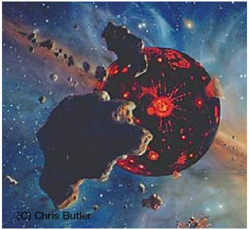
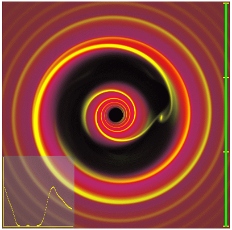
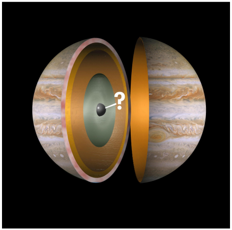
Lecture 4: Formation, migration and observations of giant planets



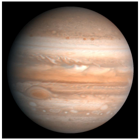
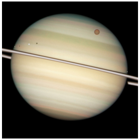
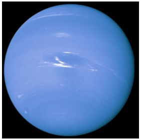



"Planet formation"

April 2016

Bertram Bitsch (Lund Observatory)

Composition of giant planets

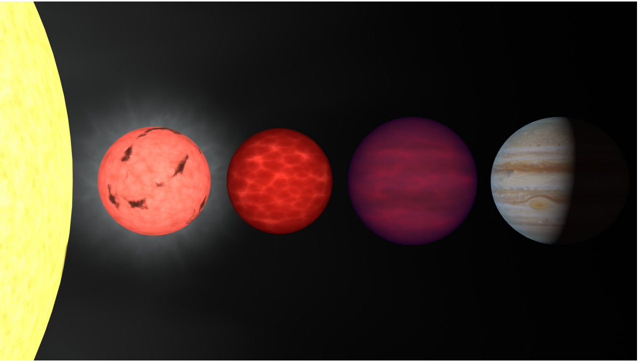
   

r/AU	5.2	9.6	19.2	30.1
R/km	69,911	58,232	25,362	24,622
$\rho/(\mathrm{g/cm^3})$	1.33	0.70	1.30	1.76
$M/M_\oplus$	318	95.2	14.5	17.1
$M_Z/M_\oplus$	$\sim$30	$\sim$24	$\sim$13	$\sim$15

$\Rightarrow$ All four giant planets are enriched in heavy elements (present in solid form in the core and in gaseous form in the atmosphere)

$\Rightarrow$ Total mass in heavy elements rather similar

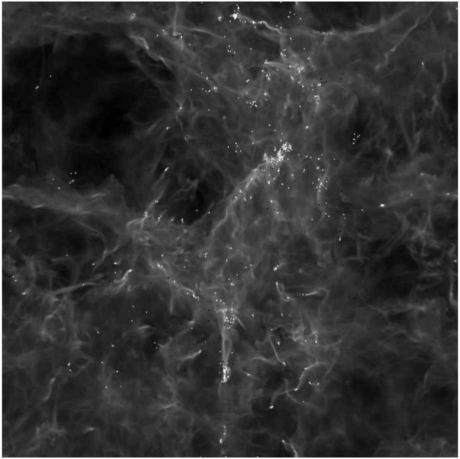
Brown dwarfs



- Brown dwarfs are low-mass objects which form like stars (gravitational fragmentation of molecular cloud)
- Too low mass to initiate H fusion
- Masses less than 75 $M_{\rm J}$
- Form with surface temperature ~3000 K, but cool quickly due to lack of internal energy source

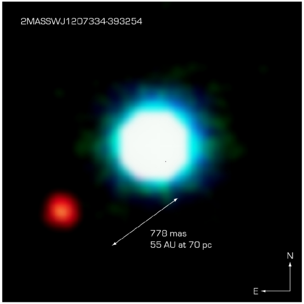
Lower mass limit of brown dwarfs

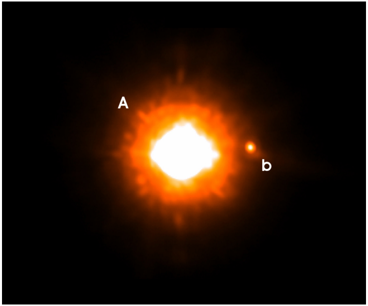
- *Jeans mass* of gravitationally unstable molecular cloud:
  $M_{\rm Jeans} \sim c_{\rm s}^3/(G^{3/2}\rho^{1/2})$
- Jeans mass decreases as clump contracts isothermally
  $\Rightarrow$ further fragmentation
- At high densities the contraction changes from *isothermal* to *adiabatic*
  $\Rightarrow$ no more fragmentation
- Minimum mass of fragments forming in gravitational instability of molecular cloud: 3–7 $M_{\rm J}$
  (Low & Lynden-Bell 1976)



(Padoan & Nordlund 2004)

2M1207 and GQ Lup





Two of the lowest-mass brown dwarf companions are:

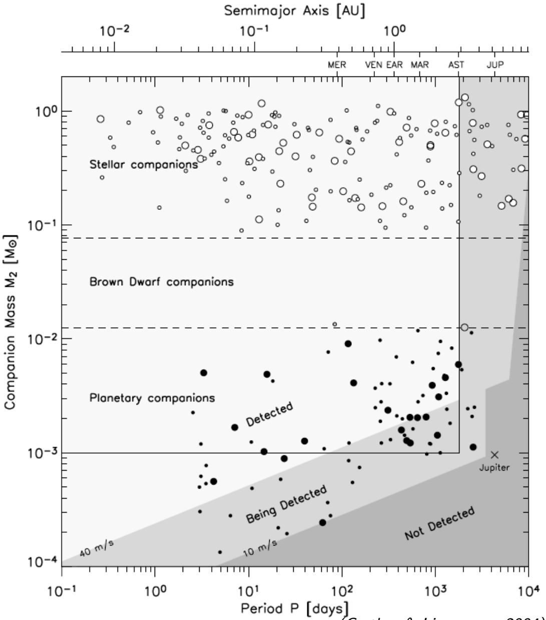
- 2M1207: $M = 3 \ldots 10\ M_{\rm J}$, $r = 40$ AU
- GQ Lup: $M = 1 \ldots 36\ M_{\rm J}$, $r = 103$ AU

$\Rightarrow$ Did Jupiter form like a brown dwarf?

Brown dwarf desert

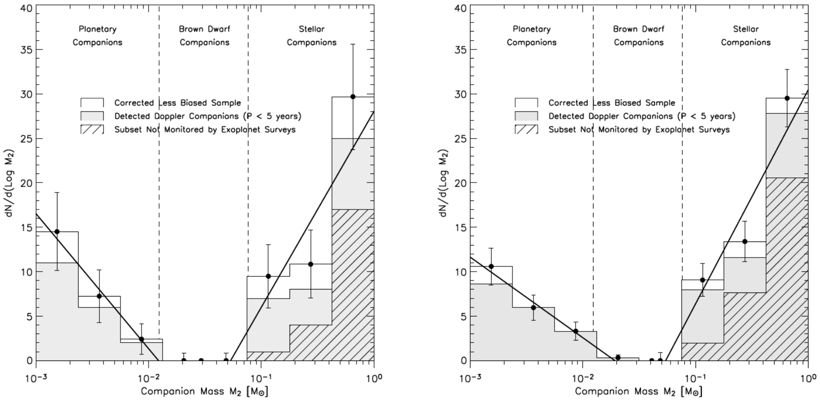
- Plot shows all detected close companions to sun-like stars within 25 pc (big dots) and from 25–50 pc (small dots)
- Large-mass companions detected visually
- Low-mass companions detected by doppler-shifted spectral lines
- Complete down to $M \approx 1\ M_{\rm J}$ $(10^{-3}\ M_{\odot})$ and up to $r \approx 3$ AU

⇒ **Clear lack of brown dwarf companions**



(Grether & Lineweaver 2004)

Brown dwarfs and planets

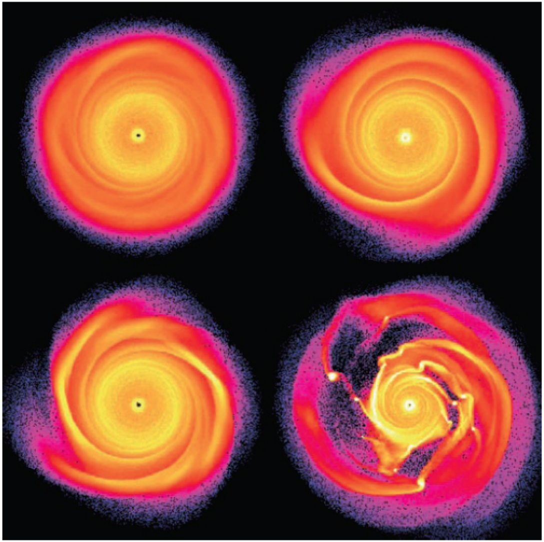


- Mass function shows two clearly distinguishable populations: *stellar companions* which become more rare with decreasing mass and *planetary companions* which become more frequent with decreasing mass
- Very few companions with brown-dwarf masses (*brown dwarf desert*)

⇒ **Giant planets and brown dwarfs do not form in the same way**

Gravitationally unstable protoplanetary discs

Massive protoplanetary discs are *gravitationally unstable*:



Al Cameron and Alan Boss

- Al Cameron (1925-2005)
- First proponent of formation of gas giants by gravitational instabilities in a protoplanetary disc

- Alan Boss
- Pioneered thorough numerical studies showing disc instabilities as a way to form gas giants rapidly

Disc instability

- Protoplanetary discs are gravitationally unstable if the Toomre parameter (or Safronov parameter)

$$Q = \frac{\Omega c_\mathrm{s}}{\pi G \Sigma}$$

  is less than one

- Rotation (Ω) and pressure (c_s) are stabilising forces, while gravity ($G\Sigma$) is destabilising

- Toomre parameter in MMSN:

$$Q = 55.8 \left(\frac{r}{\mathrm{AU}}\right)^{-1/4} \left(\frac{M_\star}{M_\odot}\right)^{1/2}$$

- MMSN has $\Sigma = 150\,\mathrm{g\,cm^{-2}}$ at $r = 5\,\mathrm{AU}$ – would need $\Sigma \approx 5000\,\mathrm{g\,cm^{-2}}$ to be gravitationally unstable

Disc mass versus planet mass

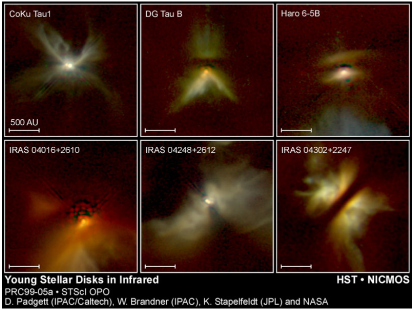
- The criterion $Q = \frac{\Omega c_\mathrm{s}}{\pi G \Sigma} = 1$ sets a minimum mass for a disc to be gravitationally unstable:

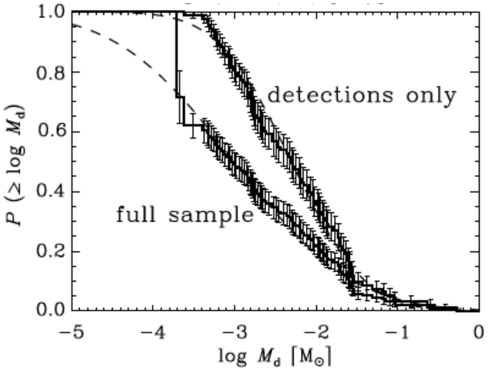
$$
\begin{aligned}
M_\mathrm{disc} &\sim \pi (2r)^2 \Sigma \\
&= 4\pi r^2 \frac{\Omega c_\mathrm{s}}{\pi G} = 4r^2 \frac{\Omega^2 H}{G} \\
&= 4r^2 \frac{G M_\star H}{r^3 G} = 4 M_\star \frac{H}{r} \\
&\Rightarrow M_\mathrm{disc}/M_\star \sim 4 \frac{H}{r}
\end{aligned}
$$

$\Rightarrow$ **Need discs that contain at least 20% of the mass of the star for gravitational instability**

