r}\right]$$

$$\delta\nu_{nl} = \nu_{n,l} - \nu_{n-1,l+2} \approx -(4l+6)\frac{\Delta\nu_0}{4\pi^2\nu_{n,l}}\int_0^R \frac{dc}{r}$$



Asymptotic analysis

Sun as a star