Disc masses in Taurus-Auriga

- *Taurus-Auriga complex* is one of the nearest active star forming regions ($d = 140$ pc, $M \sim 3.5 \times 10^4\ M_\odot$)
- Andrews & Williams (2005) monitored 153 young stars for dust emission and found significant dust discs around 93 of them





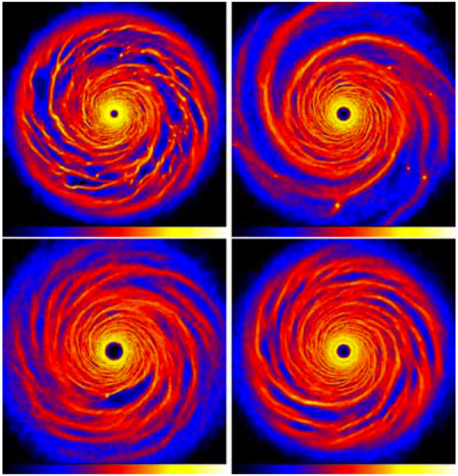
- Minimum mass solar nebula has $M_{\rm disc} = 0.013\ M_\odot$

$\Rightarrow$ Self-gravitating discs are extremely rare

Evolution of gravitationally unstable discs

$\Omega t_{\rm cool} = 3$ $\Omega t_{\rm cool} = 5$

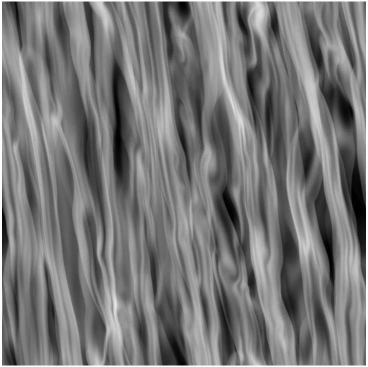
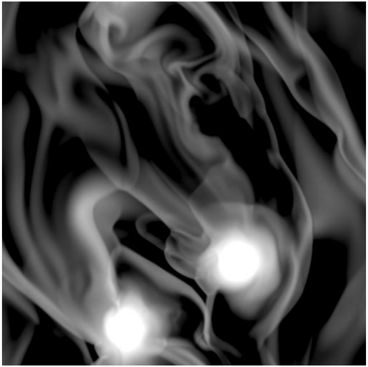
$\Omega t_{\rm cool} = 6$ $\Omega t_{\rm cool} = 8$



(Rice et al. 2005)

- A disc with $Q < 1$ develops first spiral arms
- Spiral arms heat up by compression and shock dissipation
- Arms fragment into gravitationally bound gas "blobs" if cooling is fast enough

Cooling criterion

- Gammie (2001) found:
  - $t_{\rm cool} > 3\Omega^{-1}$: spiral arms heat up and do not fragment
  - $t_{\rm cool} < 3\Omega^{-1}$: spiral arms fragment into Jupiter-mass gravitationally bound blobs

$\Rightarrow$ We need to satisfy both *Toomre criterion* (disc mass) and *Gammie criterion* (cooling time) to form gas giants by gravitational instability

Cooling in discs

- The cooling time is given by the thermal energy per unit surface $c_p T \Sigma$ divided by the cooling per unit surface $\sigma_{\rm SB} T_{\rm eff}^4$

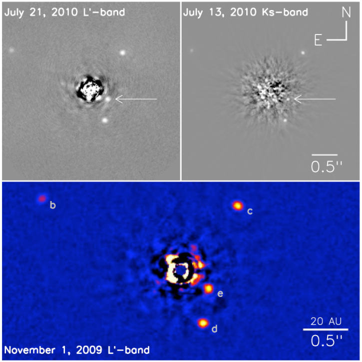
$$t_{\rm cool} \sim \frac{c_p T \Sigma}{\sigma_{\rm SB} T_{\rm eff}^4}$$

- Counterintuitively the cooling time decreases when the temperature increases

- Fragmentation possible beyond 40 AU where disc is optically thin (Rafikov 2005)

- Results in clump masses $M_{\rm clump} \sim 10\ M_{\rm J}$

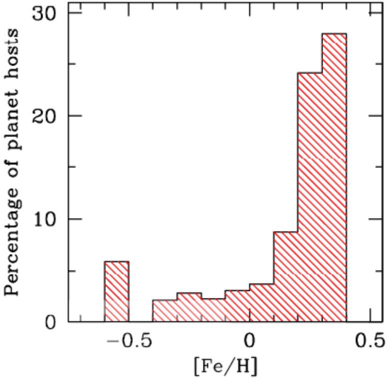
HR 8799 system



- Planets orbiting HR8799 are one of the first directly imaged planetary systems (Marois et al. 2008, 2010)
- Four planets orbiting at ≈14.5, ≈24, ≈38, ≈68 AU and all have masses $\approx 5 - 7 M_{\rm J}$

$\Rightarrow$ Prime candidates for planets formed by gravitational instability

Planets and metallicity



- Fraction of stars that host giant exoplanets is a steeply rising function of metallicity (Santos et al. 2005; Fischer & Valenti 2005)

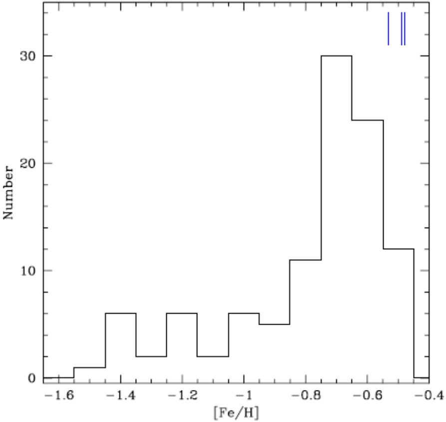
⇒ Natural correlation if gas giants form by *core accretion*

- The ability of protoplanetary discs to form gas giants by gravitational instability has not been shown to benefit from higher disc metallicity

Planets around low-metallicity stars

Santos et al. (2010) targeted $\sim$100 metal poor stars for planets

(Sozzetti et al. 2009: no planets around 160 metal poor stars with -2.0<[Fe/H]<-0.6)

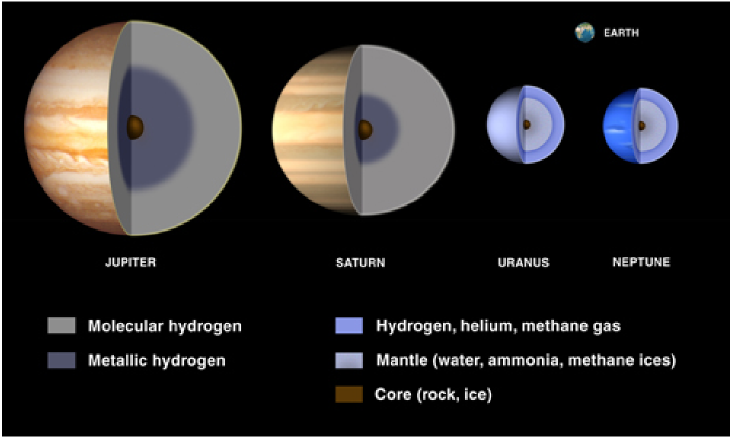


$\Rightarrow$ Three planets found

$\Rightarrow$ All three planets orbit the most metal rich stars of the sample

$\Rightarrow$ This is a spectacular confirmation that metallicity matters even for protoplanetary discs of intrinsically low metallicity

Core accretion



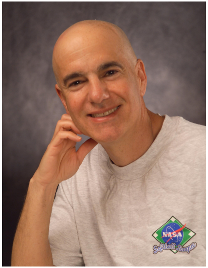
1. Planetesimals form from dust grains
2. Core of $\sim 10 M_{\oplus}$ builds by run-away accretion of planetesimals
3. Collapse of several hundred $M_{\oplus}$ protoplanetary disc gas onto the core

James B. Pollack and Jack Lissauer

Two of the scientists who developed the *core accretion model*:

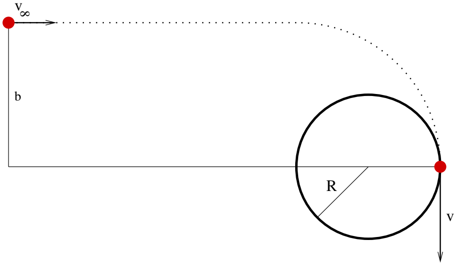
(we should not forget Safronov, Hayashi, Mizuno, and many others!)

- James B. Pollack (1939-1994)
- NASA Ames Research Center



- Jack J. Lissauer
- NASA Ames Research Center

Run-away accretion of planetesimals



- The radius of the core grows as

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \sqrt{\frac{3}{\pi}} \frac{\Sigma_{\mathrm{s}} \Omega}{4\rho_{\bullet}} (1 + 2\theta_{\mathrm{S}})$$

- Using MMSN column densities of rock and ice yields

$$\begin{aligned} \frac{\mathrm{d}R}{\mathrm{d}t} &\approx 5.5\,\mathrm{cm\,yr^{-1}} \left(\frac{r}{\mathrm{AU}}\right)^{-3} (1 + 2\theta_{\mathrm{s}}) \quad \text{for} \quad 0.27 < r < 2.7 \\ \frac{\mathrm{d}R}{\mathrm{d}t} &\approx 23.1\,\mathrm{cm\,yr^{-1}} \left(\frac{r}{\mathrm{AU}}\right)^{-3} (1 + 2\theta_{\mathrm{s}}) \quad \text{for} \quad 2.7 < r < 36 \end{aligned}$$

Formation time-scales

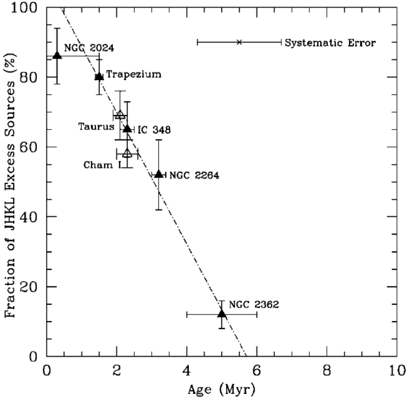
$$
\begin{aligned}
\frac{\mathrm{d}R}{\mathrm{d}t} &\approx 5.5\,\mathrm{cm\,yr^{-1}}\left(\frac{r}{\mathrm{AU}}\right)^{-3}(1+2\theta_\mathrm{s}) \quad \text{for} \quad 0.27 < r < 2.7 \\
\frac{\mathrm{d}R}{\mathrm{d}t} &\approx 23.1\,\mathrm{cm\,yr^{-1}}\left(\frac{r}{\mathrm{AU}}\right)^{-3}(1+2\theta_\mathrm{s}) \quad \text{for} \quad 2.7 < r < 36
\end{aligned}
$$

- Time-scale to build Earth at 1 AU:
$$t_\oplus \approx 162\,\mathrm{Myr}\left(\frac{r}{\mathrm{AU}}\right)^{3}(1+2\theta_\mathrm{S})^{-1}$$
- Time-scale to build 10-Earth-mass core at 5 AU:
$$t_\mathrm{core} \approx 10400\,\mathrm{Myr}\left(\frac{r}{5\,\mathrm{AU}}\right)^{3}(1+2\theta_\mathrm{S})^{-1}$$

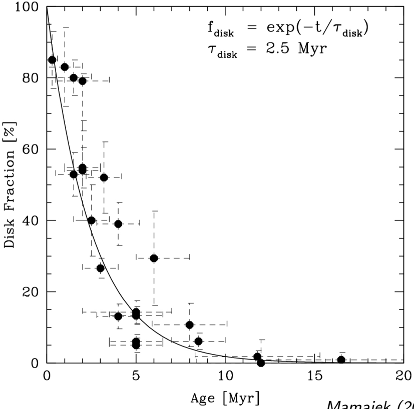
$\Rightarrow$ We need *lots* of gravitational focusing to form cores while the gas disc is still present

Life-times of protoplanetary discs

- Stars in same star-forming region are pretty much the same age
- Compare instead *disc fraction* between regions of different age



Haisch et al. (2001)



Mamajek (2009)

⇒ Protoplanetary discs live for 1–10 Myr

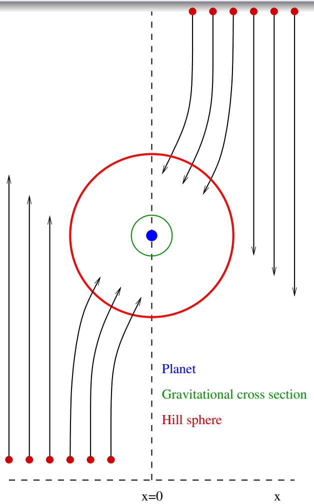
⇒ Need to form cores of gas giants in a few Myr

Safronov number

Gravitational cross section

$$\sigma = \pi b^2 = \pi R_{\mathrm{p}}^2 \left(1 + \frac{v_{\mathrm{esc}}^2}{v_\infty^2}\right) = \pi R_{\mathrm{p}}^2 (1 + 2\theta_{\mathrm{S}})$$

- Difference in Keplerian speed between planet at orbital radius r and planetesimals at $r + x$:
  $\delta v \approx -\Omega(r) x$
- Planetesimals enter the Hill sphere of the core with the *Hill speed*
  $v_{\mathrm{H}} \approx \Omega(r) R_{\mathrm{H}}$
- We can estimate θ_{S} by setting $v_\infty = v_{\mathrm{H}}$



Size of Hill sphere

- The size of the Hill sphere is

$$R_{\rm H} = \left(\frac{GM_{\rm p}}{3\Omega^2}\right)^{1/3}$$

- Use $M_{\rm p} = (4/3)\pi\rho_\bullet R_{\rm p}^3$

$$R_{\rm H} = \left(\frac{G(4/3)\pi\rho_\bullet}{3\Omega^2}\right)^{1/3} R_{\rm p}$$

- Use $\Omega^2 = GM_\star/r^3$ and $M_\star = (4/3)\pi\rho_\odot R_\odot^3$

$$R_{\rm H} = \left(\frac{G(4/3)\pi\rho_\bullet r^3}{3G(4/3)\pi\rho_\odot R_\odot^3}\right)^{1/3} R_{\rm p}$$

- This gives us the planet radius in relation to the Hill radius

$$R_{\rm p}/R_{\rm H} = \left(\frac{3\rho_\odot}{\rho_\bullet}\right)^{1/3} \frac{R_\odot}{r} \equiv \alpha$$

- The constant α only depends on the planet's density, the stellar mass and the orbital distance (Goldreich et al. 2004) – not on planet radius

$$\alpha = 0.006 \left(\frac{r}{\rm AU}\right)^{-1} \left(\frac{\rho_\bullet}{2\,{\rm g\,cm^{-3}}}\right)^{-1/3} \left(\frac{M_\star}{M_\odot}\right)^{1/3}$$

Gravitational radius versus Hill sphere

- In the limit $v_\infty \ll v_{\rm esc}$ we can write the gravitational radius b as

$$b = R_{\rm p} \frac{v_{\rm esc}}{v_{\rm H}}$$

- Insert escape speed $v_{\rm esc} \approx \sqrt{G M_{\rm p}/R_{\rm p}}$

$$b = R_{\rm p} \sqrt{\frac{G M_{\rm p}}{R_{\rm p}}} \frac{1}{v_{\rm H}} = \sqrt{R_{\rm p}} \sqrt{\frac{G M_{\rm p}}{v_{\rm H}^2}}$$

- Insert Hill speed $v_{\rm H} = R_{\rm H} \Omega$

$$b = \sqrt{R_{\rm p}} \sqrt{\frac{G M_{\rm p}}{R_{\rm H}^2 \Omega^2}}$$

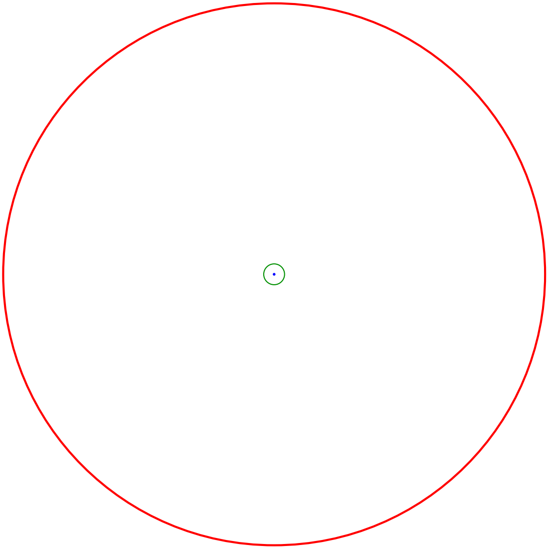
- Insert Hill radius $R_{\rm H}^3 = G M_{\rm p}/(3\Omega^2)$

$$b = \sqrt{R_{\rm p}} \sqrt{\frac{3 R_{\rm H}^3}{R_{\rm H}^2}} \approx \sqrt{R_{\rm p}} \sqrt{R_{\rm H}}$$

- Insert $R_{\rm p} = \alpha R_{\rm H}$ to finally get

$$b = \alpha^{1/2} R_{\rm H}$$

Relative size of Hill sphere, gravitational radius, and planet radius



Growth rate with gravitational focusing

- Gravitational cross section

$$\sigma = \pi b^2 = \pi R_{\mathrm{p}}^2 \left(1 + \frac{v_{\mathrm{esc}}^2}{v_\infty^2}\right) = \pi R^2 (1 + 2\theta_{\mathrm{S}})$$

- The gravitational focusing factor is

$$\mathcal{F} = 1 + 2\theta_{\mathrm{S}} = \frac{b^2}{R_{\mathrm{p}}^2} = \frac{\alpha R_{\mathrm{H}}^2}{\alpha^2 R_{\mathrm{H}}^2} = \alpha^{-1}$$

$\Rightarrow$ Gravitational focusing can speed up formation by three orders of magnitude or more

Core accretion time scales

- The size of the protoplanet relative to the Hill sphere:

$$\frac{R_\mathrm{P}}{R_\mathrm{H}} \equiv \alpha \approx 0.001 \left(\frac{r}{5\mathrm{AU}}\right)^{-1}$$

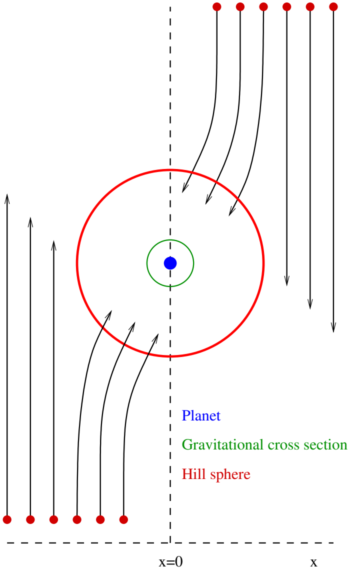
- Maximal growth rate by gravitational focussing

$$\dot{M} = \alpha R_\mathrm{H}^2 \mathcal{F}_\mathrm{H}$$

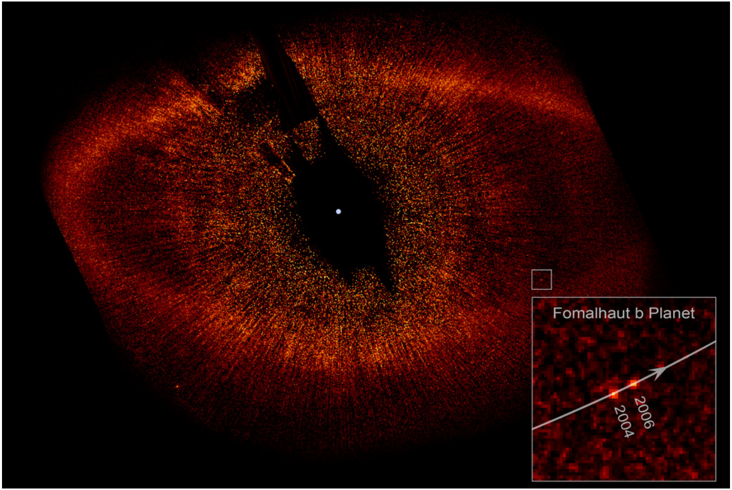
⇒ Only 0.1% (0.01%) of planetesimals entering the Hill sphere are accreted at 5 AU (50 AU)

⇒ Time to grow to 10 M_Earth is

$\sim$ 10 Myr at 5 AU
$\sim$ 50 Myr at 10 AU
$\sim$ 5000 Myr at 50 AU



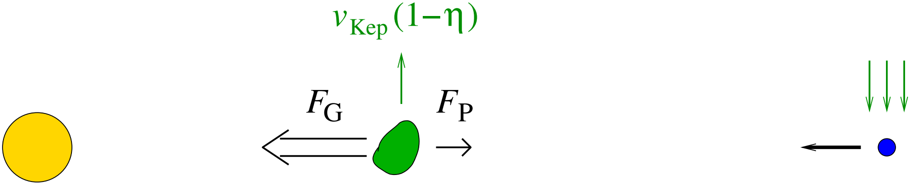
Fomalhaut system



- Planet orbiting Fomalhaut one of the first directly imaged (Kalas et al. 2008)
- Planet orbits at $\sim$115 AU and has mass $\sim 3 M_{\rm J}$

$\Rightarrow$ Do not even try to calculate the core accretion time-scale at 115 AU...

Remember radial drift

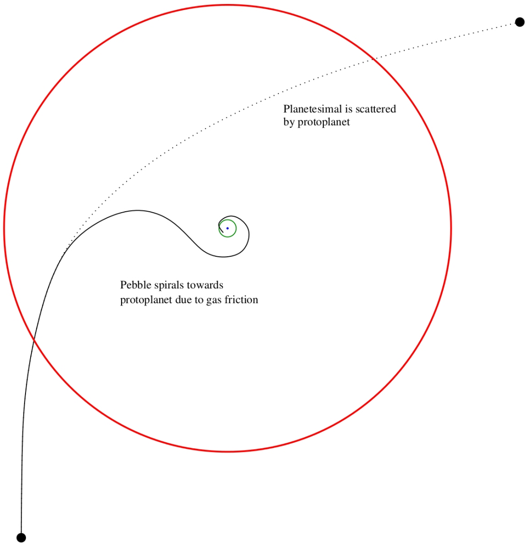


- Disc is hotter and denser close to the star: radial pressure gradient
- Radial pressure gradient force mimics decreased gravity
  $\Rightarrow$ gas orbits slower than Keplerian:

$$\eta = -\frac{1}{2}\left(\frac{H}{r}\right)^2 \frac{\partial \ln(P)}{\partial \ln(r)}$$

- Particles do not feel the pressure gradient force and want to orbit Keplerian
- Headwind from sub-Keplerian gas drains angular momentum from particles, so they spiral in through the disc

Pebble accretion



- Most planetesimals are simply scattered by the protoplanet
- Pebbles spiral in towards the protoplanet due to gas friction

⇒ Pebbles are accreted from the entire Hill sphere

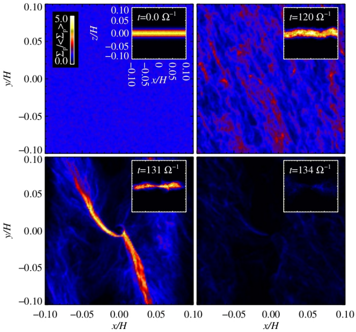
- Growth rate by planetesimal accretion is

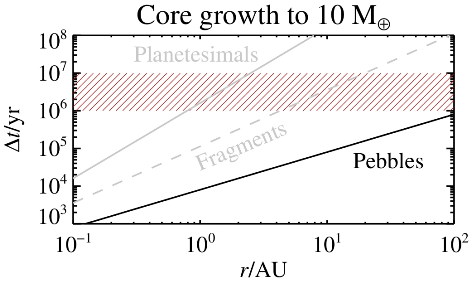
$$\dot{M} = \alpha R_{\rm H}^2 \mathcal{F}_{\rm H}$$

- Growth rate by pebble accretion is

$$\dot{M} = R_{\rm H}^2 \mathcal{F}_{\rm H}$$

Time scale of pebble accretion





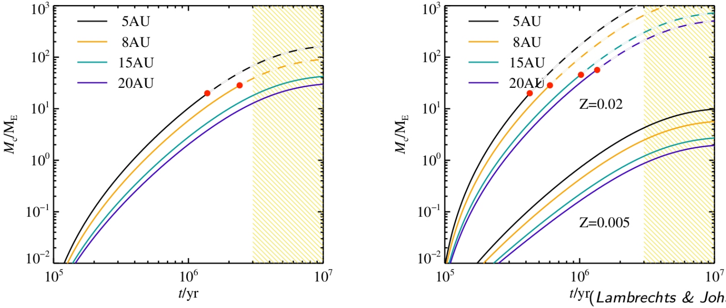
⇒ Pebble accretion speeds up core formation by a factor 1,000 at 5 AU and a factor 10,000 at 50 AU

(Ormel & Klahr, 2010; Lambrechts & Johansen, 2012; Morbidelli & Nesvorny, 2012)

⇒ Cores form well within the life-time of the protoplanetary gas disc, even at large orbital distances

- Requires large planetesimal seeds to accrete in Hill regime, consistent with planetesimal formation by gravitational collapse

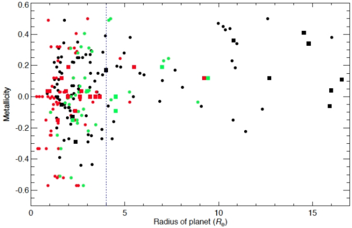
From planetesimals to planets



(*Lambrechts & Johansen*, 2014)

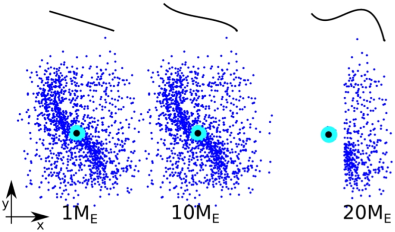
- The largest planetesimals accrete the remaining pebbles and grow to planets in the next 1-5 Myr
- Growth depends strongly on the amount of heavy elements in the protoplanetary disc ($Z = 0.01$ in the Sun's photosphere)

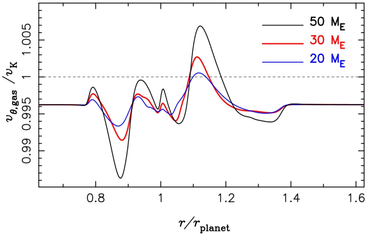
  (Lambrechts & Johansen, 2014)
- Gas-giant planets like Jupiter form if Z is high, in agreement with exoplanet surveys



(*Buchhave, Latham, Johansen, et al.*, 2012)

Halting pebble accretion

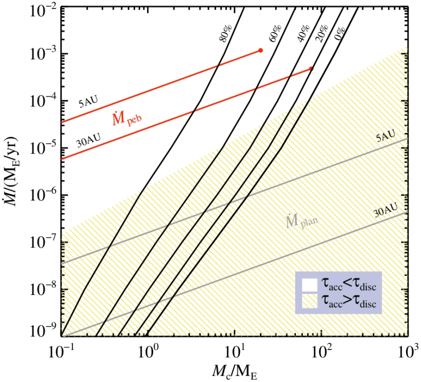


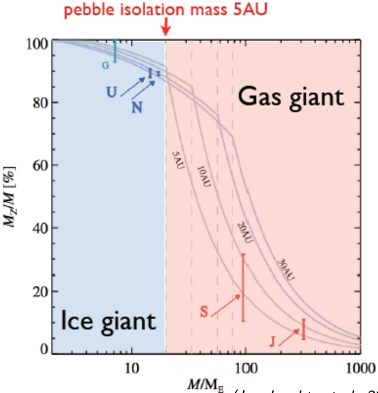


- Pebble accretion is stopped when the protoplanet grows massive enough to carve a gap in the pebble distribution
- Gap formation known for Jupiter-mass planets *(Paardekooper & Mellema, 2006)*
- *Lambrechts et al.* (2014) demonstrate that pebble accretion is stopped already at $20\mathrm{M}_\mathrm{Earth}$ at 5 AU, with isolation mass scaling as

$$M_\mathrm{iso} = 20 \left(\frac{H/r}{0.05}\right)^3 \mathrm{M}_\mathrm{Earth}$$

The critical core mass

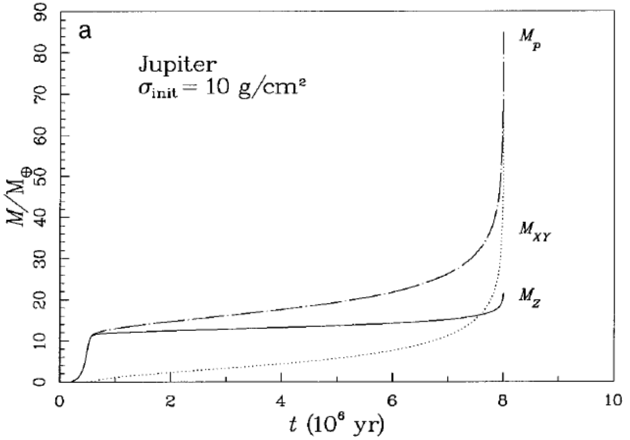




(*Lambrechts et al.*, 2014)

- Protoplanets grow at the pebble accretion rate until pebble accretion is halted *abruptly*
- The envelope is then supercritical and collapses onto the core
- Gives an excellent fit to Jupiter's and Saturn's heavy elements (Lambrechts et al., 2014)
- Gas giants in wide orbits must have large cores masses (50-100 ME)
- Explains dichotomy between ice giants and gas giants

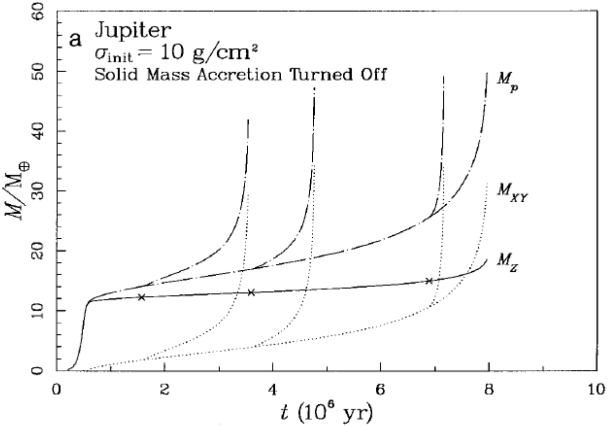
Core accretion models



(Pollack, Hubickyj, Bodenheimer, & Lissauer 1996)

- Phase 1: accretion of solid core until isolation
- Phase 2: hydrostatic atmosphere kept hot by remnant planetesimals
- Phase 3: rapid accretion of gas onto core

Heating of the envelope



- Phase 2 is frustrated by the heating due to infalling planetesimals
- Heat prevents atmosphere from cooling and contracting
- Phase 2 can be made arbitrarily shorter by turning off planetesimal accretion ($\times$ symbols in plot) or by just accreting pebbles!

Migration and detection of planets

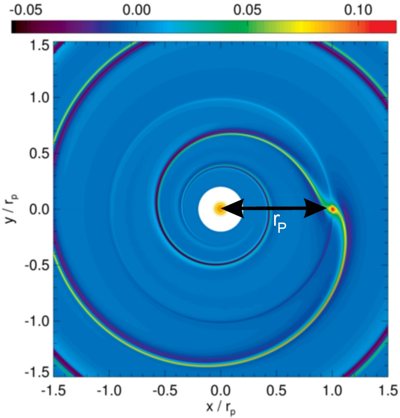
The wake

The gravity of the planet perturbs the fluid (gas).
Spiral pressure wave: the wake, corotating with the planet
Overdensity $\Rightarrow$ Force from the disc $\Rightarrow$ Torque: $\Gamma = \mathrm{d}J_\mathrm{P}/\mathrm{dt}$

where $J_\mathrm{P} = M_\mathrm{P}(G\mathrm{M}_\star r_\mathrm{P})^{1/2}$ (orbital angular momentum for a circular orbit, r_P = orbital radius)

$\Rightarrow \mathrm{d}r_\mathrm{P}/\mathrm{dt} \sim \Gamma/M_\mathrm{P}$

Positive torque: increase of r_P
Negative torque: decrease of r_P



(Picture: Baruteau et al. 2014, PPVI review)

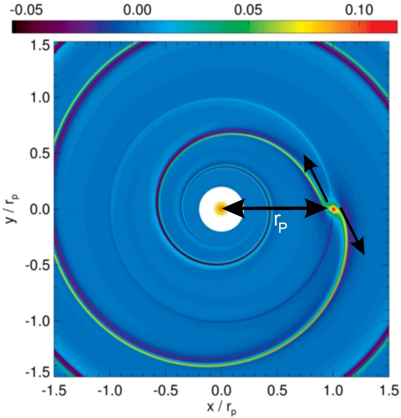
Type-I migration

Outer disc: wake trails behind the planet: negative torque

Inner disc: wave leads the planet: positive torque

The torque scales with $\boxed{\Gamma_0 = (q/h)^2 \Sigma_{\rm P} r_{\rm P}^4 \Omega_{\rm P}^2}$

(*e.g. Lin & Papaloizou, 1980; Goldreich & Tremaine, 1979*)



$q = M_{\rm P}/{\rm M}_\star$ planet/star mass ratio

$\Sigma_{\rm P}$= gas surface density

Ω= angular frequency

h= aspect ratio H/r

The torque due to the wave is in a 2D adiabatic disc:

$$\boxed{\gamma\Gamma_{\rm L}/\Gamma_0 = -2.5 - 1.7\beta_{\rm T} + 0.1\alpha_\Sigma}$$

where γ=adiabatic index,

$\Sigma \sim r^{-\alpha_\Sigma}$, $T \sim r^{-\beta_{\rm T}}$

(Paardekooper et al. 2010)

Type-I migra

Outer disc: wak

Inner disc: wav

The torque scal

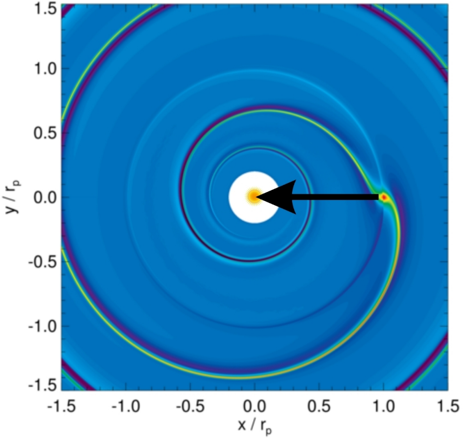
(*e.g. Lin & Papaloizou, 19*

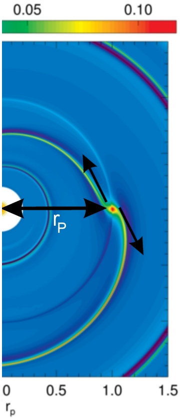
$q = M_\mathrm{P}/\mathrm{M}_\star$ plan

$\Sigma_\mathrm{P}=$ gas surface

$\Omega=$ angular frequ

$h=$ aspect ratio H





The torque due

2D adiabatic dis

migration time [years] ~1/q

In MMSN (@1AU):

→ 300000 yr for an Earth

→ 20000 yr for a Neptune

$\gamma\Gamma_\mathrm{L}/\Gamma_0 = -2.5$

where γ=adiaba

$\Sigma \sim r^{-\alpha_\Sigma}$, $T \sim$

(Paardekooper et al. 2010)

Corotation region

Around the planetary orbit, the gas corotates with the planet.
The streamlines of the velocity field have horseshoe shapes.

Picture: C. Baruteau

Corotation region

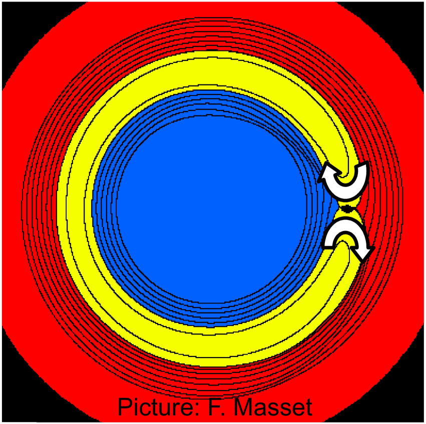
Around the planetary orbit, the gas corotates with the planet.
The streamlines of the velocity field have horseshoe shapes.

Corotation region: yellow

Interior to the planet: blue
$\Rightarrow$ hotter and denser!

Exterior to the planet: red
$\Rightarrow$ colder and less dense

The torque arising from this horseshoe region x_s is the corotation torque Γ_C, where $x_s = 1.16 r_P \sqrt{(q/h)}$



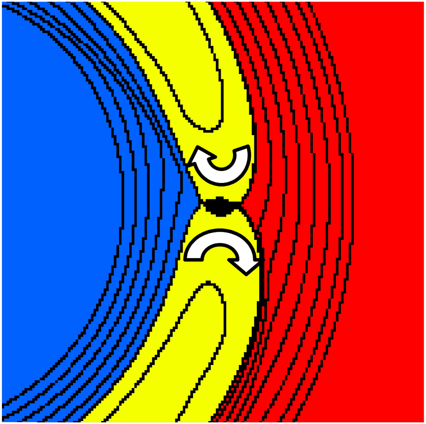
Corotation torque

$$\boxed{\gamma\Gamma_{\mathrm{C}}/\Gamma_0 = 1.1(3/2-\alpha_\Sigma)+7.9\xi/\gamma} \qquad \xi = \beta_{\mathrm{T}}-(\gamma-1)\alpha_\Sigma$$

(Paardekooper et al. 2010)

<u>1st term:</u> barotropic part

(Ward 1991, Masset 2001, Paardekooper & Papaloizou 2009)



Corotation torque

$$\gamma\Gamma_{\rm C}/\Gamma_0 = 1.1(3/2 - \alpha_\Sigma) + 7.9\xi/\gamma \qquad \xi = \beta_{\rm T} - (\gamma - 1)\alpha_\Sigma$$

(Paardekooper et al. 2010)

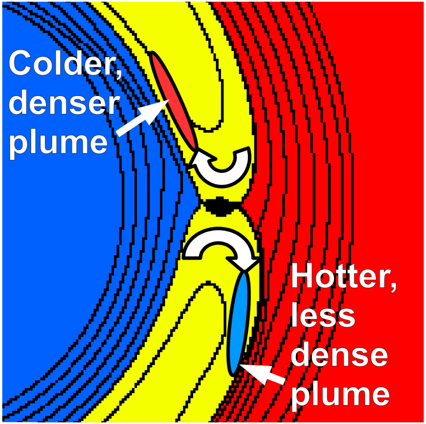
<u>1st term:</u> barotropic part

(Ward 1991, Masset 2001, Paardekooper & Papaloizou 2009)

<u>2nd term:</u> thermal part, due to the advection of the entropy :

$$\xi = -{\rm d}\log({\rm entropy})/{\rm d}\log(r)$$

(Paardekooper & Mellema 2008, Baruteau & Masset 2008)



Type-I migration

In summary, with $\gamma =$ adiabatic index, and $\Sigma \propto r^{-\alpha_\Sigma}$, $T \propto r^{-\beta_\mathrm{T}}$:

$$\boxed{\gamma\Gamma_\mathrm{L}/\Gamma_0 = -2.5 - 1.7\beta_\mathrm{T} + 0.1\alpha_\Sigma} \qquad \Gamma_0 = (q/h)^2 \Sigma_\mathrm{P} r_\mathrm{P}^4 \Omega_\mathrm{P}^2$$

$$\boxed{\gamma\Gamma_\mathrm{C}/\Gamma_0 = 1.1(3/2 - \alpha_\Sigma) + 7.9\xi/\gamma} \qquad \xi = \beta_\mathrm{T} - (\gamma - 1)\alpha_\Sigma$$

Total *unsaturated* torque (assuming $\gamma = 1.4$):

$$\boxed{(\Gamma_\mathrm{L} + \Gamma_\mathrm{C})/\Gamma_0 = -0.64 - 2.3\alpha_\Sigma + 2.8\beta_\mathrm{T}}$$

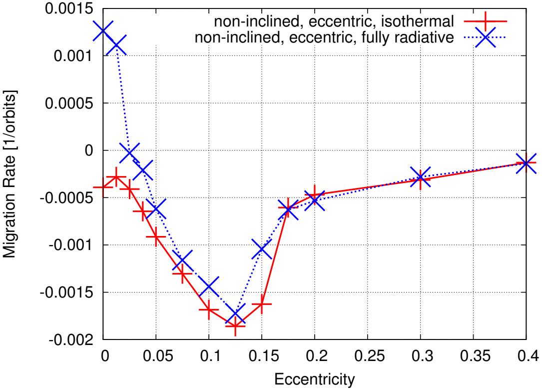
Can be positive or negative.

Depends on the local disc properties.

$\Rightarrow$ Torques can saturate!

Torque saturation: eccentricity

$$\gamma\Gamma_{\mathrm{C,circ,unsaturated}}/\Gamma_0 = 1.1(3/2 - \alpha_\Sigma) + 7.9\xi/\gamma$$



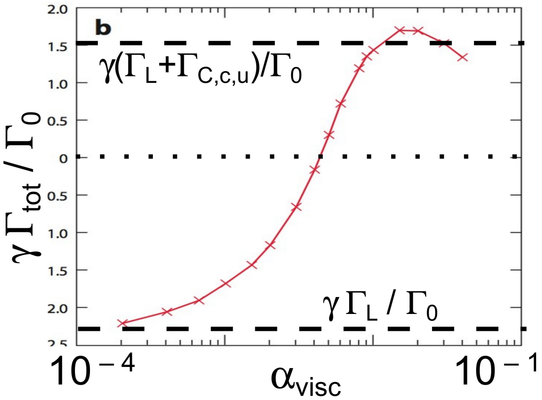
(*Bitsch & Kley 2010, Fendyke & Nelson 2014*)

⇒ for large e, $\Gamma_{\mathrm{C}} \to 0$, but the discs damps e for small planets!

⇒ important for N-body simulations (see next lecture)!

Torque saturation: viscosity

$$\gamma \Gamma_{\mathrm{C,circ,unsaturated}} / \Gamma_0 = 1.1(3/2 - \alpha_\Sigma) + 7.9\xi/\gamma$$

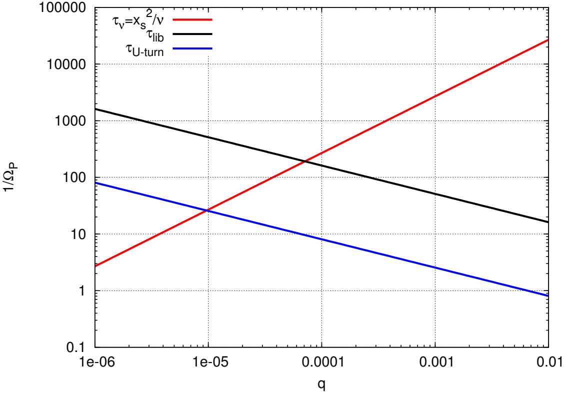


The horseshoe region only has a limited angular momentum to exchange!
Needs to be refreshed, through viscosity, otherwise $\Gamma_{\mathrm{C}} \to 0$!

(see Masset & Casoli 2010, Paardekooper et al. 2011, Fig: Kley & Nelson 2012, ARAA, 52, 211)

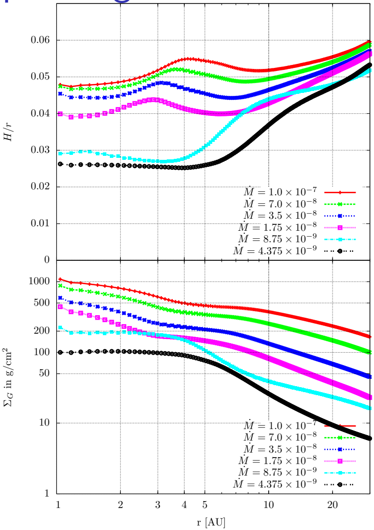
Saturation: planetary mass

- libration time-scale: $\tau_{\rm lib} = 8\pi r_{\rm P}/(3\Omega_{\rm P} x_{\rm s})$, with $x_{\rm s} = 1.16 r_{\rm P}\sqrt{(q/h)}$
- U-turn time-scale: $\tau_{\rm U-turn} = h \times \tau_{\rm lib}$
- viscous time-scale: $\tau_\nu = x_{\rm s}^2/\nu$

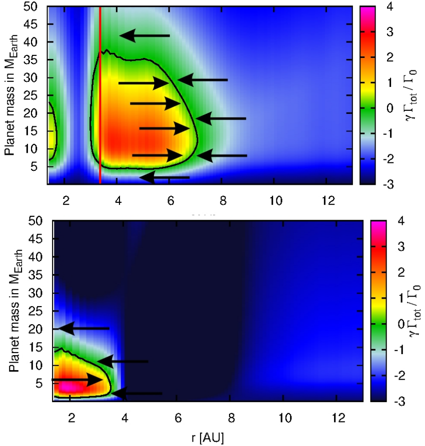


$\Rightarrow$ Outward migration only for $\boxed{\tau_{\rm U-turn} < \tau_\nu < \tau_{\rm lib}}$!

Type-I migration in evolving discs



$\dot M = 3.5 \times 10^{-8} M_\odot/\mathrm{yr}$

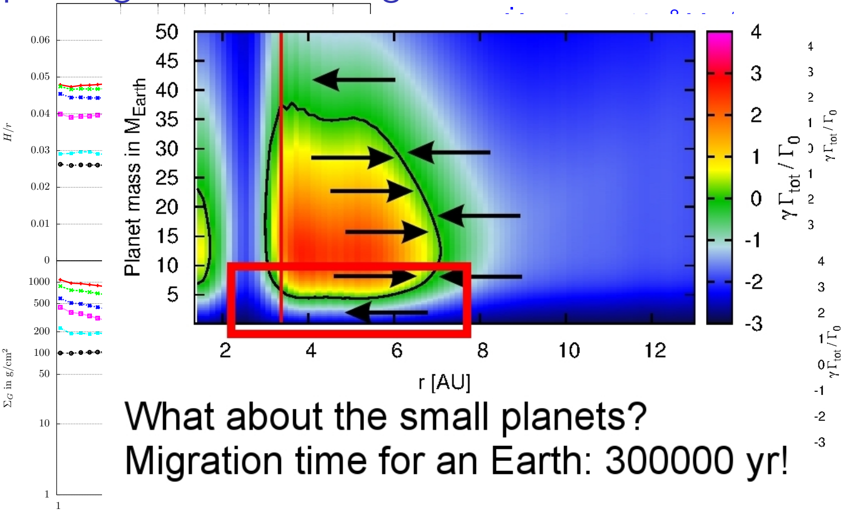


$\dot M = 8.75 \times 10^{-9} M_\odot/\mathrm{yr}$

Outward migration in regions where H/r decreases with increasing r!

(*Bitsch et al., 2015a*, torque formula from *Paardekooper et al. 2011*)

Type-I migration in evolving discs

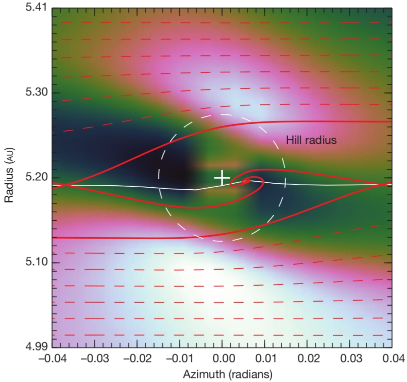


Outward migration in regions where H/r decreases with increasing r!

(*Bitsch et al., 2015a*, torque formula from *Paardekooper et al. 2011*)

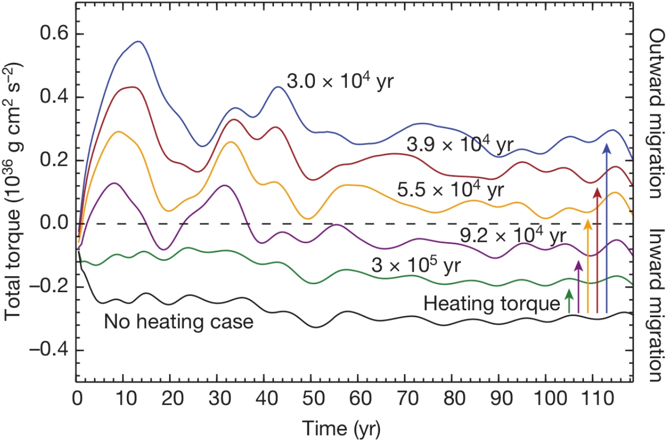
Migration of accreting planets

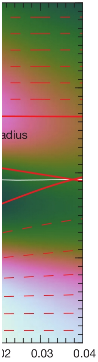
- Accreting planet releases heat
- Pressure equilibrium:
  hotter regions are less dense
- Sub-Keplerian disc:
  material behind planet is closer:
  hotter $\Rightarrow$ less dense
- Asymmetry in torque:
  $\Rightarrow$ Outward migration



(*Benitez-Llambay et al. 2015*)

Migration of accreting planets





mbay et al. 2015)

⇒ Heating depends on the accretion rate!

⇒ Therefore the heating torque depends on the accretion rate!

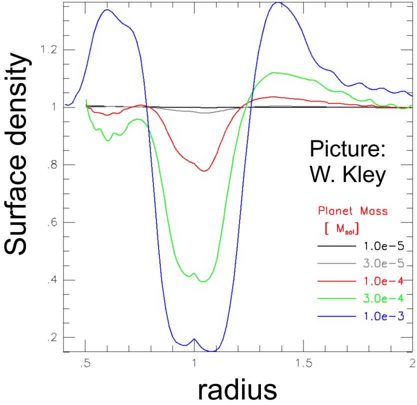
Gap opening

The planet exerts also
a negative torque on the inner disc $\rightarrow$ promotes its accretion,
a positive torque on the outer disc $\rightarrow$ repels it away from the star.

Competition with pressure and viscosity, who tend to smooth the gas profile. A gap opens if :

$$\boxed{P = h/q^{1/3} + 50\alpha_\nu/qh^2 \leq 1}$$

(*Crida et al., 2006*)
$h = 0.05,\ \alpha_\nu = 0.004$
$\Rightarrow$ gap if $q > 10^{-3}$



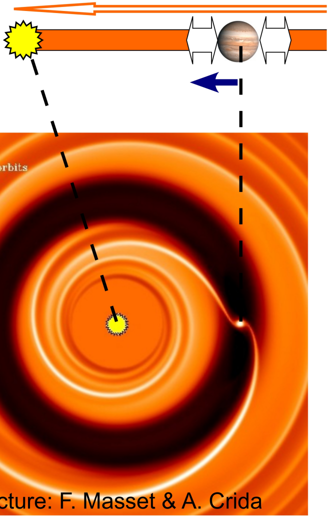
Type II migration in 2D disc

Locked inside its gap, the planet must follow the disc accretion onto the star.

Migration with viscous time-scale, $\tau_\nu = r_\mathrm{P}^2/\nu$ as long as the planet is not too massive, otherwise it slows down the disc. Two regimes:

- Disc dominated:
  $\tau_{II} = \tau_\nu$
- Planet dominated:
  $\tau_{II} = \tau_\nu \times M_\mathrm{P}/M_\mathrm{disc}$

(*Crida & Morbidelli, 2007*)



Two massive planets in resonance: outward migration

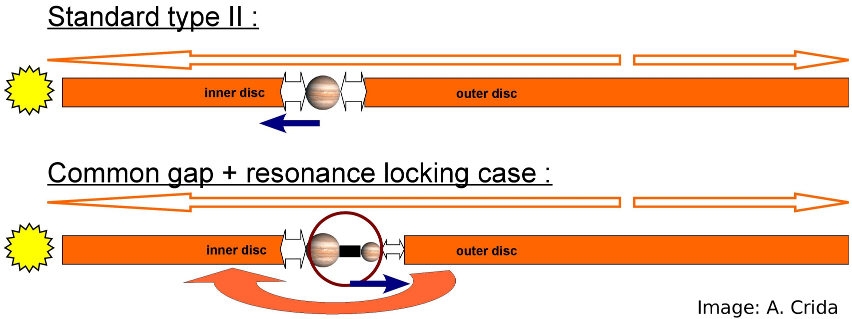
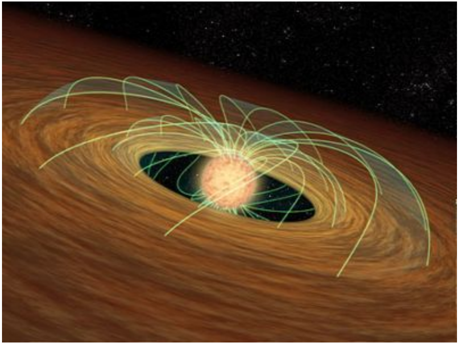


Image: A. Crida

- $M_{out} < M_{in}$ a smaller negative torque acts from the outer disc than a positive torque from the inner disc (*Masset & Snellgrove, 2001*)
- Works best if mass ratio is between $1/4$ and $1/2$ (*Morbidelli & Crida, 2007*)
- Application to the Solar System: Grand Tack scenario (*Walsh et al. 2011*)

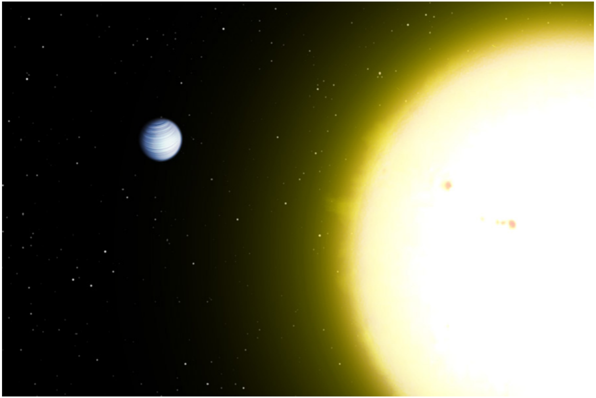
Where does Type II migration end?

- Planets migrate at same time-scale as viscously accreted gas
- Gas disc truncated by the magnetic field of the star at orbital distances of a few stellar radii
- Gas is accreted along magnetic field lines



$\Rightarrow$ Type II migration stops as planet enters the inner cavity

Type II migration and exoplanets

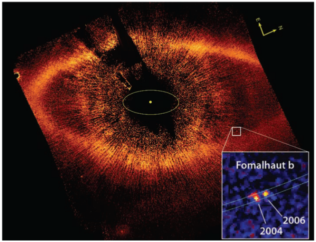


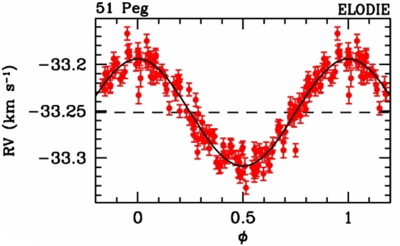
- First planet ($M \approx 0.5 M_{\rm J}$) around solar-type star discovered in 1995 (Mayor & Queloz 1995)
- Orbits only 0.05 AU from the star 51 Peg
- Must have formed further from the star and migrated in to its current position (Lin et al. 1996)

Direct and indirect detection of exoplanets

1. Direct method: collect light from the planet (intrinsic or reflected)

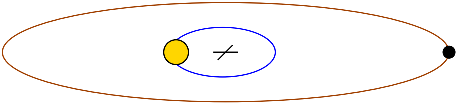
2. Indirect method: infer presence of planet from its effect on host star

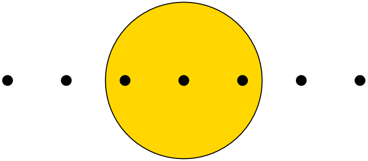




Indirect methods

The two most common indirect methods are:





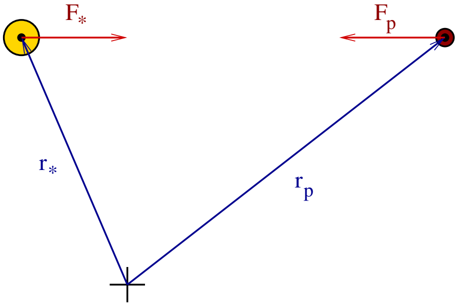
1 Star's motion due to gravitational tug by unseen planet

⇒ Measuring the radial velocity yields the orbital distance of the planet and the minimum mass

2 Star's luminosity change due to transiting planet blocking out part of the radiation

⇒ The transit depth gives the radius of the planet

Kepler problem



- Star and planet interact only by gravity

$$\mathbf{F}_\star = m_\star \ddot{\mathbf{r}}_\star = \frac{G m_\star m_\mathrm{p}}{r_{\star\mathrm{p}}^2} \frac{\mathbf{r}_{\star\mathrm{p}}}{r_{\star\mathrm{p}}}$$

$$\mathbf{F}_\mathrm{p} = m_\mathrm{p} \ddot{\mathbf{r}}_\mathrm{p} = \frac{G m_\star m_\mathrm{p}}{r_{\mathrm{p}\star}^2} \frac{\mathbf{r}_{\mathrm{p}\star}}{r_{\mathrm{p}\star}}$$

- $\mathbf{r}_{\star\mathrm{p}} = \mathbf{r}_\mathrm{p} - \mathbf{r}_\star \quad , \quad \mathbf{r}_{\mathrm{p}\star} = \mathbf{r}_\star - \mathbf{r}_\mathrm{p}$

$\Rightarrow$ *Kepler problem*

Centre-of-mass in the solar system

- Centre-of-mass between Sun and planets with semi-major axis a

$$r_\star \approx \frac{m_\mathrm{p}}{m_\star} a$$

Planet	CM with Sun [km]	CM with Sun [$R_\odot$]
Mercury	9.6 km	1.4×10^{-5}
Venus	265 km	3.8×10^{-4}
Earth	449 km	6.4×10^{-4}
Mars	74 km	1.1×10^{-4}
Jupiter	743,000 km	1.07
Saturn	408,000 km	0.59
Uranus	125,000 km	0.18
Neptune	232,000 km	0.33

Stellar velocity

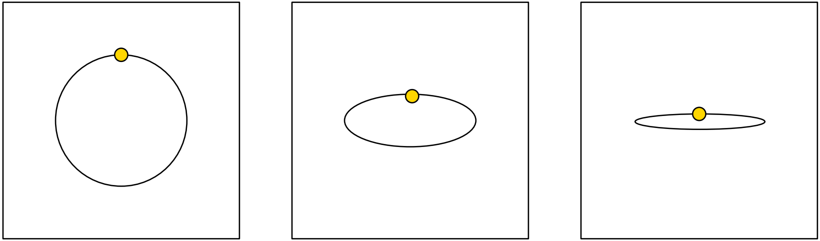
- The star's speed around the centre of mass is

$$v_\star = \frac{2\pi r_\star}{P}$$

- The centre of mass of the system is located a distance $r_\star = (m_\mathrm{p}/m_\star)a$ from the centre of the star

Planet	Speed of Sun
Mercury	0.008 m/s
Venus	0.09 m/s
Earth	0.09 m/s
Mars	0.008 m/s
Jupiter	12.5 m/s
Saturn	2.8 m/s
Uranus	0.3 m/s
Neptune	0.3 m/s

Radial velocity



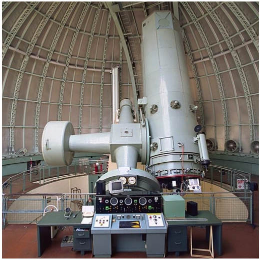
- The angle i measures the angle between the plane of the sky and the orbit of the star
  - $i = 0^\circ$: orbit seen *face on*
  - $i = 90^\circ$: orbit seen *edge on*

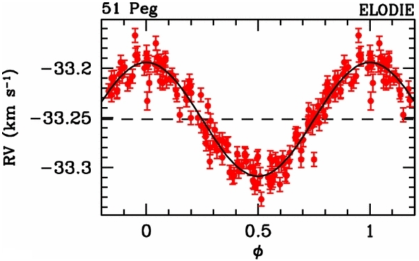
- The *radial velocity* of the star with respect to the observer is reduced to

$$v_r = v_\star \sin(i)$$

$\Rightarrow$ We can only determine the minimum mass of the plant, $M \sin(i)$

51 Peg b





- Mayor & Queloz (1995):
  "A Jupiter-mass companion to a solar-type star"
- 51 Peg's radial velocity has period 4.23 days and amplitude 59 m/s
- The inferred planet has an orbital distance of 0.053 AU and a minimum mass of 0.428 $M_{\rm Jup}$
- Comparison of the projected rotation speed of 51 Peg to magnetic activity sets an upper mass of 1.2 $M_{\rm Jup}$

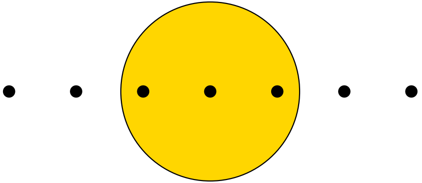
First planets

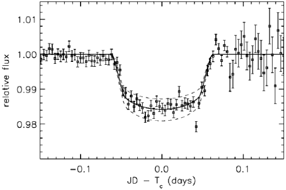
- Exoplanets named after their host star with an additional lower case letter (b for first discovered, c for second, etc.)

Planet	$m_{\rm p}$ [$M_{\rm Jup}$]	a [AU]	Reference
51 Peg b	0.468	0.052	Mayor & Queloz (1995)
70 Vir b	7.44	0.44	Marcy & Butler (1996)
47 Uma b	2.53	2.1	Butler & Marcy (1996)
55 Cnc b	0.824	0.115	Butler, Marcy, et al. (1997)
tau Boo b	3.9	0.046	Butler, Marcy, et al. (1997)
ups And b	0.69	0.059	Butler, Marcy, et al. (1997)
16 Cyg B b	1.68	1.68	Cochran et al. (1997)

$\Rightarrow$ New class of planet: *Hot Jupiters*

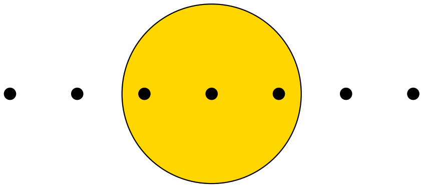
Transits





- Radial velocity gives no information about the physical properties of the planet and only yields the minimum mass (although stellar rotation and astrometric measurements can give good upper limits to the mass)
- Planetary transits give the radius of the planet $R_{\rm p}$
- Combined with mass from radial velocity we obtain planet's mass density $\rho_{\rm p} = m_{\rm p}/[(4\pi/3)R_{\rm p}^3]$

Planet radius

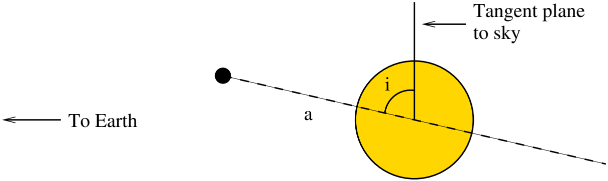


- The planet blocks out the stellar light fraction

$$\frac{\Delta L}{L} = \frac{\pi R_{\mathrm{p}}^2}{\pi R_\star^2} = \left(\frac{R_{\mathrm{p}}}{R_\star}\right)^2$$

- Jupiter's radius is roughly 10% of the solar radius and would give a transit depth of 1%
- 1% precision easily obtainable with ground-based photometry

Transit probability



- A planet transits under favourable conditions where the inclination is close to 90° (edge on)
- We require

$$a \cos i < R_\star + R_\mathrm{p} \quad \Rightarrow \quad \cos i < \frac{R_\star + R_\mathrm{p}}{a}$$

- The probability for such an alignment is

$$P_\mathrm{geom} = \frac{\int_0^{2\pi} \int_{\cos^{-1}(R_\star + R_\mathrm{p})/a}^{\pi/2} \sin i \, \mathrm{d}i \mathrm{d}\theta}{2\pi} = \frac{R_\star + R_\mathrm{p}}{a} \approx \frac{R_\star}{a}$$

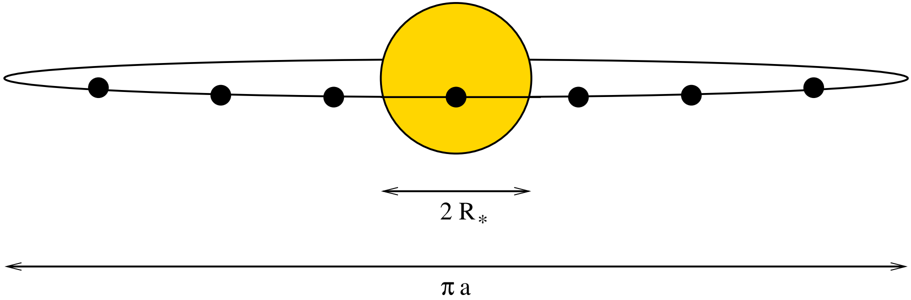
Transit probabilities

- The probability to be aligned right to see a transit is

$$P_{\rm geom} = \frac{R_\star}{a} = 0.46\% \left(\frac{R_\star}{R_\odot}\right) \left(\frac{a}{\rm AU}\right)^{-1}$$

Planet	$P_{\rm geom}$
Mercury	1.15%
Venus	0.62%
Earth	0.45%
Mars	0.29%
Jupiter	0.094%
Saturn	0.051%
Uranus	0.024%
Neptune	0.015%
51 Peg b	9.95%

Transit duration



- The maximum time that a planet spends in transit is the half orbital time-scale divided by the ratio of half orbit to stellar diameter

$$t_{\rm transit} = \frac{P}{2}\frac{2R_\star}{\pi a} = \frac{PR_\star}{\pi a}$$

Transit durations

- Duration of exactly edge-on transit

$$t_{\rm transit} = \frac{PR_\star}{\pi a} = 0.54\,{\rm d}\left(\frac{a}{\rm AU}\right)^{1/2}\left(\frac{R_\star}{R_\odot}\right)$$

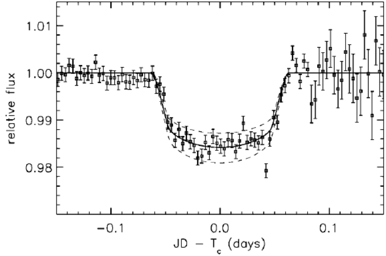
- Relative to the orbital period

$$t_{\rm transit}/P = \frac{R_\star}{\pi a} = 0.0015\left(\frac{a}{\rm AU}\right)^{-1}\left(\frac{R_\star}{R_\odot}\right)$$

Planet	$t_{\rm transit}$	$t_{\rm transit}/P$
Mercury	0.34 d	0.0038
Venus	0.46 d	0.0021
Earth	0.54 d	0.0015
Mars	0.67 d	0.00099
Jupiter	1.23 d	0.00029
Saturn	1.67 d	0.00016
Uranus	2.36 d	0.000078
Neptune	2.96 d	0.000050
51 Peg b	0.12 d	0.028

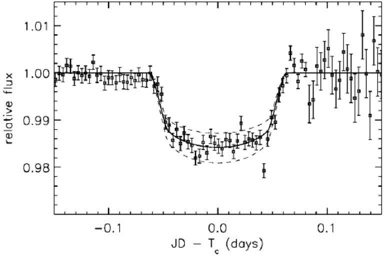
$\Rightarrow$ Close-in planets have short transits, but the transit is a big fraction of the orbit, so they are more probable to catch while transiting

HD 209458 b



- 51 Peg b does not transit, but HD 209458 b does (Charbonneau et al. 2000)
- Radial velocity signal – independently discovered from Keck (HIRES spectrograph) and Observatoire de Haute Provence (ELODIE spectrograph) – gives minimum mass $m_{\rm p} \sin i = 0.64 M_{\rm Jup}$
- Transit period of 3.52 d gives semi-major axis of 0.047 AU – in good agreement with radial velocity measurements
- Expected transit probability of ≈10% and transit duration ≈0.1 d
- Transit depth of 1.6% consistent with radius like Jupiter

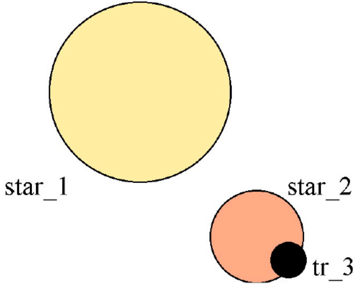
HD 209458 b



- Transit depth of 1.6% gives $R_\mathrm{p} = 1.38 R_\mathrm{Jup}$
- Inclination of 86.7° gives $m_\mathrm{p} = 0.64 M_\mathrm{Jup}$
- Mass density $\rho = 0.37\,\mathrm{g\,cm^{-3}}$, less than Jupiter ($\rho_\mathrm{Jup} = 1.326\,\mathrm{g\,cm^{-3}}$)

⇒ *First proof that Jupiter-mass exoplanets are gas giants*

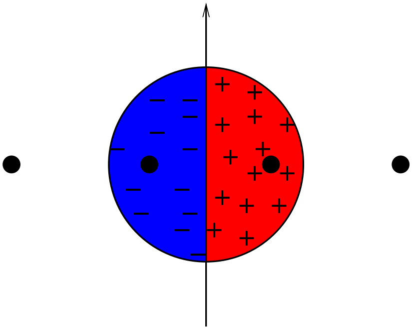
The problem with transits



- Hot Jupiters have approximately 10% probability of transiting
- Around 1% of all solar-like stars have hot Jupiters, so we expect to see a transit after observing 1000 stars
- But there are many *false positives*, such as *blending* with a faint background binary star

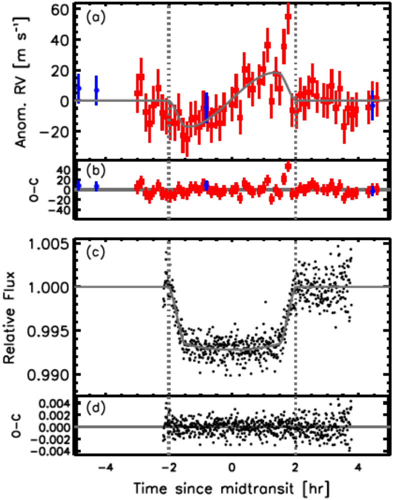
⇒ Transiting planets are not considered confirmed before radial velocity follow-up

Rossiter-McLaughlin effect



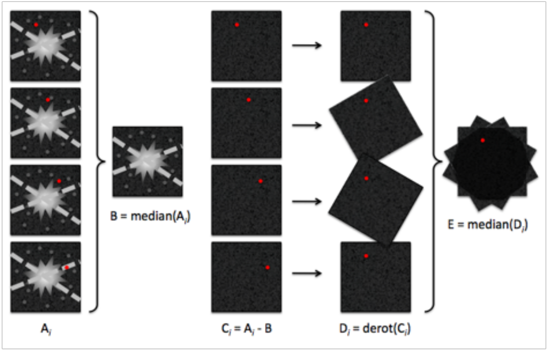
- A transiting planet blocks the red-shifted or blue-shifted radiation from the star
- A *prograde* planet blocks blue-shifted light first and then red-shifted
- A *retrograde* planet blocks red-shifted light first and then blue-shifted
- All planets in the solar system have prograde orbits

HAT-P-7b – a retrograde planet



- Winn et al. (2009) found Rossiter-McLaughlin effect on HAT-P-7
- Blueshift followed by redshift indicates retrograde orbit

Angular Differential Imaging



(Image by C. Thalmann)

Angular Differential Imaging method: *(Marois et al. 2006)*

A Take consecutive images of star and planet rotating with the sky

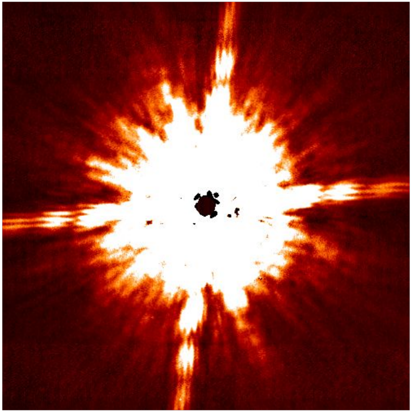
B Take the mean to find average image of target star

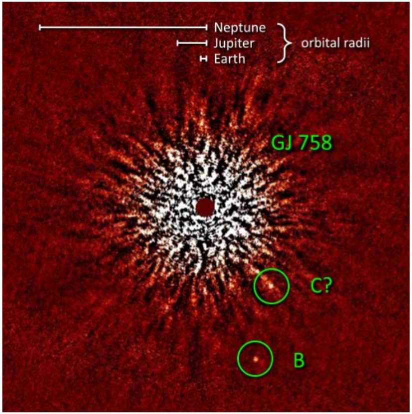
C Subtract mean image from individual frames

D Rotate resulting images to same alignment

E Average rotated images to obtain high contrast image of planet

Example of Angular Differential Imaging

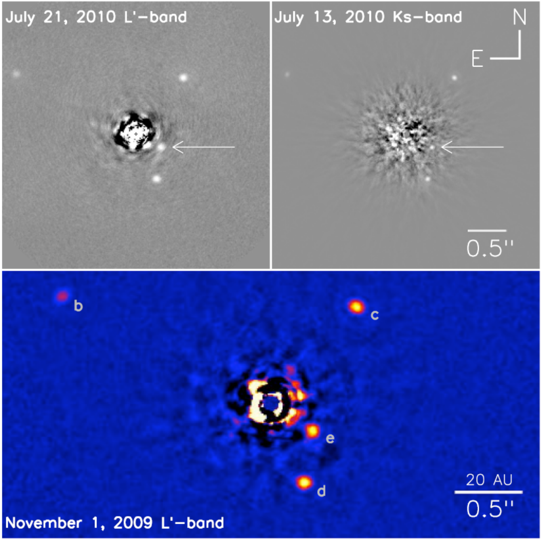




(Thalmann et al. 2009)

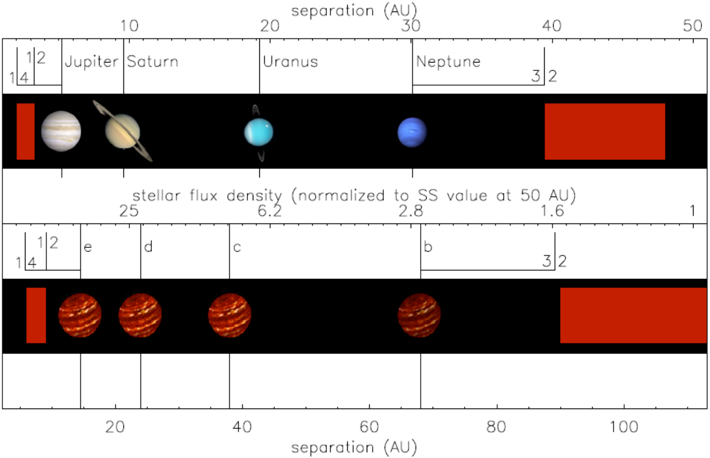
- Thalmann et al. (2009) observed the young star GJ 758 (G9) with Angular Differential Imaging
- Found 10–40 $M_{\rm Jup}$ companion with semimajor axis 54.5 AU

HR8799 planetary system



(Marois et al. 2010)

Comparison to solar system

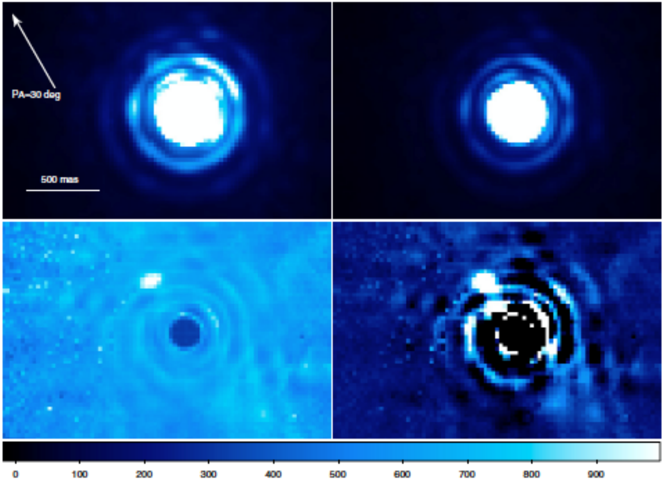


- Effective temperature of planet irradiated by star (lectures 1 and 4):

$$T_{\rm eff} = \left[\frac{(1-A)F_\star}{4\sigma_{\rm SB}}\right]^{1/4} = 280\,{\rm K}\,(1-A)^{1/4}\left(\frac{r}{\rm AU}\right)^{-1/2}\left(\frac{L_\star}{L_\odot}\right)^{1/4}$$

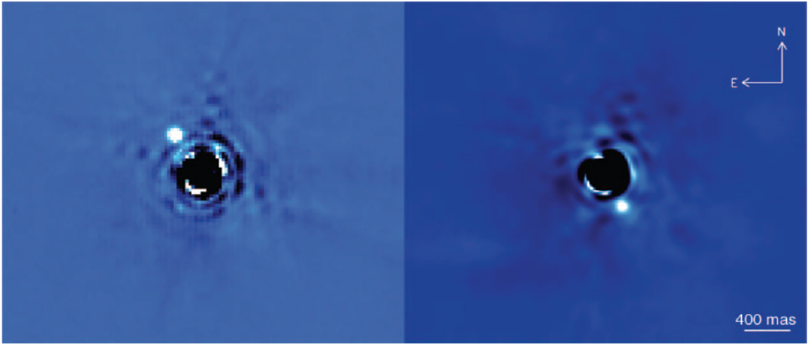
- HR 8799 has $L_\star/L_\odot = 4.92$, so constant temperature implies $r_{\rm HR8799} = 2.2 r_\odot$

Beta Pictoris



- Beta Pictoris: A6V, $M = 1.75 M_{\odot}$, $\tau = 12^{+8}_{-4}$ Myr, $d = 19.4$ pc
- Lagrange et al. (2009) found companion at 8 AU orbital separation in data from 2003 (by subtracting image of an A star with no planet)

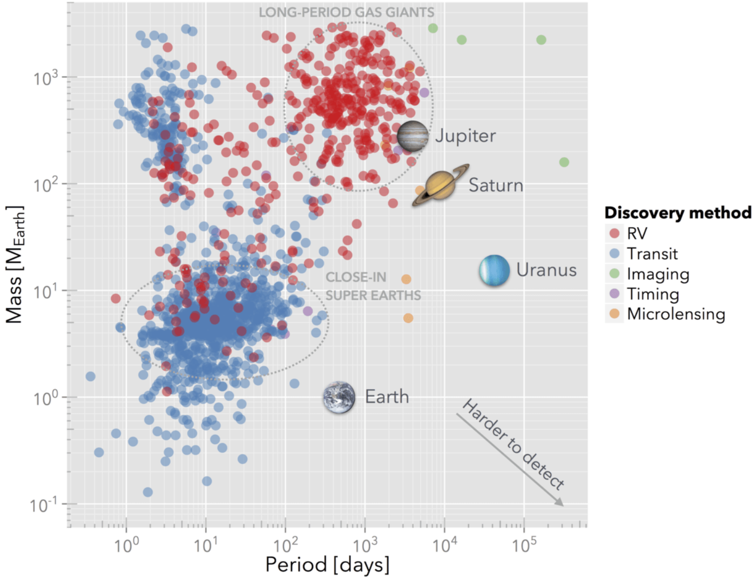
Beta Pictoris planet confirmed



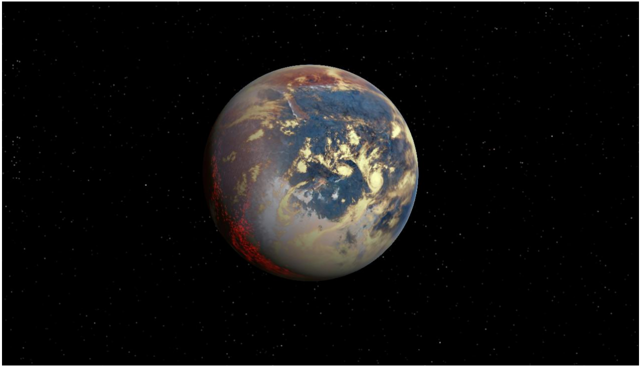
2003 2009

- Lagrange et al. (2010) recovered the β Pictoris planet at the other side of the star
- Semi-major axis between 8 and 13 AU
- Mass of $9 \pm 3\ M_{\rm Jup}$

Observations of exoplanets

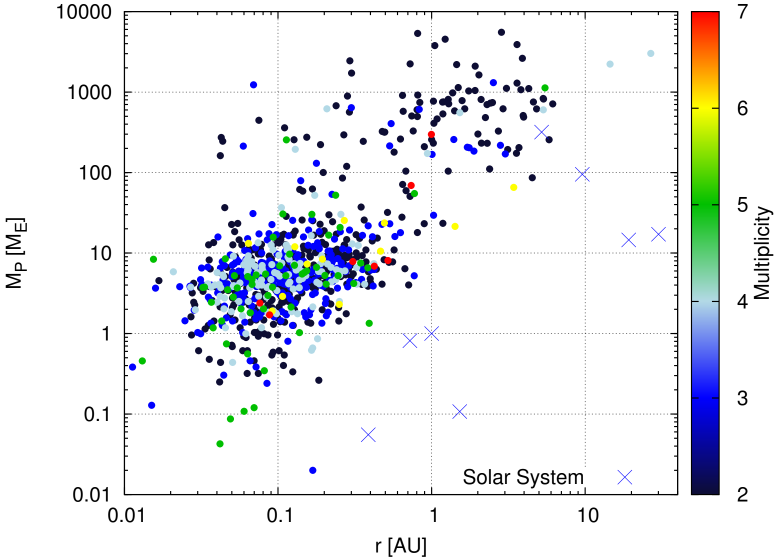


Definition of super-Earth



- Rocky planet with little or no H/He atmosphere
- Mass range 2–10 Earth masses
- Class of planet not present in the solar system
- *Super-Earths are interesting to study because undoubtedly super-Earths will be discovered and characterised in detail before lower-mass terrestrial planets*

Multiplicity



(created on exoplanets.org)

Summary

- Accretion of massive planetary cores is too slow with just planetesimals - but pebble accretion can significantly accelerate this process
- Planetary cores that are *not* bombarded by planetesimals any more can accrete a gaseous envelope
- Planets embedded in protoplanetary discs interact with those and move through them
- Observations detected two new classes of planets that do not exist in our solar system:
  - Hot Jupiters
  - Super Earths
- Exoplanet systems with mainly small planets are most often in multiple systems

$\Rightarrow$ A successful planet formation theory has to combine all those things!